Astronomy GR6001: Final Project

Due ONLINE on Sunday, December 19 (no extensions)

In this project you will investigate how the temperature of the baryons evolves in the universe as a function of redshift, due to Compton scattering between baryons and the CMB. Use the following facts from cosmology:

- The cosmic microwave background radiation follows the black-body form, with temperature $T_\gamma(z) = 2.725(1+z)^\circ$ Kelvin.
- The baryon density is $n = 2 \times 10^{-7}(1+z)^3$ cm^{-3}, i.e. simply inversely proportional to the redshift-dependent comoving volume $V \propto (1+z)^3$. For simplicity, you could assume that the baryons are composed only of Hydrogen (no Helium or heavier elements).
- Following recombination, i.e. at redshifts $z \lesssim 1100$, the ionized fraction of baryons $x_e = n_e/(n_e + n_H)$ is $x_e \sim 2 \times 10^{-5}$.
- The Hubble timescale, on which the universe expands is $t_H \approx 1/H(z)$, where $H(z) = H_0[\Omega_m(1+z)^3 + \Omega_\Lambda]^{1/2}$ is the Hubble parameter at redshift z, $H_0 = 70$ km s^{-1} Mpc^{-1} is the present-day Hubble constant, and $\Omega_m = 0.31$ and $\Omega_\Lambda = 0.69$ are the densities of matter and dark energy, respectively, in units of the critical density ($\rho_{\rm crit} = 3H_0^2/8\pi G$).

In the absence of any interactions, the baryons would cool adiabatically, and their temperature would scale with redshift as $T_b \propto (1+z)^2$. This follows from the ideal gas law ($p \propto \rho T$) with an adiabatic equation of state $p \propto \rho^\gamma$ with $\gamma = 5/3$, which implies $T \propto p/\rho \propto \rho^{2/3} \propto (1+z)^2$. This could be regarded as adiabatic cooling of the cosmic baryons on a timescale of the expansion, t_H.

The temperature-evolution is, however, modified by the presence of the CMB. Consider the electrons to be nonrelativistic particles in thermal equilibrium with the rest of the baryons at temperature $T_e = T_b$ (maintained through $e - p$ Coulomb interactions and $e - H$ collisions). The electrons exchange energy with the CMB photons through Compton scattering. Find an equation that governs the rate of energy flow from the electrons to the CMB (or vice versa) as a function of u_γ (the CMB energy density), n_e (the electron density) and the temperatures T_e and T_γ of the electrons and photons. Find the redshift z_* at which this rate matches t_H^{-1}, and therefore describe how the baryon temperature would evolve with redshift from $z = 1100$ to $z = 0$ in a homogeneous universe (ignoring the formation of stars and corresponding heating and reionization).

For extra credit: (1) Do you expect distortions in the spectral shape of the CMB to be significant, due to the interaction with the background baryons? (i.e. what is, roughly, the Compton y parameter?) (2) How does z_* depend on the value of the baryon density at $z = 0$?