Astronomy GR6001: Problem Set #4

Due in my office by Tuesday, November 2, 2021

Problem 1 (25 points):

Consider the most general form of a single monochromatic wave propagating in the direction $\hat{k}$ at frequency ω, i.e. an elliptically polarized wave. Compute $d\sigma/d\Omega$, the differential cross–section for scattering such a wave off a free charge as a function of the direction towards the observer $\hat{n}$ (i.e. the Thomson cross–section for an elliptically polarized wave).

Problem 2 (25 points):

Consider two particles, with electric charges q_1 and q_2, and masses m_1 and m_2, orbiting at non-relativistic speeds in each other's Coulomb fields. Show that if $q_1/m_1 = q_2/m_2$, then the dipole radiation vanishes.

Problem 3 (25 points):

The magnetic dipole field has exactly the same form as the electric dipole field, except that the electric dipole $\vec{d}$ is replaced by the magnetic dipole $\vec{M}$. Here we consider a spinning neutron star that is slowing down due to magnetic dipole radiation. Let us assume that the neutron star is a uniform sphere, with mass M, radius R, spinning at angular velocity ω. Let us also assume that its magnetic field is a dipole, with a strength B at the magnetic pole. In general, the angle between the magnetic and rotation axes is θ, i.e. the dipole is rotating.

(a) Show that the magnitude of the magnetic dipole moment M, is $M = BR^3/2$, and find an expression for the power radiated by the magnetic dipole, in terms of B, R, ω, and θ. (Hint: compute the E field of a dipole – two charges separated by a small distance.)

(b) Find an expression for the time it takes for the neutron star to loose all of its rotational energy. What is this "spin–down" time for a typical neutron star (assume $M = 1\mathrm{M}_\odot$, $R = 10^6$cm, $B = 10^{12}$ gauss, $\omega = 10^3$s, and $\theta = \pi/2$).

Problem 4 (25 points):

Consider the semi–classical Bohr model of the hydrogen atom, with the electron in circular orbit around a proton, radiating continuously according to the Larmor formula.

(a) Starting from an orbit with the energy equal to the $n = 2$ level (i.e. the first excited quantum state), how long would it take for the electron to descend to the ground state ?

(b) Compare this with the actual quantum-mechanical lifetime (hint: you will have to look up the Einstein coefficient A_{21} for the Lyman α transition).