Inflation and the Early Universe

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This is the third in a series of 3 talks.

July 5: Dark energy and the homogeneous universe

July 11: Dark matter and the large scale structure of the universe

Today: Inflation and the early universe
Outline

Review: expansion dynamics and light propagation

Inflation: the horizon problem and its solution

Inflation: predictions for flatness and large scale structure

Inflation: problems
Energy conservation

\[ \frac{1}{2} \dot{a}^2 - \frac{GM}{a} = E \]

For simplicity set \( E = 0 \):

\[ \frac{1}{2} \dot{a}^2 = \frac{GM}{a} \]
**Fundamental equation:** \[\frac{1}{2} \dot{a}^2 = \frac{GM}{a}\] where \[M = \frac{4\pi}{3}a^3 \rho\]

**Example 1 - ordinary matter**
\[\rho \propto \frac{1}{a^3} \quad (M = \text{constant}) \quad \implies \dot{a}^2 \propto \frac{1}{a}\]

Therefore: \(\dot{a} \downarrow \iff a \uparrow\)

**Example 2 - cosmological constant \(\Lambda\)**
\[\rho = \text{constant} \implies \dot{a}^2 \propto a^2\]

Therefore: \(\dot{a} \uparrow \iff a \uparrow\) **Acceleration!**

**Dark Energy:** \(\rho\) drops slower than \(\frac{1}{a^2}\)

i.e. \(\rho \propto a^{-3(1+w)}\) with \(w < -1/3\)
matter: $\log \rho \sim -3 \log a$

\begin{align*}
\Lambda: & \quad \log \rho \\
\log a & 
\end{align*}
\[ \log a \sim \frac{2}{3} \log t \]
Scale factor $a(t)$

Distance $= a(t) \Delta x$

Distance $= a(t') \Delta x$
Photons travel at speed of light $c$:

$$a(t) \, dx = c \, dt$$

$$\int dx = \int c \, dt / a(t)$$

Horizon $= a(t) \int dx = a(t) \int _{t_{\text{early}}}^{t} c \, dt' / a(t')$

$$= 3 \, c \, t \left( 1 - \frac{t_{\text{early}}^{1/3}}{t^{1/3}} \right)$$

$$\sim 3 \, c \, t \propto a^{3/2}$$

Contrast:

Physical distance btw. galaxies $\propto a$
$\log(\text{hor.}) \sim \frac{3}{2} \log a$

$\log(\text{phys. dist.}) \sim \log a$

Horizon problem!
Sloan Digital Sky Survey

scale $\sim 10^{26}$ cm
Horizon problem: 2 sides of the same coin

- On the scale of typical galaxy separations: how did the early universe know the density should be fairly similar?
- On the scale of typical galaxy separations: how did the early universe know the density should differ by a small amount?
Another way to view the horizon problem:
\[ \log a \sim t \]

\[ \log a \sim \frac{2}{3} \log t \]
Inflation

\[ \log a \sim \frac{2}{3} \log t \]

Very early time:

\[ t \approx t_{\text{early}} \]
Horizon \( = a(t) \int dx = a(t) \int_{t_v \text{ early}}^{t} \frac{c \, dt'}{a(t')} \)

\[= a(t) \int_{t_v \text{ early}}^{t \text{ early}} c \, dt' / a(t') + a(t) \int_{t \text{ early}}^{t_v \text{ early}} c \, dt' / a(t') \]

\[\approx \frac{a(t)}{a(t_v \text{ early})} \frac{c}{H} + 3c \, t \]

Note: \( a(t') \propto e^{Ht'} \), \( a(t') \propto t'^{2/3} \)

\( t_v \text{ early} < t' < t \text{ early} \), \( t \text{ early} < t' \)
log(hor.) $\sim \frac{3}{2} \log a$

log(phys. dist.)

$\sim \log a$

Horizon problem solved!
Inflation’s prediction for flatness

Energy conservation

\( \frac{1}{2} \dot{a}^2 - \frac{GM}{a} = E \)

\( M = 4 \pi \rho \frac{a^3}{3} \)

\( 1 - \frac{2GM}{a \dot{a}^2} = \frac{2E}{\dot{a}^2} \rightarrow 0 \)

Data tell us \( |2E/\dot{a}^2| < 0.03. \)
Inflation’s prediction for large scale structure

‘Hawking’ radiation from inflation seeds
structure formation.

Inflation predicts nearly equal power on all scales: amplitude of fluctuations $\propto \text{scale}^{n-1}$

Data tell us $n \sim 0.95 \pm 0.02$. 
\[ \log \left( \frac{c a}{\dot{a}} \right) \]

\[ \log(\text{phys. dist.}) \]

\[ \sim -\log a \]

\[ \text{inflation} \]

\[ \text{quantum fluctuation} \]
Inflation: problems

\[ \text{inflation} \ ho \]

\[ \text{matter: } \log \rho \sim -3 \log a \]

\[ \Lambda : \log \rho \]

\[ \log a \]
Problems of inflation:

Why is the $p$ associated with inflation so constant?
Why did inflation stop?
Why did inflation start?