Astronomy 3602: Homework #6

Due Monday, December 13

Problem 1: Horizon Size at Radiation-Matter Equality (25 points).

Show that the Hubble distance at the time of radiation-matter equality is

$$\frac{c}{H(t_{\rm rm})} = \frac{c}{\sqrt{2}H_0}\frac{\Omega_{\rm r,0}^{3/2}}{\Omega_{\rm m,0}^{2}}. \quad (1)$$

What is the value of this Hubble distance in Mpc? In comoving Mpc ? Similarly, what was the Hubble distance at the time of matter - dark energy equality in the fiducial 'Benchmark' model? What is its value in comoving Mpc? How much mass is contained in the Hubble volume at matter - dark energy equality?

Problem 2: Equation of Motion for the Inflaton (25 points).

The expressions for the energy density and pressure of a scalar field ϕ are (Ryden, Chapter 10):

$$\epsilon_\phi = \frac{1}{2}\frac{1}{\hbar c^3}\dot{\phi}^2 + V(\phi) \quad (2)$$

$$p_\phi = \frac{1}{2}\frac{1}{\hbar c^3}\dot{\phi}^2 - V(\phi) \quad (3)$$

By subsituting these expressions into the fluid equation ($\dot{\epsilon}+3H(t)[\epsilon+p] = 0$), show that you obtain the "equation of motion", analogous to a ball rolling down a hill, with friction proportional to its speed,

$$\ddot{\phi} + 3H(t)\dot{\phi} = -\hbar c^3\frac{dV}{d\phi}. \quad (4)$$

Problem 3: Update on Inflation (50 points).

Current measurements of the curvature of space from the cosmic microwave background anisotropies indicate $\Omega_0 = 1.00 \pm 0.005$, where $\Omega_0 = \Omega_{\Lambda,0} + \Omega_{m,0} + \Omega_{r,0}$ is the present day energy density in units of the critical density (from the *Planck* satellite). The remarkable closeness of Ω_0 to unity can be explained by an inflationary epoch in the early universe.

Suppose that a future satellite makes a 10 times more accurate measurement and finds $\Omega_0 = 1.00 \pm 0.0005$.

(a) Show that there is a 'fine-tuning problem' in the standard cosmological model, without inflation. In particular, if $\Omega_{\Lambda,0} = 0.691416$, $\Omega_{m,0} = 0.308$, and $\Omega_{r,0} = 8.4 \times 10^{-5}$, which is a slightly negatively curved universe, but still within the range allowed by *Planck*'s measurement, then what was the value of the energy density, $\Omega(t) = \Omega_\Lambda + \Omega_m + \Omega_r$, at the following epochs: (i) when the energy densities of matter and the cosmological constant were equal? (ii) when the energy densities of matter and radiation were equal? (iii) at the Planck time ($t_P = 5 \times 10^{-44}$ seconds, when the scale factor, without inflation, would be $a_P = 2 \times 10^{-32}$) ?

(b) How many e-foldings of inflation would be needed to solve the fine-tuning problem? Assume that the inflationary epoch starts at some initial time t_i during the radiation-dominated epoch, and that $|1-\Omega(t)| \sim 1$ before inflation. Assume further that inflation lasts for N Hubble times, so that it ends at $t_f = (N+1)t_i$, with $t_f = 4 \times 10^{-36}$ seconds. Compare your answer to $N = 60$ mentioned on page 187 of Ryden. Would the 10-fold increase in precision from $|1 - \Omega_0| \leq 0.005$ to $|1 - \Omega_0| \leq 0.0005$ necessitate many more e-foldings?

(c) Since the present-day energy density is dominated by a cosmological constant, the Universe is also 'inflating' at the present time. What will be the value of the scale factor, $a(t)$ when the age of the universe is 137 Gyr, or 10 times its current value (in estimating this value, ignore the contribution from matter, and assume a flat universe with cosmological constant only). What will be the value of the energy density, $\Omega(t)$ at this epoch ? Does the fine-tuning problem become better or worse?