Astronomy GR6001: Problem Set #5

Due in my office by Monday, November 22, 2021

Problem 1 (25 points):

At what column density does the curve of growth begin to become nonlinear?

(a) First, express the equivalent width (EW) of Lyα absorption (in Å) as a function of neutral hydrogen column density $N_{\rm HI}$ in the range in which the curve of growth is linear ($\tau_0 \ll 1$).

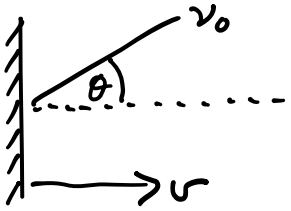
(b) Then, find the value of $N_{\rm HI}$ (in cm^{-2}) corresponding to line-center optical depth $\tau_0 = 0.1$ in the linear part of the curve of growth if $b = 6$ km s^{-1}.

(c) If this b is a thermal Doppler width, what is the temperature?

(d) What are $N_{\rm HI}$ and the EW (in Å) of Lyα absorption when $\tau_0 = 100$ and $b = 6$km s^1 ?

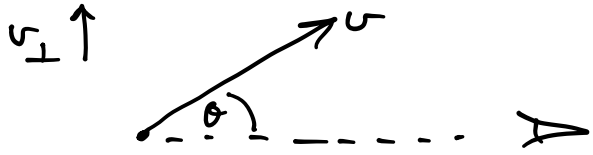
Problem 2 (25 points):

In the frame of an observer, a mirror moves perpendicular to its plane with velocity v. If light of frequency ν_0 is incident at the angle θ from the normal, at what angle will the observer see it reflected? What will be the frequency of the reflected ray? (Hint: one approach is to Lorentz-transform the wave/momentum four-vector of the light to and from the mirror's frame before and after the reflection.)



Problem 3 (25 points):

(a) Derive the expression for the apparent superluminal velocity in the transverse direction of the sky (as seen for some radio jets),



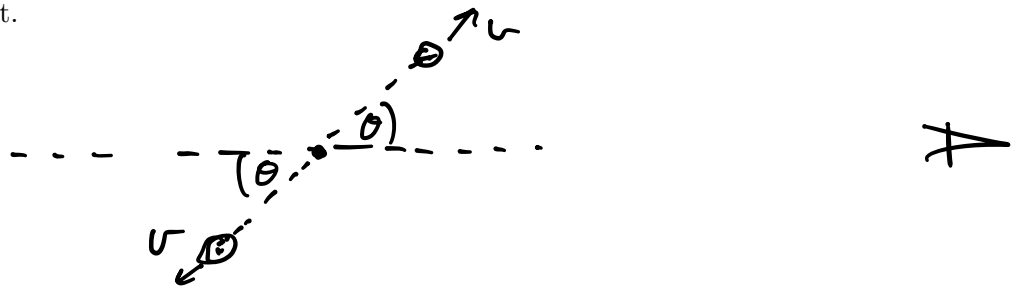
$$v_\perp = \frac{v \sin\theta)}{1 - \beta \cos\theta}, \tag{1}$$

where v is the (assumed constant) speed of the jet and θ is its angle with respect to the line of sight:

(b) Assume v is fixed, and maximize $v_\perp$ with respect to θ. What is the maximum value, and for what θ does it occur?

Problem 4 (25 points):

A quasar ejects a pair of blobs from its core in opposite directions at equal speed v, at an angle θ from the line of sight.



(a) Show that v and θ can both be inferred from the proper motion (angular velocity $v_\perp/d$) of the blobs if the distance d to the quasar is known.

(b) Show that if Doppler shifted emission lines are detected from the blobs, then the distance d can be measured independently, as well.