Dynamics of the isotope exchange reaction of D with $\text{H}_3^+$, $\text{H}_2\text{D}^+$, and $\text{D}_2\text{H}^+$
Dynamics of the isotope exchange reaction of D with H$_3^+$, H$_2$D$^+$, and D$_2$H$^+$

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ABSTRACT

We have measured the merged-beams rate coefficient for the titular isotope exchange reactions as a function of the relative collision energy in the range of ∼3 meV–10 eV. The results appear to scale with the number of available sites for deuteration. We have performed extensive theoretical calculations to characterize the zero-point energy corrected reaction path. Vibrationally adiabatic minimum energy paths were obtained using a combination of unrestricted quadratic configuration interaction of single and double excitations and internally contracted multireference configuration interaction calculations. The resulting barrier height, ranging from 68 meV to 89 meV, together with the various asymptotes that may be reached in the collision, was used in a classical over-the-barrier model. All competing endoergic reaction channels were taken into account using a flux reduction factor. This model reproduces all three experimental sets quite satisfactorily. In order to generate thermal rate coefficients down to 10 K, the internal excitation energy distribution of each H$_3^+$ isotopologue is evaluated level by level using available line lists and accurate spectroscopic parameters. Tunneling is accounted for by a direct inclusion of the exact quantum tunneling probability in the evaluation of the cross section. We derive a thermal rate coefficient of $<1 \times 10^{-12}$ cm$^3$ s$^{-1}$ for temperatures below 44 K, 86 K, and 139 K for the reaction of D with H$_3^+$, H$_2$D$^+$, and D$_2$H$^+$, respectively, with tunneling effects included. The derived thermal rate coefficients exceed the ring polymer molecular dynamics prediction of Bulut et al. [J. Phys. Chem. A 123, 8766 (2019)] at all temperatures.

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I. INTRODUCTION

Bimolecular ion–molecule reactions involving H-bearing species are among the most fundamental of all gas-phase chemical processes. Such reactions feature prominently in many different chemical situations, including astrochemistry, combustion, fusion plasmas, planetary atmospheres, and plasma processing. Our understanding of these reactions builds on decades of benchmark laboratory measurements and complementary theoretical studies. However, experimental challenges limit the specific systems and reactions that can be studied. Theory must be relied on to fill in the gaps; but, theoretical and computational challenges necessitate the use of approximations to make the calculations tractable. To help guide the development of more reliable theoretical methods, experimental studies are needed for systems of slowly increasing complexity.
The simplest and most fundamental ion–molecule reactive scattering processes proceed through $\text{H}_3^+$, $\text{H}_4^+$, and $\text{H}_5^+$ intermediates and their isotopologues. The bulk of the related published experimental studies has focused on isotopic variants of the reaction systems,

\begin{align}
\text{H}^+ + \text{H}_2, & \quad (1) \\
\text{H}_2^+ + \text{H}_2, & \quad (2) \\
\text{H}_3^+ + \text{H}_2, & \quad (3)
\end{align}

These prototype systems have often been studied with their deuterated variants, as that enables the experimental ease of using mass spectrometry to identify the reactants and products. Brief reviews of the published experimental studies have been given by others for reaction (1),\textsuperscript{18,21} reaction (2),\textsuperscript{22} and reaction (3).\textsuperscript{23–25}

Less well studied experimentally are the reactions,

\begin{align}
\text{H} + \text{H} + \text{H}_2, & \quad (4) \\
\text{H} + \text{H} + \text{H}_3, & \quad (5)
\end{align}

and their isotopic variants. The dearth of laboratory studies for these two reactions has been due, in part, to the experimental challenge of generating controlled and well-quantified beams or ensembles of atomic H.\textsuperscript{17} To the best of our knowledge, there are no published reactive scattering measurements for reaction (4); only recently have experimental results been reported by us\textsuperscript{26} for the isotopic variant of reaction (5), namely,

$$D + \text{H}^+_3 \rightarrow \text{H}_2D^+ + H.$$  \hspace{1cm} (6)

Here, we build on our published experimental and theoretical work\textsuperscript{26} and present new results for

$$D + \text{H}_2D^+ \rightarrow \text{D}_2H^+ + H,$$  \hspace{1cm} (7)

$$D + \text{D}_2H^+ \rightarrow \text{D}_3^+ + H.$$  \hspace{1cm} (8)

The rest of this paper is organized as follows. In Sec. II, the experimental apparatus is briefly described. Section III gives an overview of the measurement and data analysis procedures. The theoretical description of the reaction paths is presented in Sec. IV. The experimental results are reported and discussed in Sec. V. A summary is given in Sec. VI.

II. EXPERIMENTAL DESCRIPTION

The experiments were performed using a dual-source, merged-fast-beams apparatus depicted in Fig. 1. With this, we can perform absolute measurements of the reaction dynamics between neutral atoms and molecular cations by measuring the number densities of the parent beams and their energies and overlap and by detecting the charged daughter products. The apparatus has been described in detail elsewhere.\textsuperscript{26–29} Here, we provide only a brief overview, with an emphasis on those aspects that are most specific to the present results.
A. Neutral D beam

We used a Peabody Scientific duoplasmatron source, operated with D₂ gas to produce D⁻ and floated to a voltage of \( V_{D^−} = −12.00 \text{kV}, -9.60 \text{kV}, \) and \( -8.00 \text{kV} \) for studies of reactions (6)–(8), respectively. The resulting negative particle beam was sent through a Wien filter and mass-to-charge \((m/q)\) filtered to generate a pure beam of \( D^−(S_0)\), as \( D^− \) has only one bound level. The translational energy in the laboratory frame was \( E_{D^−} = |eV_{D^−}| \), where \( e \) is the elementary charge. The beam was, then, guided using a series of electrostatic optics. Typical \( D^− \) currents after the Wien filter were \( 3.7 \mu A \) for reactions (6) and (7) and \( 3.0 \mu A \) for reaction (8). These \( D^− \) currents, and those measured further downstream, do not scale as expected with the acceleration voltage for a couple of reasons. One is that the ion optics were optimized to maximize the neutral D beam particle current in the interaction region. The second is that the filament coating in the ion source aged over the course of the measurements, which were carried out first for reaction (6), then for reaction (8), and finally for reaction (7).

The \( D^− \) beam was, then, deflected by 90° and directed into a floating cell. Typical \( D^− \) currents entering the floating cell were \( 3.3 \mu A \) for reaction (6), \( 1.7 \mu A \) for reaction (7), and \( 2.5 \mu A \) for reaction (8). A voltage \( U_L \) was applied to the floating cell, resulting in an anion beam energy in the cell of \( E_{D^−} + eU_L \). Within the floating cell, ground level D beams are produced via photodetachment,

\[
D^−(S_0) + hv \rightarrow D^0(S_{1/2}) + e^−, \tag{9}
\]

using an \( \sim 1 \text{kW} \) laser beam of wavelength \( \lambda = 808 \text{nm} \). Typically, a few percent of the anion beam is photodetached to form the desired fast neutral D beam.

The neutral beam leaves the floating cell with an energy of \( E_n = E_{D^−} + eU_L \). Apertures before and after the floating cell collimate the beam. The \( D^− \) remaining after the second aperture was electrostatically deflected into a beam dump. The pure ground level D beam, then, continued ballistically into the interaction region.

B. Cation beam

The \( H_2^+ \) beam was generated by introducing pure H₂ into a Peabody Scientific duoplasmatron source. Beams of \( H_2D^+ \) and \( D_2H^+ \) were produced by introducing a mixture of H₂ and D₂ gas into the duoplasmatron source in roughly equal proportion. The molecular cations are produced internally excited, with an internal temperature of \( \sim 1200 \text{K} \), the lowest that we could achieve with the current source configuration. The cation beam energies were chosen so as to match the neutral D beam velocity at \( U_L = 0 \text{ V} \). This resulted in \( E_{H_2^D^+} = 18.02 \text{keV} \) for reaction (6), \( E_{H_2D^+} = 19.21 \text{keV} \) for reaction (7), and \( E_{D_2H^+} = 20.01 \text{keV} \) for reaction (8). The cation beam then passed through a Wien filter to select the desired beam, according to its \( m/q \) ratio. After the Wien filter, the cation beam was electrostatically deflected 90° and steered through two collimating apertures. A retractable Faraday cup after the second aperture was used to measure the ion beam current before it was merged onto the neutral beam. Typical beam currents after the second aperture were \( 1.1 \mu A \) for \( H_2^+ \), \( 1.2 \mu A \) for \( H_2D^+ \), and \( 0.46 \mu A \) for \( D_2H^+ \). The beam was then steered by using a cylindrical deflector onto the path of the neutral beam. The beam current after this merging deflector was monitored by using a retractable Faraday cup in the interaction region. Current measurements before and after the merging cylindrical deflector indicated that 100% of the cation beam was transmitted through the merger.

The purity of the beams used was determined in several ways. The purity of the \( D_2H^+ \) beam was 100%; since we use gases of H₂ and D₂, the only \( m/q = 5 \) ion that could be formed was \( D_2H^+ \). To determine the purity of the \( H_2^+ \) beam, we used only H₂ gas and measured the \( m/q = 3 \) and 5 currents output from the source. The lack of a measurable \( m/q = 5 \) signal indicated that there was no D contamination in the source that could produce \( HD^+ \). Hence, our \( m/q = 3 \) beam was pure \( H_2^+ \).

To determine the \( H_2D^+ \) and \( D_2^+ \) fractions in the \( m/q = 4 \) beam, we developed a novel beam diagnostic. We used the second leg of the cation beamline, between the collimating apertures, as a gas cell that was filled with an \( \sim 0.775 \text{mTorr} \) of Ar, as measured using an ion gauge calibrated for Ar. As the cation beam passed through the first half of the gas cell, a portion of the beam was neutralized, forming \( H_2D \) and \( D_2 \). Neutral \( H_2D \) is unstable and fragments, whereas \( D_2 \) is stable. Midway between the collimating apertures, we located an electrostatic deflector with an applied voltage of \( 1 \text{kV} \) that was used to remove the cations from the beam, leaving a pure beam of neutrals. A portion of these neutralized as they passed through the second half of the gas cell, forming \( D_2^+ \), \( H_2^+ \), \( D^+ \), and \( H^+ \) with laboratory translational energies of \( 19.21 \text{keV}, 9.62 \text{keV}, 9.62 \text{keV}, \) and \( 4.81 \text{keV} \), respectively. We, then, used the beam merger as an electrostatic energy analyzer to select for and direct \( D_2^+ \) into the interaction region, where we measured the current using the retractable Faraday cup.

For a pure \( D_2^+ \) beam, ensured to be free of \( H_2D^+ \) contamination by the lack of a concurrent \( m/q = 5 \) beam, the neutralization and reionization efficiency, \( r_{D_2^+} \), was determined by the ratio of the currents measured in the retractable Faraday cup with and without gas in the cell,

\[
r_{D_2^+} = \frac{I_{D_2^+}^g}{I_{D_2^+}^{ng}}, \tag{10}
\]

where the superscripts \( g \) and \( ng \) indicate the presence of gas or no gas in the cell, respectively. Our measurements yielded \( r_{D_2^+} = 0.40 \pm 0.02\% \). Here and throughout, all uncertainties are quoted at an estimated one-standard-deviation statistical accuracy.

For a \( m/q = 4 \) beam containing both \( H_2D^+ \) and \( D_2^+ \), the ratio of the currents measured in the retractable Faraday cup with and without gas in the cell can be represented as

\[
\frac{I_{m/q=4}^g}{I_{m/q=4}^{ng}} = \frac{I_{D_2^+}^g}{I_{D_2^+}^{ng}} + \frac{I_{H_2D^+}^g}{I_{H_2D^+}^{ng}}. \tag{11}
\]
Here, we have used the arguments given above to set \( n^P_{H,D^+} = 0 \). As a result, the fraction of \( D^+_2 \) in a mixed \( m/q = 4 \) beam, \( f_{D^+_2} \), can be determined by

\[
f_{D^+_2} = \frac{r_{\text{max}}}{r_{D^+_2}^2}
\]  

(12)

Typically, \( f_{D^+_2} < 3\% \). This low level of contamination results from the source parameters having been optimized for the production of triatomic cations.\(^{35} \) The actual \( I_{H,D^+} \) was given by the measured current reduced by \( f_{D^+_2} \).

### C. Interaction region

The two beams overlap for \( L = 1215 \) mm, as measured from the exit of the beam merger to the entrance of an electrostatic chicane (described below). The overlap of the beams is determined using two single-wire beam profile monitors (BPMs)\(^{34} \) placed near the beginning and the end of the interaction region, \( \sim 280 \) mm and \( 1090 \) mm from the exit of the merging cylindrical deflector, respectively.

The product ions for reactions (6)–(8) of \( H_2D^+, D_2H^+, \) and \( D_3^+ \), respectively, are formed at a laboratory translational energy equal to the sum of the energies of the parent beams minus the translational energy of the replaced \( H \) atom. For parent beams at matched velocities, this corresponds to a laboratory translational energy of 24.01 keV for the product ions in reactions (6)–(8).

### D. Signal detection

The end of the interaction region is located within the first of four pairs of parallel plate electrodes of a chicane, which is designed to deflect the charged particles horizontally. The voltage on the first set of plates was cycled between the voltage necessary to allow the products to pass through the chicane and the voltage necessary to direct the parent cation beam into a Faraday cup, where the ion current \( I_i \) was measured. The transmission from the interaction region to this Faraday cup was 100%.

The voltage applied to the rear set of plates was used to optimize the horizontal transmission of the product beam through the final analyzer.

The final analyzer consists of a series of three \( 90^\circ \) cylindrical parallel plate deflectors, guiding the product ions along a perpendicular vertical plane. The first two deflectors, referred to as the lower cylindrical deflector (LCD) and middle cylindrical deflector (MCD), respectively, result in a \( 180^\circ \) bend. A 5 mm slit after the exit of the MCD helps remove any parent beam that makes it into the final analyzer, a potential source of background. The third deflector, the upper cylindrical deflector (UCD), bends in the opposite direction and guides the products onto a channel electron multiplier (CEM). The transmission from the interaction region to the exit of the UCD was measured to be \( T_{uc} = 90\% \pm 5\% \) using a proxy cation beam at the energy of the signal ions and a Faraday cup after the exit of the UCD.

The CEM counts the products with an efficiency of \( \eta = 99\% \pm 3\% \).\(^{32,33} \) The grid in front of the CEM has a geometric transmittance of \( T_g = 90\% \pm 1\% \) and was biased negatively to prevent electrons from leaving the CEM. Voltage scans of the rear chicane, LCD, MCD, and UCD were performed to determine the range of voltages over which the product signal was essentially constant and non-zero. The voltage center of the signal plateau for each deflector was used for data collection. Our data acquisition cycle typically consisted of scanning \( U_i \) to vary \( E_i \), which results in a different product energy for each \( U_i \). For this reason, it was necessary to scale the chicane, LCD, MCD, and UCD voltages for each step in the data acquisition cycle, as described in detail elsewhere.\(^{32} \) The measured signal rate, \( S \), was \( \sim 20 \) s\(^{-1} \) for reactions (6) and (7) and \( \sim 4 \) s\(^{-1} \) for reaction (8).

### E. Neutral current

The neutral beam travels ballistically through the interaction region, chicane, and LCD, exiting through a hole in the outer plate of the LCD and continuing into a neutral detector, referred to as the neutral cup (NC). The NC can also be reconfigured to function as a Faraday cup for ion current measurements. The transmission from the interaction region to the NC, measured using proxy ion beams, was \( T_n = 95\% \pm 3\% \). The total neutral current was measured by collecting the secondary emission of negatively charged particles from a metal target in the NC impacted by the neutral beam. The total neutral particle current, as measured in amperes, is given by

\[
I_D = \frac{I_{NC}}{yT_n},
\]  

(13)

where \( I_{NC} \) is the secondary negative particle current measured in the neutral cup and \( y \) is the mean number of negative particles emitted by a neutral particle striking the target. A detailed account of the procedure used for determining \( y \) can be found elsewhere.\(^{34} \) At \( E_D \) corresponding to matched beam velocities, for reaction (6), \( y = 1.6 \pm 0.1 \), for reaction (7), \( y = 1.3 \pm 0.1 \), and for reaction (8), \( y = 1.1 \pm 0.1 \). Typical neutral particle currents were 43 nA, 52 nA, and 39 nA, respectively. We found that there was a small linear dependence of \( y \) on \( E_D \) and took this into account for the energy ranges studied.

### III. MEASUREMENT AND ANALYSIS

Here, we provide a brief description of the signal determination and the data analysis. A more detailed discussion can be found in our previous publications.\(^{26,28} \)

#### A. Signal determination

The signal was separated from the background by chopping both beams out of phase with one another. The laser beam for photoionization was cycled on for 5 ms and off for 5 ms in a square-wave pattern. The cation beam was electrostatically chopped with the same pattern but phase shifted by 2.5 ms. Hence, the 10 ms cycle consists of four phases. In the first phase, only the neutral beam is present, during which \( N_i \) counts are collected. In the second phase, both beams are on, resulting in \( N_i \) counts. The third phase consists of only the cation beam and \( N_i \) counts. In the final phase, both beams are off, and \( N_i \) counts are collected. The desired signal counts, \( N_s \), are given by

\[
N_s = N_2 - N_1 - N_3 + N_4.
\]  

(14)

Note that the beam-off background-only counts given by \( N_4 \) are also present in the other three chopping phases. Hence, \( +N_4 \) on the
right-hand side of Eq. (14) is to correct for the beam-off background-
only contribution being added once by \( N_2 \) and subtracted twice by
\(-N_1 - N_3 \). For a given value of \( U_i \), this cycle above is repeated for 10 s.
The signal rate, \( S \), is given by dividing \( N_s \) by the signal measurement
time of \( \tau = 2.5 \) s within the 10 s. The statistical uncertainty in the
signal is given by
\[
\delta N_s = (N_2 + N_1 + N_3 + N_4)^{1/2}. \tag{15}
\]
The fractional statistical uncertainty in \( S \) is given by \( \delta N_s / N_s \), and the
resulting uncertainty in \( S \) is \( \delta S = (\delta N_s / N_s)S \).

B. Data acquisition procedure

The procedure for data acquisition remains largely unchanged
from our previously published work. The floating cell was scanned
over three ranges for reactions (6)–(8). The scans consisted of 20
equally spaced voltages. For reaction (6), the scans were within the
ranges \( U_i = -0.900 \) to 1.000 kV, \(-0.450 \) to 0.500 kV, and
\(-0.225 \) to 0.250 kV. For reactions (7) and (8), the floating cell
was scanned over \( U_i = -0.720 \) kV to 0.800 kV, \(-0.360 \) to 0.400 kV,
and \(-0.180 \) kV to 0.200 kV. The voltages applied to the chicane
and final analyzer were scaled synchronously with \( U_i \). The neutral
and ion current were measured before and after signal collection for
each floating cell setting.

C. Relative translational energy and beam overlap

In the center-of-mass frame, the relative translational energy \( E_r \)
for monoenergetic beams intersecting at an angle \( \theta \) is given by
\[
E_r = \mu \left( \frac{E_D}{m_D} + \frac{E_i}{m_i} - 2\sqrt{\frac{E_D E_i}{m_D m_i}} \cos \theta \right), \tag{16}
\]
where \( m_D = 2.015 \) u is the mass of \( D \) and \( m_i \) is 3.023 u for \( H_2^+ \), 4.029 u
for \( H_2D^+ \), and 5.035 u for \( D_2H^+ \). The reduced mass, defined as
\[
\mu = \frac{m_D m_i}{m_D + m_i}, \tag{17}
\]
is 1.209 u for reaction (6), 1.343 u for reaction (7), and 1.439 u for
reaction (8). The relative velocity is, then, given by
\[
v_r = \sqrt{\frac{2E_i}{\mu}}. \tag{18}
\]
The experimental \( E_r \) and energy spread depend on the energies
and energy spreads of the beams, the divergence of the beams,
and the bulk interaction angle, \( \theta_{\text{bulk}} \), between the beams. We have
calculated \( E_r \) using a Monte Carlo particle ray tracing described in
detail elsewhere. These simulations incorporated the geometric
constraints from the various collimating apertures in the apparatus.
These constraints were adjusted slightly to reproduce the mea-
sured typical beam profiles, overlaps, \( \theta_{\text{bulk}} \), and the overlap integral
(\( \Omega(z) \)). The quantities \( \theta_{\text{bulk}} \) and (\( \Omega(z) \)) for reactions (6)–(8)
can be found in Table I. The simulations also yield a histogram of rel-
ative translational energies throughout the interaction volume. We
take the mean of this distribution as our experimental \( E_r \) and the
one-standard-deviation spread of the histogram, \( \Delta E_r \), as our relative
energy uncertainty.

<p>| Table I: Typical experimental values. |</p>
<table>
<thead>
<tr>
<th>Source</th>
<th>Symbol</th>
<th>Value for reaction</th>
<th>Uncertainty (%)</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Non-constants</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Signal rate (statistical)</td>
<td>( S )</td>
<td>20</td>
<td>20</td>
<td>4</td>
</tr>
<tr>
<td>D velocity</td>
<td>( v_D )</td>
<td>10.7</td>
<td>9.6</td>
<td>8.8</td>
</tr>
<tr>
<td>D current</td>
<td>( I_D )</td>
<td>43</td>
<td>52</td>
<td>39</td>
</tr>
<tr>
<td>Ion current</td>
<td>( I_i )</td>
<td>1.1</td>
<td>1.2</td>
<td>0.46</td>
</tr>
<tr>
<td>Bulk angle</td>
<td>( \theta_{\text{bulk}} )</td>
<td>0.39 ( \pm 0.19 )</td>
<td>0.69 ( \pm 0.18 )</td>
<td>0.69 ( \pm 0.28 )</td>
</tr>
<tr>
<td>Overlap factor</td>
<td>(( \Omega(z) ))</td>
<td>2.8</td>
<td>3.1</td>
<td>3.4</td>
</tr>
<tr>
<td>Neutral detector efficiency</td>
<td>( \gamma )</td>
<td>1.6</td>
<td>1.3</td>
<td>1.1</td>
</tr>
<tr>
<td>Constants</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ion velocity</td>
<td>( v_i )</td>
<td>10.7</td>
<td>9.6</td>
<td>8.8</td>
</tr>
<tr>
<td>Analyzer transmission</td>
<td>( T_a )</td>
<td>0.90</td>
<td>0.90</td>
<td>0.90</td>
</tr>
<tr>
<td>Grid transmission</td>
<td>( T_g )</td>
<td>0.90</td>
<td>0.90</td>
<td>0.90</td>
</tr>
<tr>
<td>Neutral transmission</td>
<td>( T_n )</td>
<td>0.95</td>
<td>0.95</td>
<td>0.95</td>
</tr>
<tr>
<td>CEM efficiency</td>
<td>( \eta )</td>
<td>0.99</td>
<td>0.99</td>
<td>0.99</td>
</tr>
<tr>
<td>Interaction length</td>
<td>( L )</td>
<td>121.5</td>
<td>121.5</td>
<td>121.5</td>
</tr>
<tr>
<td>Total systematic uncertainty</td>
<td></td>
<td></td>
<td></td>
<td>15</td>
</tr>
</tbody>
</table>
The molecular rearrangements occurring along the reaction path of the isotope exchange reactions (6)–(8) are illustrated schematically in Fig. 2. The collisions of a D atom with H$_2$ and its mono- and bi-deuterated isotopologues are described by pathways (a), (b) + (c), and (d), respectively. They follow a pathway detailed in previous ab initio investigations of the Born–Oppenheimer (BO) potential energy surface (PES) of H$_2^*$, which is the intermediate of the reactions under study.\textsuperscript{26,39–41} After the initial collision, the system stabilizes in a first minimum Min$_1$, crosses the potential energy barrier characterized by transition state TS$_1$, and reaches a second minimum Min$_2$ before dissociating into the reaction products. The nomenclature of the stationary points of the PES is adopted from previous works.

The successive steps of the paths depicted in Fig. 2 can be written in a general way as

\begin{align}
D + [XYZ]^+ &\rightarrow [D-[XYZ]]^+ \rightarrow [D-X-Y-Z]^+ \\
&\rightarrow ([DXY]-Z)^+ \rightarrow [DXY]^+ + Z, \\
D + [XYZ]^+ &\rightarrow [D-[XYZ]]^+ \rightarrow [D-X-Z-Y]^+ \\
&\rightarrow ([DXZ]-Y)^+ \rightarrow [DXZ]^+ + Y,
\end{align}

where X, Y, and Z are H or D atoms, with X denoting the atom of the H$_2^*$ isotopologue pointing in the direction of the colliding D atom. [XYZ] symbolizes a three-atom cycle, and the curly brackets indicate the range of delocalization of the positive charge. Two different paths result from a given initial orientation of the reactants. These paths are common up to Min$_1$, after which they bifurcate to form two different transition states, (D-X-Y-Z)$^+$ and (D-X-Z-Y)$^+$, depending on whether it is the XZ or XY bond of the Min$_1$ cycle that is broken, respectively. Both paths, then, follow their own route to the exit channel, releasing Z and Y as the atomic fragment, respectively. It must be stressed that the isotopic exchange takes place only if Z is an H atom in the case of path (21) or if Y is an H atom in path (22). Only paths obeying these conditions are retained for the interpretation of our experimental results.

Another point to be considered is that the H$_2^*$ species can orient itself toward the D atom as the reactants approach one another. Three orientations are possible, each of which can lead to pathway (21) or (22). This brings to six the number of potentially open
routes, each with an equal probability for a reaction to occur. Permutational symmetry of identical atoms must, however, be used to identify the symmetry equivalent paths. Each path can, then, be characterized by its multiplicity, defined as the number of symmetry equivalences. Multiplicities enable the prediction of relative probabilities.

Application of the above prescriptions to the different reactions leads to the following predictions:

- $\text{D} + \text{H}_2^+$ reaction: in this reaction, X, Y, and Z are identical hydrogen atoms. Stationary points have symmetry equivalent structures whatever the initial orientation, and all paths are symmetry equivalent, isoenergetic, and equiprobable. This reaction is described by path (a) of Fig. 2 and has a multiplicity of 6.

- $\text{D} + \text{mono-deuterated H}_2^+$ reaction: in this case, two independent collisions must be considered, $\text{D} + \text{DH}_2^+$ and $\text{D} + \text{H}_2^2D^-$, depending on if $X = \text{D}$ or $\text{H}$, respectively. When $X = \text{D}$, reactions (21) and (22) exhibit the same \{\text{D-D-H-H}^{-}\} transition state structure, give rise to the D/H exchange, and are symmetry equivalent. This path of multiplicity 2 is represented by path (b) of Fig. 2. When $X = \text{H}$, the transition states formed in reactions (21) and (22) are \{\text{D-H-H-H}^{-}\} and \{\text{D-H-D-D}^{-}\}, respectively. These reactions produce, respectively, atomic H and D at dissociation and are both of multiplicity 2, but only the former contributes, as path (c) of Fig. 2, to the isotopic exchange.

- $\text{D} + \text{bi-deuterated H}_2^+$ reaction: this reaction also involves two different initial orientations. When $X = \text{D}$, two TS\textsubscript{3} structures \{\text{D-D-D-H}^{-}\} and \{\text{D-D-D-H}^{-}\} are formed, of which only the first arrangement leads to isotopic exchange. This path of multiplicity 2, given the number of possible D permutations, corresponds to path (d) of Fig. 2. No exchange is, however, possible when $X = \text{H}$ because Y and Z are both D atoms.

In conclusion, we have identified four pathways (a)–(d), which describe the exchange reactions experimentally observed. Their multiplicities of 6, 2, 2, and 2, respectively, lead to a probability ratio of 3:2:1 for reactions (6)–(8), respectively, and a ratio of 1:1 for the two pathways characterizing reaction (7).

### B. Ab initio methods of calculation

Two complementary sets of calculations were carried out to characterize the energy profile of the reaction paths depicted in Subsection IV A. The first, presented in Sec. IV C, focuses on the characteristic points occurring along the paths (stationary points, and entrance and exit channels) and provides accurate zero-point energy (ZPE)-corrected energies at these points. The second set of calculations was carried out with the Gaussian 16 program package, in which specific methods for determining reaction paths are available. The minimum energy paths were calculated by means of a second order reaction-path-following approach\textsuperscript{51,52} using the Intrinsic Reaction Coordinate (IRC).\textsuperscript{53} Frequencies of the vibrations perpendicular to the reaction path, referred to as the projected frequencies,\textsuperscript{52,53} were, then, calculated at each point of the IRC by projecting out of the Hessian matrix the motion in the direction of the path.\textsuperscript{54} ZPE corrections related to these projected frequencies will be referred to below as $\text{ZPE}_{\text{proj}}$ to differentiate them from those arising from unprojected calculations, such as those performed in the first set of calculations. Adding $\text{ZPE}_{\text{proj}}$ corrections to the BO energies along the IRC provides the so-called vibrationally adiabatic (VA) energy curves.

The level of \textit{ab initio} theory adopted for the second set of calculations has been selected to be compatible with the IRC algorithms in Gaussian 16. As the ic-MRCI method is not implemented in that program, we have instead used the unrestricted quadratic configuration interaction of single and double excitations (UQCISD) method\textsuperscript{4} and the cc-pVQZ basis set.\textsuperscript{11} This level of theory is, however, somewhat less accurate than our reference ic-MRCI calculations mainly in the evaluation of the relative energies characterizing the energy profiles. This is due to the lack of completeness of the one- and multiple-electron basis sets in the UQCISD calculations. The latter level is, however, good enough to determine the IRCs of the reactions. Test calculations indeed show that geometrical parameters optimized at the stationary points using both approaches agree to within better than 0.2%. A similar conclusion holds for the calculation of ZPE values, which agree to within better than 0.4%. As only the energies are affected by the change in the method, this allowed us to exploit the advantages of both approaches by recalculating at the ic-MRCI/aug-cc-pVQZ level the BO energy at the geometries along the UQCISD paths and by correcting these energies by the UQCISD ZPE\textsubscript{proj} values. We show below that the relative energies resulting from this hybrid procedure agree to within 1 meV with the full ic-MRCI results.

### C. Stationary point calculations

The ZPE-corrected energies $E_{\text{corr}}$ of the characteristic points along the pathways of Fig. 2 are reported in Table II. The results have all been obtained using the ic-MRCI/aug-cc-pVQZ level of calculations.
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The absolute BO energy values used for calculating the energy differences reported in Table II can be found in our previous work.\(^\text{26}\)

The harmonic ZPE values are given in Table III. The variations of the harmonic ZPE values from one isotopologue to another depend, as explained previously,\(^\text{17}\) on the relative positions of the D atoms in the molecular structure, i.e., they are a nuclear mass effect.

For the case of \(\text{H}_2^+\), it follows from the Pauli principle applied to the three equivalent fermionic H nuclei of this system of \(D_{\text{sh}}\) symmetry that the ground rotational level \(J = K = 0\) (with \(J\) being the rotational angular momentum and \(K\) being its projection along the \(C_3\) symmetry axis) is forbidden.\(^\text{17}\) The rotational excitation of the lowest allowed level \(J = K = 1\) of \(64.12 \text{ cm}^{-1}\) (8 meV) is, thus, to be added to the ground state ZPE to form the so-called rotational ZPE of \(\text{H}_2^+\).\(^\text{26}\) For the case of the three bosonic D nuclei of \(D_3^+\), also of \(D_{\text{sh}}\) symmetry, the ground rovibrational state is Pauli allowed.

The \(E_\text{b}\) values of Table II are to be compared to those of the recent work of Bulut et al.,\(^\text{26}\) which report values of 87 meV, 108 meV, 108 meV, and 99 meV for paths (a)–(d), respectively. Note that no difference was made in their work between paths (b) and (c) for the mono-deuterated reaction. Their values, from their work on \(\text{H}_2^+\) BO-PES\(^\text{26}\) and obtained at the same level of \textit{ab initio} theory as ours, differ significantly from our values of 68 meV, 80 meV, 83 meV, and 89 meV, respectively. These discrepancies arise from several different sources. First, there is a systematic error of 6 meV in their evaluation of the \(E_\text{b}\) values, which appears when applying expression (23) using the data reported in Table I of Bulut et al.\(^\text{26}\) Another difference of 8 meV appears in the case of path (a) because they did not account for the rotational contribution to the ZPE of \(\text{H}_2^+\). Finally, discrepancies of 1.5 meV–4.5 meV are observed in the ZPE contributions to the barrier heights. Such differences are larger than the uncertainties of the corresponding harmonic frequency calculations used. For example, for the reaction of \(\text{H} + \text{H}_2\), we compared\(^\text{26}\) our ZPE value of \(\text{TS}_3\) of \(4167 \text{ cm}^{-1}\) to that calculated by Sanz-Sanz et al.\(^\text{26}\) and found agreement to within 0.1 cm\(^{-1}\), while Bulut et al.\(^26\) reported a value of 4202 cm\(^{-1}\), probably obtained indirectly from the analytical expression of the PES.

The results presented in this section show that the height of the ZPE-corrected barrier \(E_\text{b}\) increases with the number \(n\) of D atoms in the \(\text{H}_2^+\) \(D_n^+\) isotopologue: 68 meV, 80/83 meV, and 89 meV for \(n = 0, 1,\) and 2, respectively. This increase comes from the ZPE contribution to \(E_\text{b}\) (−76 meV, −64/−61 meV, and −55 meV, respectively), which is the difference between the ZPE of \(\text{TS}_3\) and the ZPE of the \(\text{H}_2^+\) isotopologue in the

### Table II. ZPE-corrected energy differences characterizing pathways (a)–(d) of Fig. 2, calculated at the ic-MRCI/aug-cc-pV5Z level of theory.

<table>
<thead>
<tr>
<th>Pathway</th>
<th>Potential wells</th>
<th>Barrier height (E_\text{b})</th>
<th>Reaction energy ΔE</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) (\text{D} + \text{H}_2)(^b)</td>
<td>Min(_1): −204.66</td>
<td>Min(_2): −245.74</td>
<td>TS(_3): 67.98</td>
</tr>
<tr>
<td></td>
<td>(39.32)</td>
<td>(−1.76)</td>
<td>(−76.01)</td>
</tr>
<tr>
<td>(b) (\text{D} + \text{DH}_2)(^c)</td>
<td>−198.34</td>
<td>−245.51</td>
<td>80.35</td>
</tr>
<tr>
<td></td>
<td>(45.64)</td>
<td>(−1.53)</td>
<td>(−63.64)</td>
</tr>
<tr>
<td>(c) (\text{D} + \text{H}_2\text{D}^\text{v})</td>
<td>−200.03</td>
<td>−241.54</td>
<td>82.90</td>
</tr>
<tr>
<td></td>
<td>(43.95)</td>
<td>(2.43)</td>
<td>(−61.10)</td>
</tr>
<tr>
<td>(d) (\text{D} + \text{D}_2\text{H}^\text{v})</td>
<td>−200.22</td>
<td>−247.32</td>
<td>89.14</td>
</tr>
<tr>
<td></td>
<td>(43.75)</td>
<td>(−3.34)</td>
<td>(−54.85)</td>
</tr>
</tbody>
</table>

\(^a\)According to (23), \(E_{\text{b}} = E_{\text{b2}} - \text{ZPE}\), with identical values for all isotopologues, i.e., −243.98 meV for Min\(_1\) and Min\(_2\), 143.99 meV for TS\(_3\), and 0.0 meV for ΔE, as already reported by Hillenbrand et al.\(^\text{26}\).

\(^b\)The rotational ZPE of \(\text{H}_2^+\) has been used for calculating all energies for this path (see the text for details).

### Table III. Harmonic ZPE values characterizing pathways (a)–(d) of Fig. 2, calculated at the ic-MRCI/aug-cc-pV5Z level of theory.

<table>
<thead>
<tr>
<th>Pathway</th>
<th>In</th>
<th>Min(_1)</th>
<th>Min(_2)</th>
<th>TS(_3)</th>
<th>Out</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) (\text{D} + \text{H}_2)(^b)</td>
<td>→ (\text{DH}_2^\text{v} + \text{H})</td>
<td>4555.7(^a)</td>
<td>4872.7</td>
<td>4541.8</td>
<td>3942.5 (785.9)</td>
</tr>
<tr>
<td>(b) (\text{D} + \text{DH}_2)(^c)</td>
<td>→ (\text{D}_2\text{H}^\text{v} + \text{H})</td>
<td>4086.3</td>
<td>4454.4</td>
<td>4074.0</td>
<td>3573.0 (788.1)</td>
</tr>
<tr>
<td>(c) (\text{D} + \text{H}_2\text{D}^\text{v})</td>
<td>→ (\text{D}_2\text{H}^\text{v} + \text{H})</td>
<td>4086.3</td>
<td>4440.8</td>
<td>4105.9</td>
<td>3593.5 (789.0)</td>
</tr>
<tr>
<td>(d) (\text{D} + \text{D}_2\text{H}^\text{v})</td>
<td>→ (\text{D}_3^+ + \text{H})</td>
<td>3648.1</td>
<td>4001.0</td>
<td>3621.1</td>
<td>3205.7 (736.4)</td>
</tr>
</tbody>
</table>

\(^a\)Rotational ZPE of \(\text{H}_2^+\) used in Table II (see the text).

\(^b\)ZPE of the forbidden rovibrational ground state of \(\text{H}_2^+\).
entrance channel. The observed trend is explained by the more rapid decrease in ZPE with increasing $n$ for the case of the strongly linked H$_2^+$ species than for the weakly bound transition state. The fact that the barrier height increases more significantly with the first increment in $n$ than for the second one results from the 8 meV rotational contribution to the ZPE of H$_2^+$. Ignoring this contribution to focus on the vibrational effects only would increase the barrier height of reaction D + H$_2^+$ to 76 meV, leading, then, to an almost linear variation of $E_b$ with $n$.

The imaginary frequencies $\omega_i$ characterizing the transition state TS$_3$ are also reported for each pathway, in parentheses in Table III. These frequencies characterize the negative curvature of the BO potential at the top of the reaction barrier and are usually related to the corresponding barrier width. They form, with the values of the barrier heights, the basic set of parameters used for estimating the tunneling probabilities by means of simple one-dimensional (1D) analytical forms of the barrier, such as Eckart barriers.63,64 These approaches only provide, however, a crude approximation of the reaction energy profile. They do not take into account the change in curvature due to the ZPE corrections nor the evolution of the curvature along the path. They ignore also the existence of intermediate minima, such as can be seen in the reactions under study.

D. Vibrationally adiabatic minimum energy paths (ZPE$_p$ corrected)

We present in this subsection the second set of calculations, which have been carried out with the Gaussian 16 program in the framework of the vibrationally adiabatic (VA) approximation. Using these calculations, we aim at obtaining more realistic energy profiles, in which a ZPE$_p$ correction is included at all points of the paths. The VA approximation, initially introduced by Marcus,65 has been widely used, namely, in the framework of variational transition state theory and reaction-path Hamiltonian methods.66 It assumes that the motion along the reaction coordinate is separable from the other motions of the colliding system and that these other motions adjust adiabatically to the former. The resulting VA effective potential $E_{VA}$ can, then, be used in a quantum mechanical treatment of the tunneling.67 A common way to implement this approach by means of \textit{ab initio} techniques is to calculate the IRC, which is the path of steepest descent in mass-weighted Cartesian coordinates beginning in the direction of the negative curvature away from the transition state and following the gradient of the PES in the direction of the reactants and of the products. The IRC provides the BO potential energy $E_{BO}$ as a function of the reaction coordinate $s$, defined as the arc length along the path. For an N-atom non-linear molecule, 3N-7 vibrations are orthogonal to the path direction at each point of the reaction coordinate. These frequencies are the so-called projected frequencies, and resulting ZPE$_p$ is given by their sum divided by 2. This value can be added to the BO energy to furnish the energy $E_{VA}$ at the point considered,

$$E_{VA}(s) = E_{BO}(s) + \text{ZPE}_p(s). \quad (24)$$

We have calculated the IRC paths and projected frequencies for pathways (a)–(d). For completeness, the calculations were also performed for the reference reaction H + H$_2^+$ → H$_2^+ +$ H. The calculated $E_{VA}(s)$ and ZPE$_p(s)$ curves are shown in Figs. 3 and 4, respectively, in which the corresponding entrance channel energies have been subtracted out. The mass-weighted reaction coordinate is expressed in the units of $a_0 u^{-1/2}$, where $a_0$ is the atomic unit of length and $u$ is the atomic mass unit. The VA curves shown in Fig. 3 are tabulated in Tables SI–SV of the supplementary material.

The VA calculations were made by applying the following computational steps to calculate each path. The central part of the profile has been first calculated. Starting from TS$_3$, as the initial geometry, the path was followed in both directions to Min$_1$ and Min$_2$. The entrance and exit channels were, then, calculated starting from a geometry where the fragments were fixed at a large distance from each other, and the other internal parameters were optimized. An IRC path was followed from this point in the direction of the descending gradient until Min$_1$ or Min$_2$ was reached. Additional points were calculated in the vicinity of the minima to improve the description of the potential wells. These various curves were, then, combined to provide the $E_{BO}(s)$ curve at the UQCISD/cc-pVQZ level of theory. The total number of calculated points per path is of about 110. The projected frequencies and the corresponding ZPE$_p(s)$ values were calculated at all points. The $E_{BO}(s)$ curve was, then, recalculated at the ic-MRCI/aug-cc-pVQZ level of theory using the Molpro program. The final $E_{VA}(s)$ curves were obtained by using in Eq. (24) the $E_{BO}(s)$/ic-MRCI and ZPE$_p(s)$/UQCISD values. For

![Graph showing vibrational adiabatic energy curves](image-url)
FIG. 4. Vibrationally adiabatic ZPE\(p\) curves corresponding to the energy curves of Fig. 3.

Figures 2 and 3 clearly show the specificities of the different pathways, in particular, in the region of the three stationary points for which enlarged views are provided in corresponding insets of Fig. 3. The comparison with the symmetrical H + H\(_3\) reaction shows the asymmetry induced by deuteration whose effects are detailed in Fig. 4 for VA ZPE\(p\). The difference between the positive and negative ranges of \(s\) shown in Fig. 3 is important, with a larger stability of Min\(_2\) with respect to Min\(_1\) and with the exothermicity, as already pointed out in our analysis of Table II. The slope of the asymmetric \(E_{VA}\) curves is found to be steeper on the Min\(_2\) side of the barrier than on the Min\(_1\) side, and a displacement of the extrema along the \(s\) axis is also observed in the insets.

The evolution of the projected frequencies and of the related ZPE\(p\) value along the path is shown illustratively in Fig. 5 for the case of the D + H\(_3\) reaction. The vibrational modes are numbered by decreasing frequency order at the geometry of the global minimum Min\(_1\). The evolution of the related normal coordinates is

...
The permeability of the barrier, the ratio of the transmitted to the incident flux, which defines the classical thermal rate coefficient. In brief, one must compute Eckart potential formulas that we introduced in our preceding curve.

Illustrated by the sketches in Fig. 5 drawn at the characteristic points of the path corresponding to \( H^+_3, \text{Min}_1, \text{TS}_3, \text{Min}_2, \) and \( \text{DH}_2^+ \). These drawings show that all modes correspond to in-plane vibrations, except \( \omega_5 \). The \( \omega_1 \) vibration frequency corresponds to a ring breathing, \( \omega_2 \) and \( \omega_7 \) to ring deformations, \( \omega_4 \) to an out-of-plane ring bending, and \( \omega_6 \) to an in-plane ring reorientation. It can also be seen that vibrations \( \omega_1, \omega_2, \) and \( \omega_7 \) of \( \text{DH}_2^+ \) correlate adiabatically with the vibrations of \( H^+_3 \) and \( \text{DH}_2^+ \) when \( s \) tends to \( -\infty \) and \( +\infty \), respectively, noting the degeneracy of \( \omega_2 \) and \( \omega_7 \) for \( H^+_3 \). We also note that at both dissociation limits, the frequencies \( \omega_1 \) and \( \omega_5 \), which involve the relative motions of the two fragments, go to 0. Figure 5 also gives some insight into the quantum mechanical information embedded in the VA model. It shows that the ZPE curve results from half the sum of the five frequency profiles, which show significant variations as a function of \( s \), due to the quantum interactions governing the vibrational degrees of freedom. In particular, the existence of avoided crossings between vibrations \( \omega_3 \) and \( \omega_4 \), which involve the relative motions of the two fragments, go to 0.

E. Tunneling

The complete knowledge of the ZPE-corrected energy curves for all isotopic reactions allows us to go beyond the approximate Eckart potential formulas that we introduced in our preceding work in order to evaluate the tunneling factor \( \Gamma(T) \) to apply to the classical thermal rate coefficient. In brief, one must compute the ratio of the transmitted to the incident flux, which defines the permeability of the barrier,

\[
\kappa = \frac{k_{\text{out}}}{k_{\text{in}}} \frac{|A_1|}{|A_2|},
\]

where \( k_{\text{in}} \) and \( k_{\text{out}} \) are the asymptotic momenta in the reactant and product channels, respectively, and \( A_1 \) and \( A_2 \) are the incident and transmitted amplitudes, respectively.

Here, we closely follow the prescription of Le Roy et al., which boils down to solving a 1D Schrödinger equation along the reaction coordinate \( s \), with the help of a renormalized Numerov algorithm. While it is difficult to perform such a forward calculation in a time-independent manner due to the presence of some reflected amplitude, it is straightforward to consider instead the problem from the product side, where the outgoing wave is well defined. The real and imaginary parts of the wavefunction \( \Psi(s) \) are propagated backward from \( s = +\infty \) by imposing asymptotic cosine and sine behavior at \( s \to +\infty \) of

\[
\phi_1(s) = A_T \cos(k_{\text{out}}s),
\]

\[
\phi_2(s) = A_T \sin(k_{\text{out}}s). \tag{27}
\]

After backward propagation, the real wave functions \( \phi_1 \) and \( \phi_2 \) are compared to the real and imaginary parts of the wavefunction on the reactant side \( s \to -\infty \),

\[
\Psi(s) = A_R \exp(ik_{\text{in}}s) + A_I \exp(-ik_{\text{in}}s), \tag{28}
\]

where \( A_R \) is the reflected amplitude. The various amplitudes are retrieved by equating the log-derivative of the propagated and asymptotic wave functions at \( s \to -\infty \).

The permeability \( \kappa \) was computed for the various reaction energy profiles (Fig. 3) by applying this algorithm in mass-weighted reaction coordinates. The result is presented in Fig. 6 as a function of the total energy available above the ZPE-corrected transition state, \( E_{\text{TS3}} \), for the various systems. The combined effect of the reduced mass and the barrier height causes this tunneling probability to be more strongly suppressed for more deuteronated systems. Note the incomplete transmission across the barrier in the immediate vicinity of \( E_{\text{TS3}} = 0 \). This is an example of how a submerged barrier may affect the overall reactivity.

V. RESULTS AND DISCUSSION

A. Experimental merged-beams rate coefficient

Our measured \( \langle \sigma v_r \rangle \) vs \( E_r \) for reactions (6)–(8) are shown in Fig. 7. The results are shown for \( E_r \approx 0.01 - 10 \) eV. The data plotted for reaction (6) have been published earlier. The data for reactions (7) and (8) are given in Tables SVI and SVII of the supplementary material, respectively. The data points are plotted in Fig. 7 with the one-standard-deviation error bars for both \( \langle \sigma v_r \rangle \) and \( E_r \). The solid curves are from fits to the data points that have been generated using the model introduced in our previous publication and adjusted to reflect the differing number of D substitution channels available for reactions (6)–(8). Further details on the model are discussed below.

B. Competing channels

For reactions (6)–(8), all competing reaction channels are endoergic. Our measurements span the thresholds for these
endothermic channels up to and including complete atomization. The opening of these channels is evidenced by the rapid decrease in the measured merged-beams rate coefficient above the relevant threshold energies. To calculate the threshold energies for these competing reactions, we used literature values for the dissociation energies of $H_i^+$, $H_iD^+$, and $D_2H^+$ and all relevant diatomic molecules, as well as the needed electron binding energies.28,66

For reaction (6), the competing channels are

$$D + H_i^+ \rightarrow HD + H_i^+ - 1.65 \text{ eV},$$  
(29a)

$$D + HD^+ \rightarrow H_2 + HD^+ - 1.67 \text{ eV},$$  
(29b)

$$D + HD^+ \rightarrow HD + H + H^+ - 4.30 \text{ eV},$$  
(29c)

$$D + HD^+ \rightarrow H_2 + [D + H]^+ - 4.34 \text{ eV},$$  
(29d)

$$D + HD^+ \rightarrow HD^+ + H + H - 6.15 \text{ eV},$$  
(29e)

$$D + HD^+ \rightarrow H_i^2 + D + H - 6.17 \text{ eV},$$  
(29f)

$$D + HD^+ \rightarrow [D + H + H]^+ - 8.82 \text{ eV}. $$  
(29g)

The square brackets indicate that the location of the final charge could be on any of the included atoms. Additional details have been given previously.28 Similarly, for reactions (7) and (8), the competing reaction channels are

$$D + H_2D^+ \rightarrow D_2 + H_2^+ - 1.66 \text{ eV}$$  
(30a)

$$D + H_2D^+ \rightarrow HD + HD^+ - 1.68 \text{ eV},$$  
(30b)

$$D + H_2D^+ \rightarrow H_2 + D_2^+ - 1.70 \text{ eV},$$  
(30c)

$$D + H_2D^+ \rightarrow D_2 + H + H^+ - 4.31 \text{ eV},$$  
(30d)

$$D + H_2D^+ \rightarrow HD + [D + H]^+ - 4.35 \text{ eV},$$  
(30e)

$$D + H_2D^+ \rightarrow H_2 + D + D^+ - 4.39 \text{ eV},$$  
(30f)

$$D + H_2D^+ \rightarrow D_2^+ + H + H - 6.18 \text{ eV},$$  
(30g)

$$D + H_2D^+ \rightarrow HD^+ + D + H - 6.20 \text{ eV},$$  
(30h)

$$D + H_2D^+ \rightarrow H_2^+ + D + D - 6.21 \text{ eV},$$  
(30i)

$$D + H_2D^+ \rightarrow [D + D + H]^+ - 8.86 \text{ eV}. $$  
(30j)

The thresholds appear to cluster around $\sim 1.65 \text{ eV}$, $\sim 4.35 \text{ eV}$, $\sim 6.2 \text{ eV}$, and $\sim 8.9 \text{ eV}$. They correspond, in energetic order and for all their isotopic variants, to the production of a pair of diatomics, the production of a neutral molecule and an atomic ion or a molecular ion and a neutral atom, and full atomization. These channels are aggregated into four distinct thresholds in the model described below.

### C. Cross section model

A semiempirical model, initially developed for reaction (6) and introduced in our previous publication, was slightly modified and applied to our experimental results for reactions (7) and (8). A brief overview of the model is provided here.

The model follows a Langevin-like formalism for scattering events with an energy barrier.20 Our model also takes into account the internal excitation of the reactant molecular cations, as well as the opening of competing endothermic reaction channels. We adopt an impact parameter formulation of the cross section, assuming that the dynamics follows the IRC reaction coordinate $s$, which describes the relative motion of D and the triatomic cation up to the transition state,

$$\sigma_\alpha(E_t, E_{\text{max}}) = 2\pi \int_0^{b_{\text{max}}} P(b, E_\alpha) b db, $$

(32)
where \( E_b \) is the total reaction energy available to overcome the barrier of the transition state at some impact parameter \( b \) and \( P(b, E_b) \) is the corresponding probability. If we assume that the internal excitation energy, \( E_{\text{int}} \), is available to combine with \( E_t \) to overcome any reaction barriers, we may write

\[
E_{\text{ts}} = E_t + E_{\text{int}} - E_b - \frac{b^2 E_t}{R_b^2},
\]

where \( R_b \) is the radial separation of the centers-of-mass of the reagents, \( R_{\text{cm}} \), at the transition state. The evolution of \( R_{\text{cm}} \) along the IRC coordinate \( s \) is shown in Fig. 8, together with that of individual interatomic distances. Reactions (6)–(8) may go forward, provided that the system overcomes the energy barrier \( E_b \), i.e.,

\[
P(b, E_b) = H(E_{\text{ts}}),
\]

where \( H \) is the Heaviside function. Simple integration yields the classical result,

\[
\sigma_b(E_b, E_{\text{int}}) = \begin{cases} 
0 & \text{for } E_t + E_{\text{int}} < E_b \\
\pi R_b^2 \left[ \frac{E_{\text{int}}}{E_b} \right] \times S(E_t, E_{\text{int}}) & \text{for } E_t + E_{\text{int}} \geq E_b,
\end{cases}
\]

(35)

where we have introduced a flux reduction factor, \( S(E_t, E_{\text{int}}) \), to take into account the other competing channels. This factor is given in the following form:

\[
S(E_t, E_{\text{int}}) = \begin{cases} 
1 & E_t' < E_1 \\
\frac{1}{1 + a_1(E_t' - E_1)} & E_1 \leq E_t' < E_2 \\
\frac{1}{1 + a_1(E_t' - E_1) + a_2(E_t' - E_2)^2} & E_2 \leq E_t' < E_3 \\
\frac{1}{1 + a_1(E_t' - E_1) + a_2(E_t' - E_2)^2 + a_3(E_t' - E_3)^3} & E_3 \leq E_t' < E_4 \\
\frac{1}{1 + a_1(E_t' - E_1) + a_2(E_t' - E_2)^2 + a_3(E_t' - E_3)^3 + a_4(E_t' + E_{\text{int}} - E_4)^4} & E_t' \geq E_4,
\end{cases}
\]

(36)

The threshold energies, \( E_i \), are defined as the energy at which the successive competing reaction channels become accessible (see Sec. VII B), \( E_t' = E_t + \delta E_{\text{int}} \) and \( a_k \) and \( f \) are adjustable parameters, the latter representing the fraction of internal energy available to overcome the various thresholds. The thresholds have been grouped when nearly degenerate, e.g., reactions (29a) and (29b), reducing their number to 4. The choice of exponent is guided by the multiplicity of the breakup channels, being set to \( n - 1 \) for \( n \)-body breakup. Note that \( f \) is set to unity for the atomization channel.

Within this potential scattering approach, the Langevin value, \( \sigma_{\text{L}} \), is the upper limit for the cross section. From the above discussion of the different reaction paths leading to deuterated products (Sec. IV A), our expression of the reaction cross section becomes

\[
\sigma(E_t, E_{\text{int}}) = \frac{1}{N} \sum_{i}^{N} \min[\sigma_{\text{L}}(E_t, E_{\text{int}}), \sigma(E_i)],
\]

(37)

where \( N \) is the number of total number of outgoing reactions and \( N_i \) is the number of reaction paths leading to deuteration. This formulation is needed to take into account the differing barrier height encountered in reaction (7) depending on whether \( H \) or \( D \) from \( \text{H}_2\text{D}^+ \) is pointing toward the colliding D atom.

Our model cross section, \( \sigma_{\text{mod}} \), is generated by convolving Eq. (37) over \( E_{\text{int}} \). The resulting expression is
The model merged-beams rate coefficients are determined by multiplying $\sigma_{\text{mod}}$ by $\gamma$, and by adjusting the parameters $a_k, f, R_0$, and $\langle E_{\text{int}} \rangle$ to reproduce the measured rate coefficients. The resulting fits are plotted in Fig. 7. The values of the derived fit parameters listed in Table IV are quite similar to those extracted from our previous study of reaction (6). The overall magnitude of $\sigma_{\text{mod}}$ is mainly determined by the value of $R_0$. As mentioned above, our experimentally derived value of $R_0$ can be compared to our theoretically derived value of $R_{\text{cm}}(s = 0) = 2.873 \text{ a}_0$ shown in Fig. 8. The latter value is identical to 2.87 $\text{a}_0$ given by Hillenbrand et al., which was deduced from the geometries of Sanz-Sanz et al. Taking into account the 15% systematic uncertainty in the absolute scale of our experimental data, $R_0$ and $R_{\text{cm}}(s = 0)$ are in reasonably good agreement. This is an important indication for the consistency of our analysis.

The parameter $f$ appears to jump from a percent (albeit with a large error bar) to unity when going from reaction (6) to reactions (7) and (8). In the case of reaction (6), the low $f$ value is indicative of a substantial internal excitation being transferred to the diatomic products of reactions (29a) and (29b). However, for all three reactions, it appears that there are an insufficient number of data points to reliably constrain $f$, making its evolution with deuteration somewhat inconclusive. The best fit values of $\langle E_{\text{int}} \rangle$ for reactions (6)–(8) correspond to $T_{\text{int}} \approx 1130 K, 1280 K$, and 1200 K, respectively.

The various fragmentation channels competing with deuteration of the triatomic cation evolve in importance along the deuteration chain, as revealed by the decreasing value of $a_1$ and the concomitant increase in $a_2$, while $a_3$ and $a_4$ remain negligible throughout. Variable $a_1$ is a measure of the importance of the first endoergic threshold and corresponds to the formation of a neutral molecule and a molecular cation: reactions (29a)–(29b), (30a)–(30c), and (31a)–(31b). The variable $a_2$ is a measure of the importance of the second endoergic threshold and corresponds to the additional fragmentation of that cation: reactions (29c)–(29d), (30d)–(30f), and (31c)–(31d). It is tempting to attribute the predominance of multiple fragmentation for heavier systems to the increased residence time in the Min potential well of the deuterated $H_3^+$. More insight could be gained with the help of advanced quantum dynamical methods such as MCTDH (Multi-configuration Time-Dependent Hartree Method).

To incorporate the effect of quantum tunneling, the Heaviside function in Eq. (34) may be replaced by the energy dependent tunnel probability $\kappa(b, E_i)$ evaluated with the help of the reaction energy profiles given in Sec. IV E.

$$\sigma_{\text{run}}(E_i, E_{\text{int}}) = \left(2\pi \int_0^\infty \kappa(b, E_i)db\right)S(E_i, E_{\text{int}}), \quad (40)$$

with $E_i$ depending on $b$ according to Eq. (33). Here, the tunneling correction is insignificant due to the high level of internal excitation in our experiments and does not improve the fit of the semiempirical model to the experimental results. It will, nevertheless, prove essential when deriving low temperature thermal rate coefficients, as discussed below.

**D. Translational temperature and thermal rate coefficients**

To be applicable to lower internal temperatures ($T_{\text{int}} < 500 K$), our model cross section, Eq. (38), needs to be adapted to the discrete character of the partition function,

$$\sigma_{\text{disc}}(E_i, T_{\text{int}}) = \frac{1}{Z} \sum_i \sigma(E_i, E_i)(2J_i + 1)g_i \exp \left(-\frac{E_i}{k_b T_{\text{int}}} \right), \quad (41)$$

where $E_i, J_i$, and $g_i$ are the energy, total angular momentum, and spin multiplicity of rovibrational state $i$ of the triatomic reactant. The corresponding line lists were taken from the work of Mizus et al., for $H_3^+$ and from the work of Sochi and Tennyson for $H_2D^+$. In the case of $D_3H^+$, no such line list was available, so we have generated a low-energy line list from the most recent spectroscopic parameters by means of the SPFIT/SPCAT package. This discrete sum is replaced above $T \approx 800 K$ by the integral over internal energy.

**TABLE IV.** Best fit parameters for the model cross section and the flux reduction factor. Energetic thresholds are the average values for reactions differing by isotopic substitution only. The uncertainties in the last one or two digits are given in parentheses.

<table>
<thead>
<tr>
<th>Reactants</th>
<th>$R_0$ (aₐ₀)</th>
<th>$\langle E_{\text{int}} \rangle$ (eV)</th>
<th>$f$</th>
<th>$E_1$ (eV)</th>
<th>$a_1$ (eV⁻¹)</th>
<th>$E_2$ (eV)</th>
<th>$a_2$ (eV⁻²)</th>
<th>$E_3$ (eV)</th>
<th>$a_3$ (eV⁻²)</th>
<th>$E_4$ (eV)</th>
<th>$a_4$ (eV⁻³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>D + H₃⁺</td>
<td>2.53⁷</td>
<td>0.183(2)</td>
<td>0.01(8)</td>
<td>1.66</td>
<td>0.296(4)</td>
<td>4.32</td>
<td>0.019(2)</td>
<td>6.16</td>
<td>0.001(6)</td>
<td>8.82</td>
<td>0.010(9)</td>
</tr>
<tr>
<td>D + H₂D⁺</td>
<td>2.57(8)</td>
<td>0.222(2)</td>
<td>1.00(13)</td>
<td>1.68</td>
<td>0.223(6)</td>
<td>4.35</td>
<td>0.078(6)</td>
<td>6.20</td>
<td>0.001(27)</td>
<td>8.86</td>
<td>0.00(12)</td>
</tr>
<tr>
<td>D + D₂H⁺</td>
<td>2.57(14)</td>
<td>0.201(4)</td>
<td>1.00(26)</td>
<td>1.70</td>
<td>0.199(8)</td>
<td>4.38</td>
<td>0.091(12)</td>
<td>6.24</td>
<td>0.00(3)</td>
<td>8.92</td>
<td>0.00(11)</td>
</tr>
</tbody>
</table>

⁷Reference 26.
in $E_{\text{int}}$, as given by Eqs. (38) and (39). With this further refinement, a model thermal rate coefficient may be computed as

$$k(T) = \left( \frac{8}{\pi \hbar^2 k_B^2} \right)^{1/2} \int_0^\infty \sigma(E_r, T_{\text{int}}) E_r \exp \left( \frac{E_r}{k_B T} \right) dE_r. \quad (42)$$

Here, $\sigma(E_r, T_{\text{int}})$ is given by Eqs. (35), (37), and (41) for our model without tunneling and Eqs. (37), (40), and (41) for our model with tunneling. $T$ refers to the translational temperature of the gas, $T_{\text{gas}}$. Setting $T_{\text{int}} = 0 \text{ K}$ yields a translational temperature rate coefficient. For thermal equilibrium between the translational and internal energy, $T_{\text{gas}} = T_{\text{int}}$. The resulting thermal rate coefficients for reactions (6)–(8) are given by the full and dashed curves in Fig. 9, depending on whether tunneling is incorporated ($k_{\text{tun}}$) or not ($k_{\text{mod}}$) in the evaluation of the reaction cross section. Their values are tabulated for all three systems in Tables SVIII–SX of the supplementary material. The values published before for reaction (6) are superseded by these newer results where we have included the quantum tunneling probability in the model cross section and have performed a more careful thermal averaging.

In order to facilitate the implementation of our results into astrochemical models, we have fit the entire set of model thermal rate coefficients, both without and with the tunneling correction for the reaction cross section, with the commonly used Arrhenius–Kooij formula, giving

$$k_{\text{arr}}(T) = \alpha \left( \frac{T}{300 \text{ K}} \right)^\beta \exp \left( \frac{-\gamma}{T} \right), \quad (43)$$

where $\alpha$, $\beta$, and $\gamma$ are fitting parameters. This fit is accurate to within 10% over the range $T = 10 \text{ K}–4000 \text{ K}$. The corresponding parameters are listed in Table V. For the classical over-the-barrier cross section model, we may interpret $\gamma$ as the effective barrier height expressed in K. We find 64.7 meV, 70.7 meV, and 79.2 meV for reactions (6)–(8), respectively, somewhat below the values of $E_b$ listed in Table II. Applying the common definition of the activation barrier for Eq. (43),

$$E_a = k_B T \frac{d \ln k(T)}{dT} = k_B (\gamma + \beta T), \quad (44)$$

we observe that $E_a$ slowly grows with temperature, given the values of $\beta$ and $\gamma$ listed in Table V.

**TABLE V.** Parameters for the fits of the Arrhenius–Kooij formula (43) to the thermal rate coefficients derived from our model without and with tunneling. All fit functions describe our results with a relative accuracy of 10% or better across the given temperature range, $T_{\text{min}} \leq T \leq T_{\text{max}}$.

<table>
<thead>
<tr>
<th>Reactants</th>
<th>Tunneling</th>
<th>$\alpha$ (cm$^3$ s$^{-1}$)</th>
<th>$\beta$</th>
<th>$\gamma$ (K)</th>
<th>$T_{\text{min}}$ (K)</th>
<th>$T_{\text{max}}$ (K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D + H_2$</td>
<td>No</td>
<td>5.43 $\times 10^{-10}$</td>
<td>0.26</td>
<td>751</td>
<td>80</td>
<td>4000</td>
</tr>
<tr>
<td></td>
<td>Yes</td>
<td>4.89 $\times 10^{-12}$</td>
<td>0.81</td>
<td>0</td>
<td>10</td>
<td>65</td>
</tr>
<tr>
<td></td>
<td>Yes</td>
<td>5.26 $\times 10^{-11}$</td>
<td>2.37</td>
<td>65</td>
<td>257</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Yes</td>
<td>3.62 $\times 10^{-10}$</td>
<td>0.43</td>
<td>573</td>
<td>257</td>
<td>4000</td>
</tr>
<tr>
<td>$D + H_2D^+$</td>
<td>No</td>
<td>3.94 $\times 10^{-10}$</td>
<td>0.20</td>
<td>821</td>
<td>80</td>
<td>4000</td>
</tr>
<tr>
<td></td>
<td>Yes</td>
<td>3.14 $\times 10^{-12}$</td>
<td>1.06</td>
<td>8</td>
<td>10</td>
<td>73</td>
</tr>
<tr>
<td></td>
<td>Yes</td>
<td>3.10 $\times 10^{-11}$</td>
<td>2.75</td>
<td>73</td>
<td>210</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Yes</td>
<td>2.26 $\times 10^{-10}$</td>
<td>0.44</td>
<td>590</td>
<td>210</td>
<td>4000</td>
</tr>
<tr>
<td>$D + D_2H^+$</td>
<td>No</td>
<td>2.13 $\times 10^{-10}$</td>
<td>0.17</td>
<td>919</td>
<td>80</td>
<td>4000</td>
</tr>
<tr>
<td></td>
<td>Yes</td>
<td>9.79 $\times 10^{-13}$</td>
<td>1.45</td>
<td>10</td>
<td>73</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Yes</td>
<td>1.24 $\times 10^{-11}$</td>
<td>3.25</td>
<td>73</td>
<td>206</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Yes</td>
<td>1.19 $\times 10^{-10}$</td>
<td>0.41</td>
<td>686</td>
<td>206</td>
<td>4000</td>
</tr>
</tbody>
</table>

$^a$X[Y] corresponds to $X \times 10^Y$.

**FIG. 9.** Rate coefficient, $k$, vs temperature, $T$, for reactions (6)–(8). For reaction (7), the two paths are separated, as discussed in Sec. IV A. The full curves ($k_{\text{tun}}$) include the tunnel corrected cross section described in Sec. V C, while the dashed curves ($k_{\text{mod}}$) use the classical expression for the cross section given by Eq. (35). The energy of the barrier for each reaction, $E_b/k_B$, is indicated on the $T$ axis by a vertical marker. The light green marker overlaps the red marker.

**E. Comparison to theory**

As we have reevaluated the thermal rate coefficient for reaction (6) and extended our model to reaction (7), we may compare our classical over-the-barrier thermal results with the predictions of Bulut et al.$^{59}$ obtained with the Ring Polymer Molecular Dynamics (RPMD) approach for these two reactions. Such a comparison is given in Fig. 10, together with the pure translational rate coefficients obtained by setting $T_{\text{int}} = 0 \text{ K}$ in the evaluation of the cross section. The RPMD rate coefficient lies below our modeled thermal results but close to the pure translational case. Bulut et al. attributed the difference between the RPMD results and experimental values to the possible existence of non-thermal vibrational excitation of $H_2$ in their original model.$^{59}$ The now explicit treatment of internal excitation embodied in Eq. (41) should prevent such spurious excitations from affecting our modeled rate coefficient. Hence, it is unclear why the RPMD results lie below our thermal model results. A partial explanation may be that there are a large number of trapped trajectories in the RPMD calculations that form long-lived complexes with lifetimes longer than the 1 ns simulated in the calculations. RPMD is also expected to incorporate tunneling, but no obvious enhancement of the rate coefficient is visible at the lower end of the temperature range studied. We expect that some unknown fraction of the trapped trajectories will contribute to tunneling. It is clear that additional
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The authors thank Octavio Roncero for stimulating conversations and Anthony Roucou for having computed the level list (Tables SVIII–SX). 

SUPPLEMENTARY MATERIAL

See the supplementary material for a tabulation of the curves shown in Fig. 3 (Tables SI–SV), Fig. 7 (Tables SVI–SVII), and Fig. 9 (Tables SVIII–SX).

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DATA AVAILABILITY

The data that support the findings of this study are available within the article and its supplementary material.

REFERENCES
