Physics 8048
Problem Set 3, due 10/2/14
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Readings: if you are interested in quantizing a massive vector field and then sending mass to zero as a way to treat photons, see Zee I.5; for quantization of massless photons in Coulomb gauge, see Srednicki chapters 55 - 57; for Faddeev-Popov quantization, see Coleman 5.2.

1. Srednicki problem 55.1.

2. Justify the following claim made in the class:

$$\int D\phi e^{i\int d^4x \frac{1}{2}(\partial_i\phi)^2 - \rho\phi} = e^{\frac{i}{2}\int dt d^3x d^3y \rho(\mathbf{x},t)\rho(\mathbf{y},t)G(\mathbf{x}-\mathbf{y})} \tag{1}$$

where

$$G(\mathbf{x}-\mathbf{y}) = -\int \frac{d^3k}{(2\pi)^3}\frac{1}{|\mathbf{k}|^2}e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{y})} = \frac{-1}{4\pi|\mathbf{x}-\mathbf{y}|} \tag{2}$$

paying special attention to the fact that we have only one integration over t in the result. Here ϕ is what we called A^0 in the class. One way to proceed is to carefully write out the Gaussian integral in discretized form. The fact that G comes with a negative sign is crucial in establishing that the Coulomb energy is positive (using the comment in problem 1), and thus the force between like charges is repulsive. Zee I.5 has a nice discussion about this, contrasting it with the scalar force which is attractive.

3. Show that $\partial_\mu \tilde{F}^{\nu\mu} = 0$ (these are the source-free Maxwell equations) is automatically guaranteed by the definition of F and its dual $\tilde{F}$.

4. Show that the Gauss constraint ($\partial_\mu F^{0\mu} = J^0$) is respected by the evolution ($\partial_\mu F^{i\mu} = J^i$), meaning once the Gauss constraint is respected at the initial time, it will be respected at all times.

5. This problem functions more as notes. In the class, we showed that the Maxwell Lagrangian corresponds to a massless particle, by adopting the Lorenz gauge $\partial \cdot A = 0$. You might wonder how one can show the massless nature of the photon in a gauge independent way. Here's how you might proceed. Consider a plane wave solution of the Maxwell equation (source set to zero) $\partial_\mu F^{\nu\mu} = 0$, without any gauge fixing, i.e. try a solution of the form $A^\mu = \xi^\mu e^{ik\cdot x}$. *Show that*

$$-k^\nu(k\cdot\xi) + k^2\xi^\nu = 0 \tag{3}$$

There are two possibilities: $k^2 = 0$ or $k^2 \neq 0$. For $k^2 = 0$, *show that* ξ^μ has three linearly independent solutions: $\propto k^\mu$, and the two standard transverse polarizations $\propto \epsilon^\mu_\pm(k)$. For $k^2 \neq 0$, *show that* $\xi^\nu \propto k^\nu$ is the only solution. This way, one can see that, yes, there is an apparently massive solution ($k^2 \neq 0$), but it is a gauge mode. And in the massless case ($k^2 = 0$), only two independent solutions are physical.