Physics 8048
Problem Set 2, due 9/24/14
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1. In our discussion of loops, we considered the theory:

$$\mathcal{L} = i\bar{\Psi}\gamma^\mu\partial_\mu\Psi - m\bar{\Psi}\Psi - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}M^2\phi^2 + ig\phi\bar{\Psi}\gamma_5\Psi - \frac{1}{4!}\lambda\phi^4\,, \tag{1}$$

plus suitable counter terms. Let's consider the loop correction to the fermion propagator: the left diagram of Srednicki's Fig. 51.2. Let us express the (full) two-point fermion correlation function in terms of the (full) fermion propagator:

$$\langle\Psi_\alpha(x_1)\bar{\Psi}_\beta(x_2)\rangle_{\rm full} = \int \frac{d^dp}{(2\pi)^d}\frac{1}{i}\tilde{S}^{\rm full}_{\alpha\beta}(p)\,e^{ip\cdot(x_1-x_2)}\,. \tag{2}$$

By 'full', we mean including both free and interacting parts. The average $\langle\,\rangle_{\rm full}$ denotes averaging using the path integral of the full theory. As discussed in class, we can expand out the interacting part of the action and obtain perturbatively corrections to the two-point function. By expanding out the cubic scalar-fermion-fermion interaction quadratically, *show that*:

$$\frac{1}{i}\tilde{S}^{\rm full}_{\alpha\beta}(p) = \frac{1}{i}\tilde{S}^{\rm free}_{\alpha\beta}(p) + \frac{1}{i}S^{\rm free}_{\alpha\rho}(p)\,i\Sigma_{\rho\kappa}(\gamma^\mu p_\mu)\,\frac{1}{i}S^{\rm free}_{\kappa\beta} + ... \tag{3}$$

with $i\Sigma$ given by Srednick's eq. 51.27. Here $\tilde{S}^{\rm free}_{\alpha\beta}$ is the free fermion propagator (Srednicki simply uses $\tilde{S}_{\alpha\beta}$ to denote the same). *Show also that*:

$$\tilde{S}^{{\rm full}\,-1}{}_{\beta\sigma} = \tilde{S}^{{\rm free}\,-1}{}_{\beta\sigma} - \Sigma_{\beta\sigma}\,, \tag{4}$$

where $\tilde{S}^{-1}$ denotes the (spinor) matrix inverse of $\tilde{S}$. Lastly, apply the usual set of tricks – Feynman's trick, gamma matrix algebra, and dim. reg. – to derive Srednick's eq. 51.32 from eq. 51.27. Recall that $\tilde{S}^{{\rm free}-1} = \gamma^\mu p_\mu + m - i\epsilon$ has a pole at $\gamma^\mu p_\mu = -m$. The on-shell renormalization scheme (where m is identified with the physical fermion mass) thus requires $\Sigma(\gamma^\mu p_\mu = -m) = 0$, and $\Sigma'(\gamma^\mu p_\mu = -m) = 0$. This is the analog of requiring $\Pi(k^2 = -M^2) = \Pi'(k^2 = -M^2) = 0$ for the loop correction to the scalar propagator.

2. In the class, we claimed that the theory described by Eq. (1) will never generate a ϕ^3 term from loop diagrams. Our argument was based on symmetry: that the theory is parity invariant (recall ϕ transforms as a pseudo-scalar under partiy). Here, I want you to verify this by explicit computation, i.e. show that no ϕ^3 term is generated at the one-loop level.

3. Srednicki's problem 52.1. You might want to remind yourself the meaning of anomalous dimensions by referring to chapter 28. You will need to know the coefficents of the $1/\epsilon$ poles in Z_m, Z_M and so on; you can find them in chapter 51.