Inflation and the Early Universe

早期宇宙之加速膨脹

Lam Hui 許林
Columbia University

This is the third in a series of 3 talks.

July 5: Dark energy and the homogeneous universe

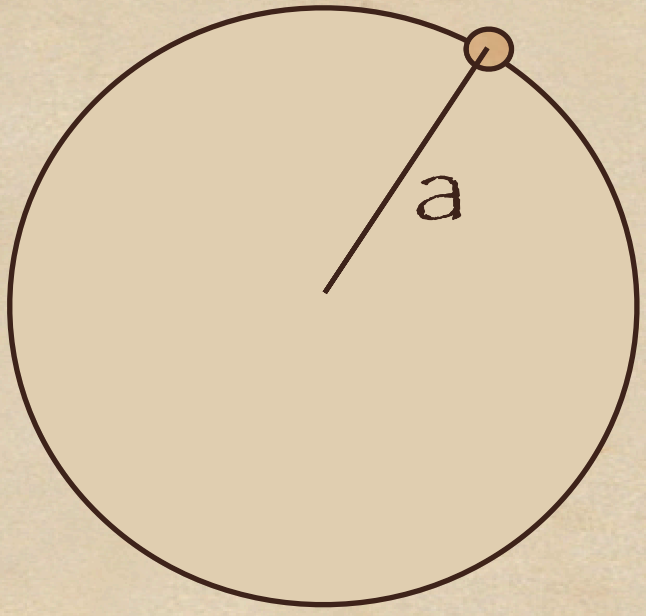
July 11: Dark matter and the large scale structure of the universe

Today: Inflation and the early universe

Outline

- **Review:** expansion dynamics and light propagation
- **Inflation:** the horizon problem and its solution
- **Inflation:** predictions for flatness and large scale structure
- **Inflation:** problems

Cosmology 101



Energy conservation

$$\frac{1}{2}\dot{a}^2 - \frac{GM}{a} = E$$

For simplicity set $E = 0$: $\frac{1}{2}\dot{a}^2 = \frac{GM}{a}$

Fundamental equation: $\frac{1}{2}\dot{a}^2 = \frac{GM}{a}$ where $M = \frac{4\pi}{3}a^3\rho$

Example 1 - ordinary matter

$$\rho \propto \frac{1}{a^3} \quad (M = \text{constant}) \quad \Longrightarrow \dot{a}^2 \propto \frac{1}{a}$$

$$\text{Therefore}: \dot{a} \downarrow \Longleftrightarrow a \uparrow$$

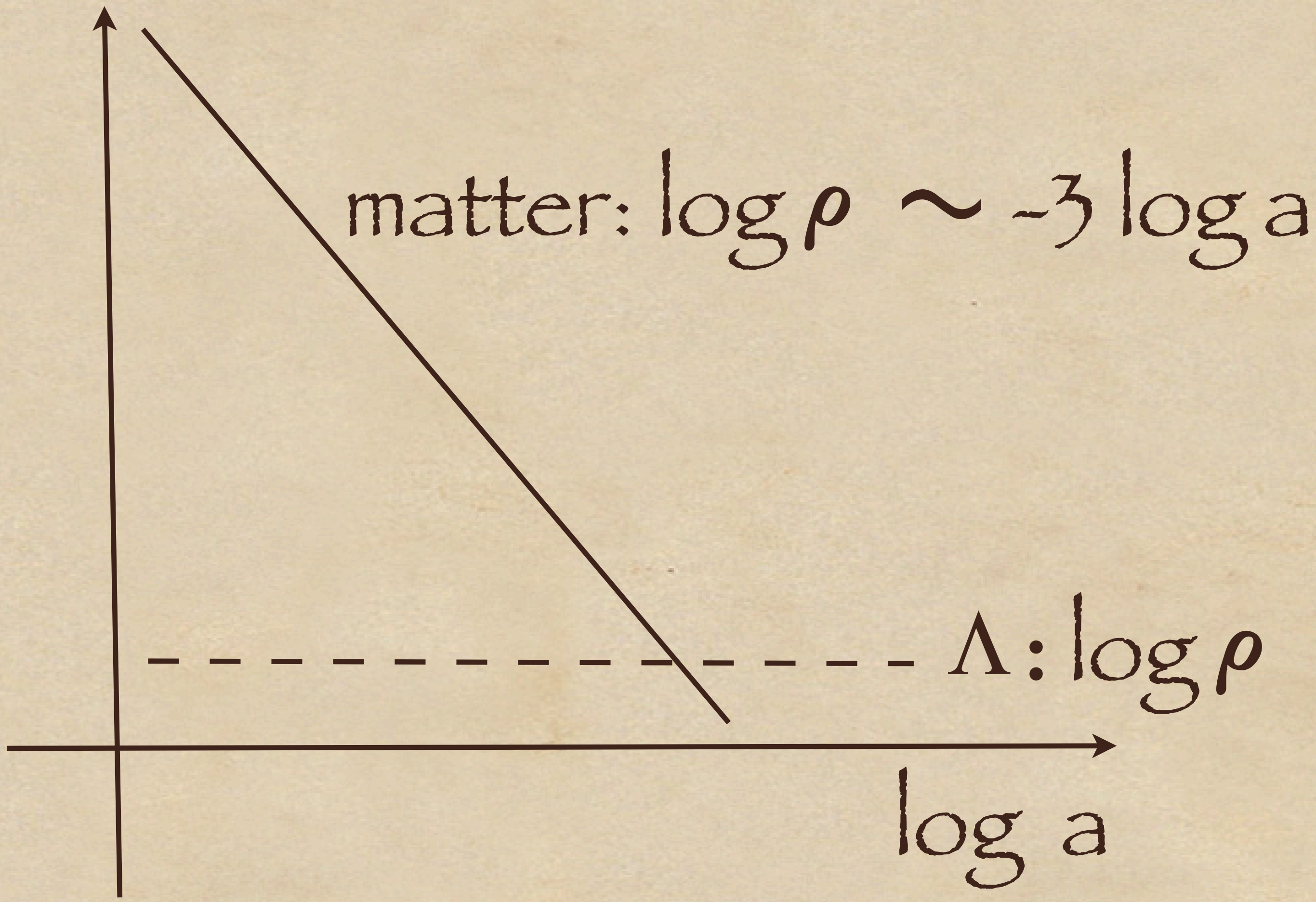
Example 2 - cosmological constant Λ

$$\rho = \text{constant} \Longrightarrow \dot{a}^2 \propto a^2$$

$$\text{Therefore}: \dot{a} \uparrow \Longleftrightarrow a \uparrow$$ Acceleration!

Dark Energy: ρ drops slower than $\frac{1}{a^2}$

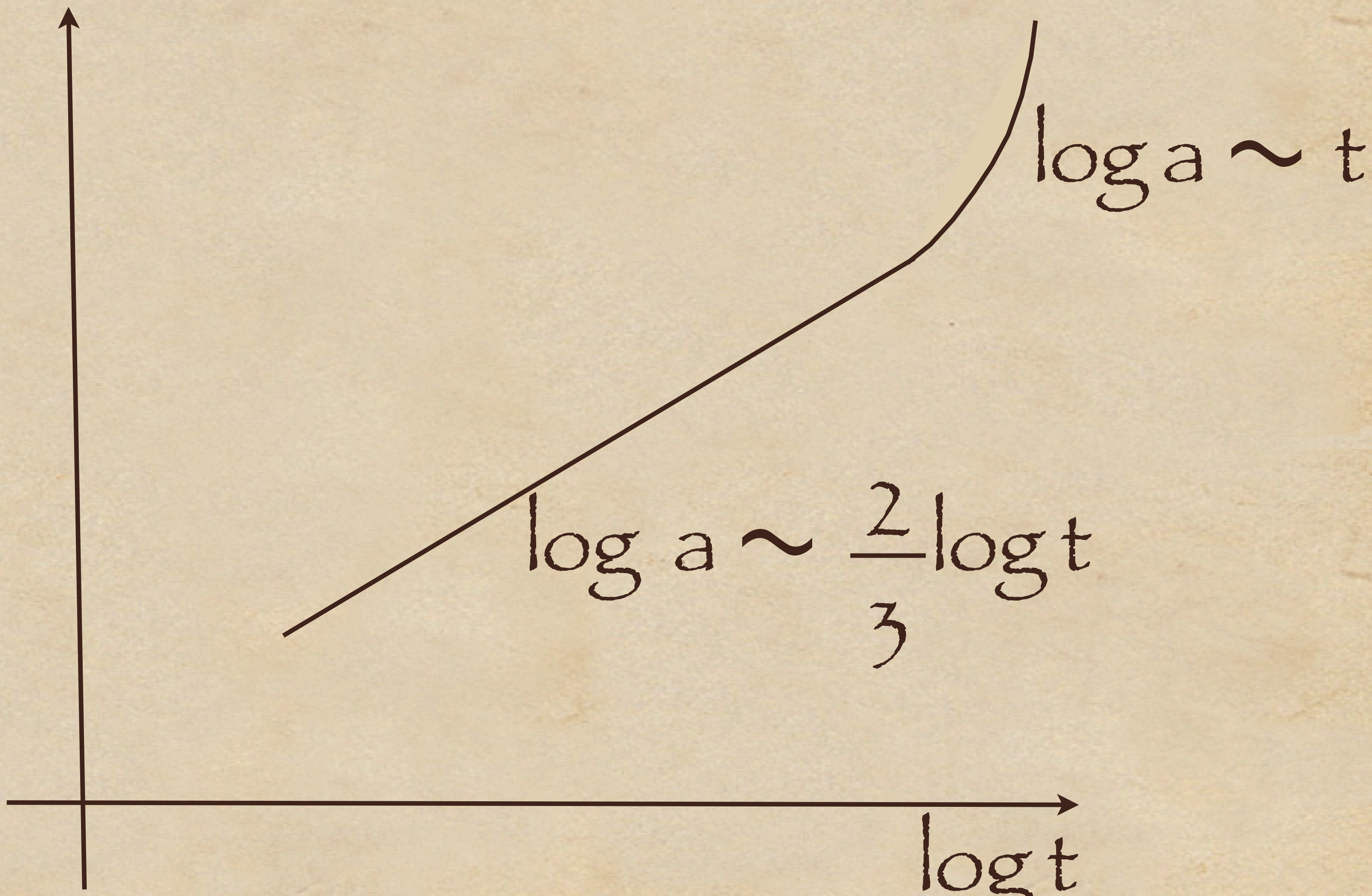
i.e. $\rho \propto a^{-3(1+w)}$ with $w < -1/3$

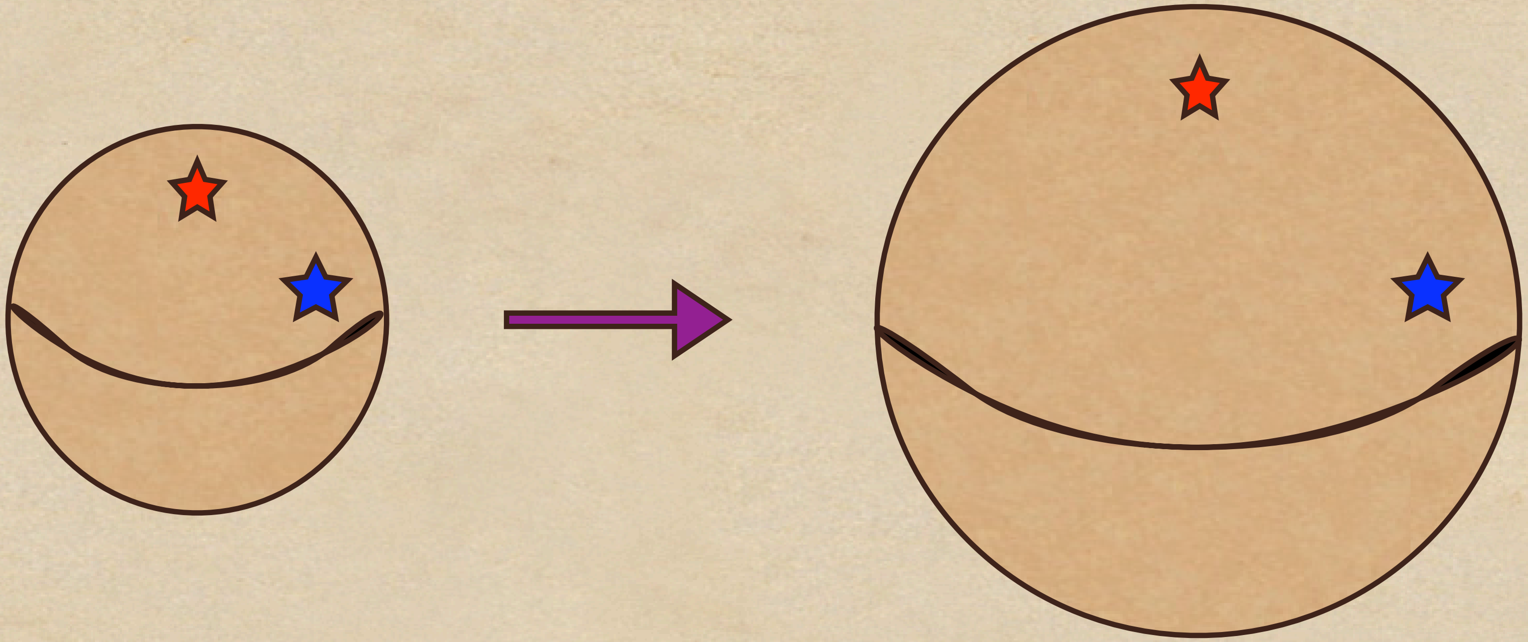


matter: $\log \rho \sim -3 \log a$

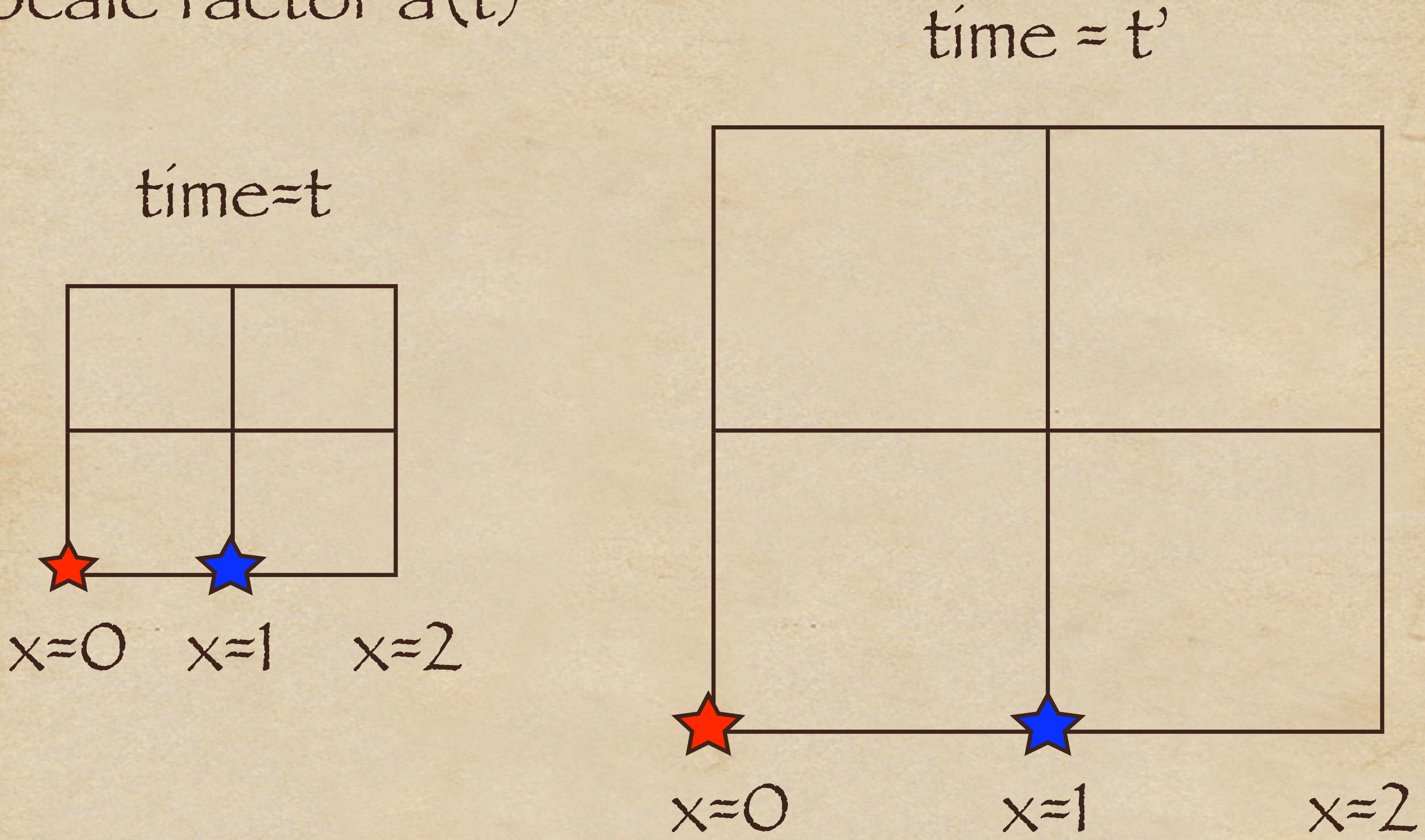
Λ : $\log \rho$

$\log a$


$\log a \sim t$
$\log a \sim \frac{2}{3} \log t$
$\log t$



Scale factor $a(t)$



distance = $a(t)\,\Delta x$

distance = $a(t')\,\Delta x$

Photons travel at speed of light c :

$$a(t)\,dx = c\,dt$$

$$\int dx = \int c\,dt/a(t)$$

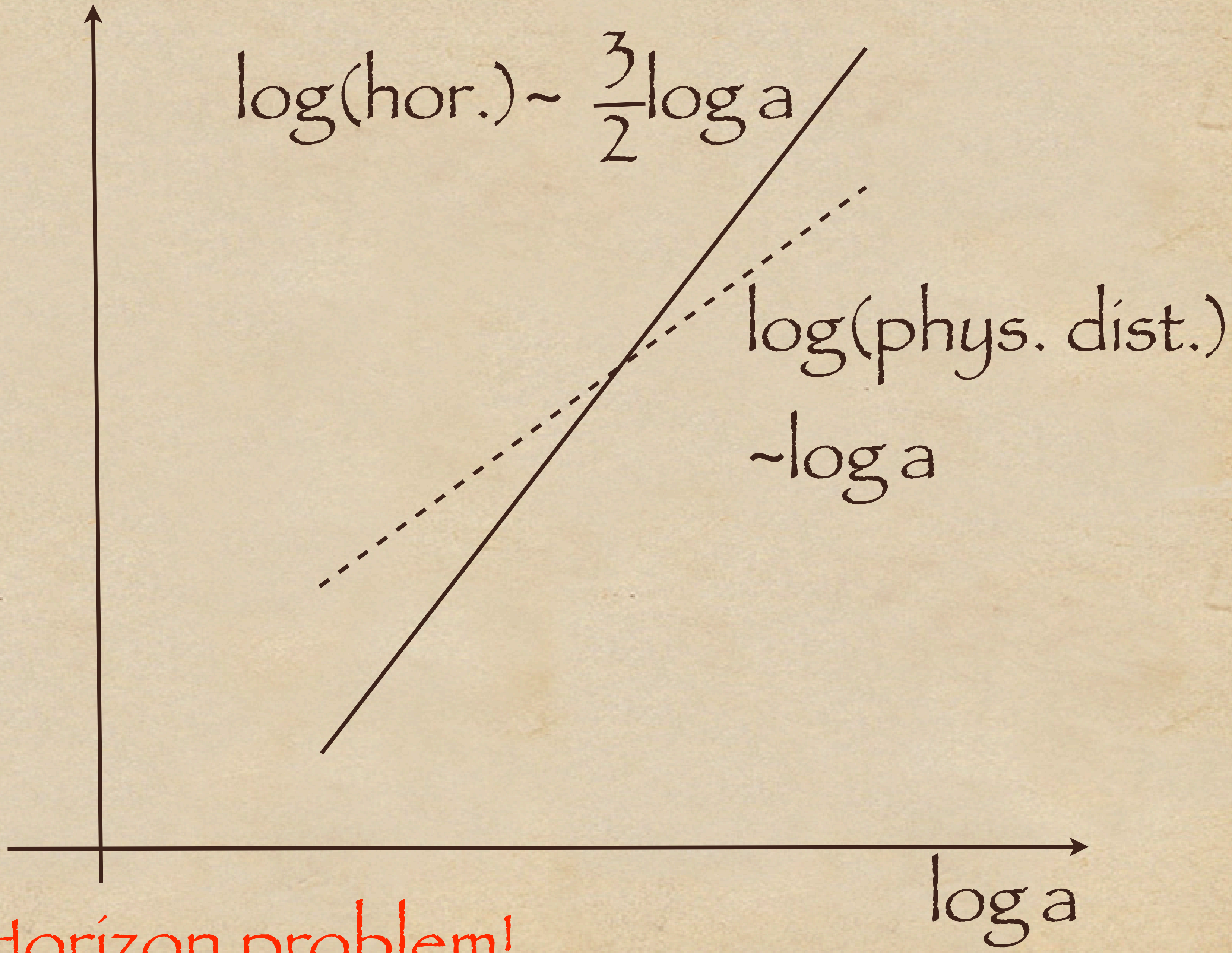
Horizon $= a(t)\int dx = a(t)\int_{t_{early}}^{t} c\,dt'/a(t')$

$$= 3ct\left(1 - \frac{t_{early}^{1/3}}{t^{1/3}}\right)$$

$$\sim 3ct \propto a^{3/2}$$

Contrast:

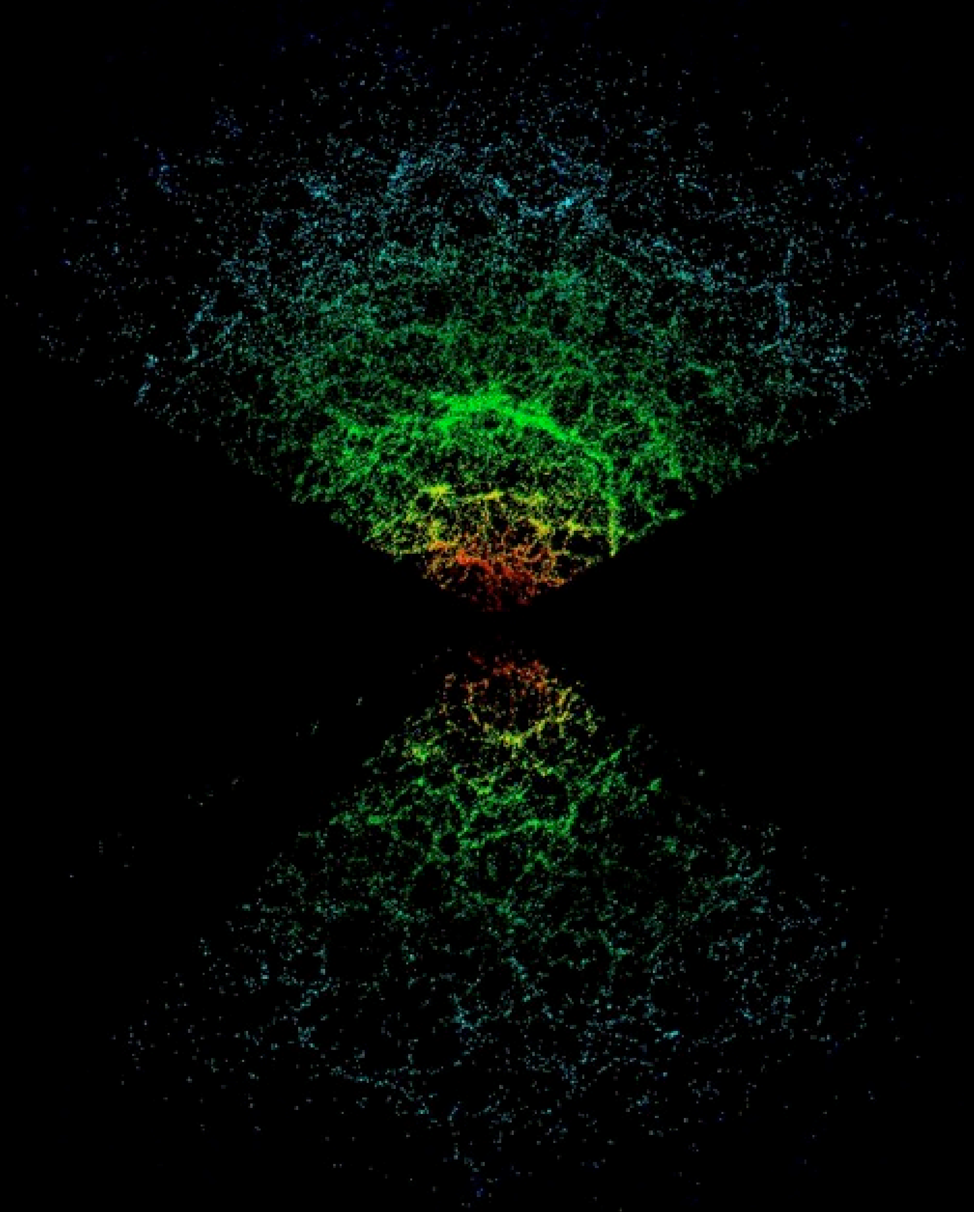
Physical distance btw. galaxies $\propto a$

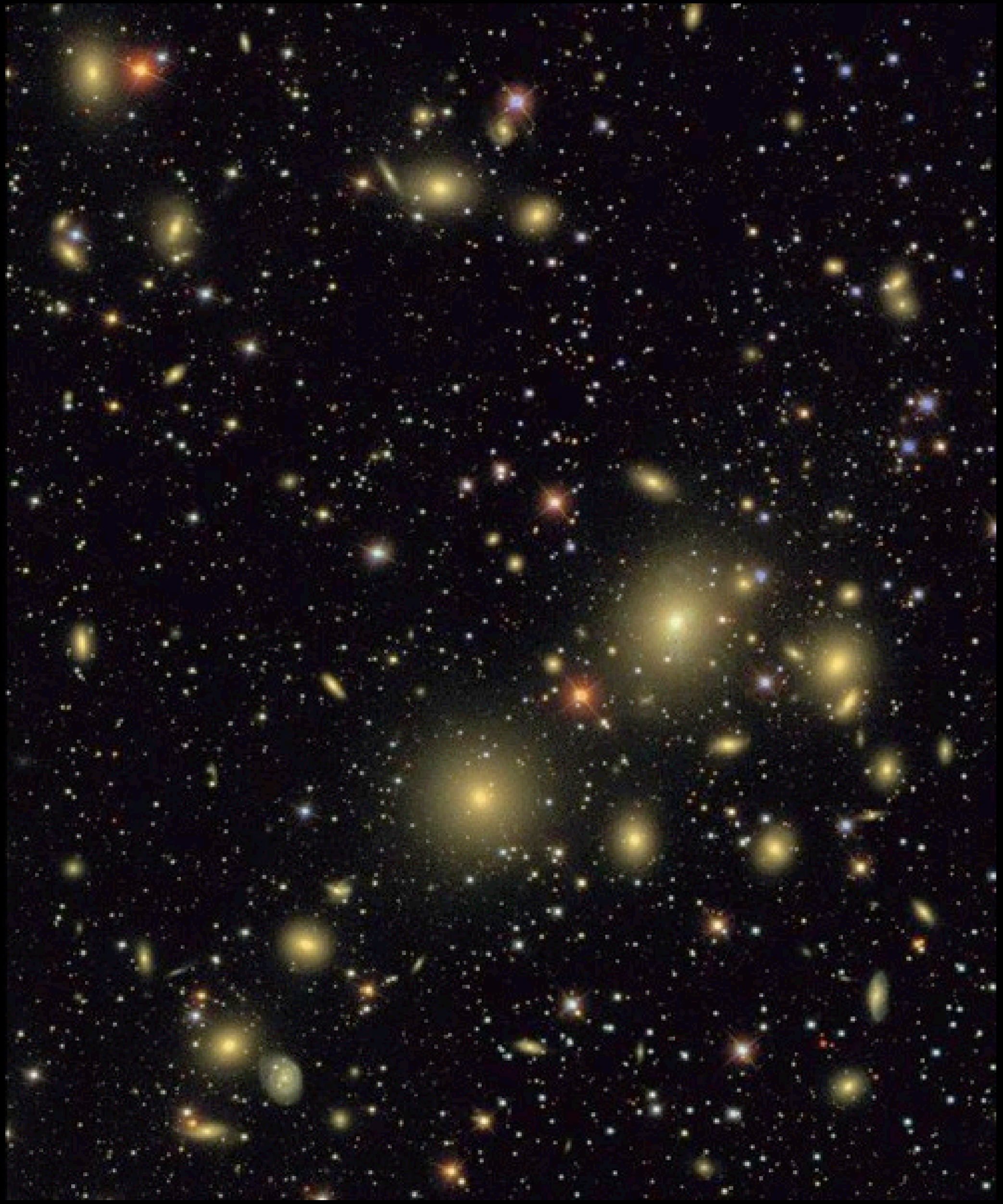


Horizon problem!

Sloan Digital Sky Survey

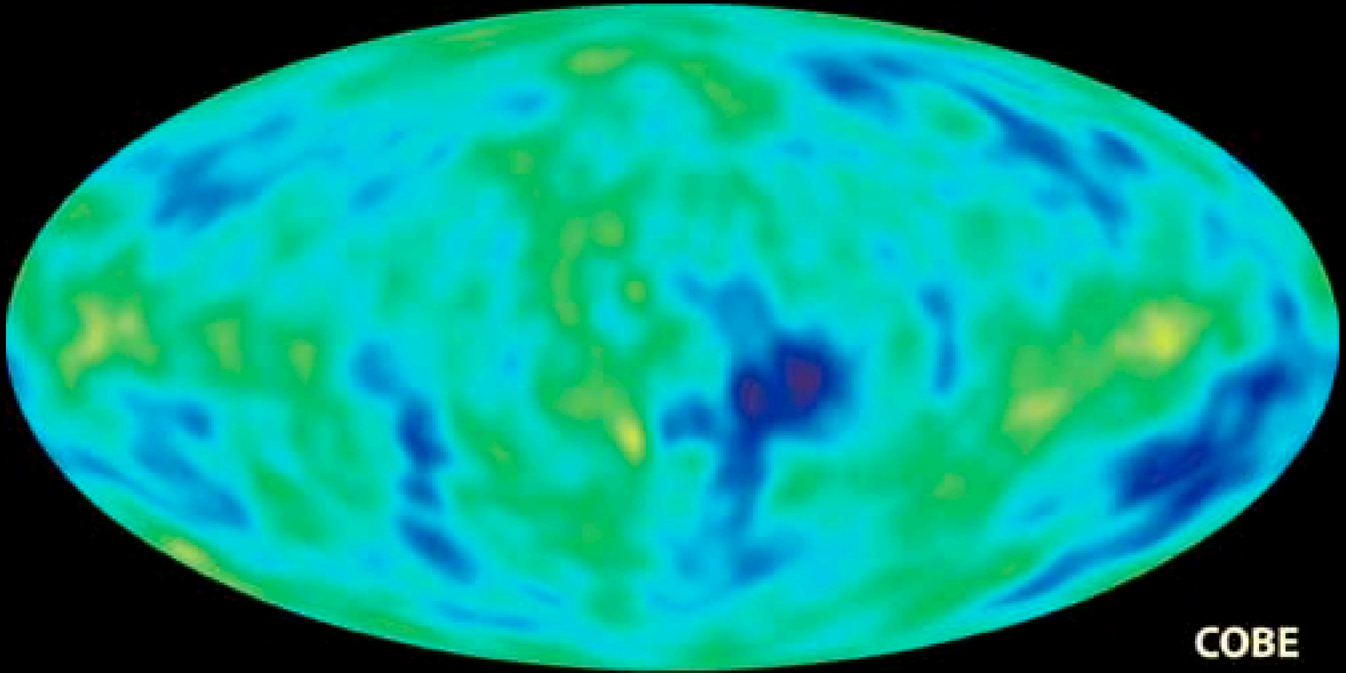
scale $\sim 10^{26}$ cm



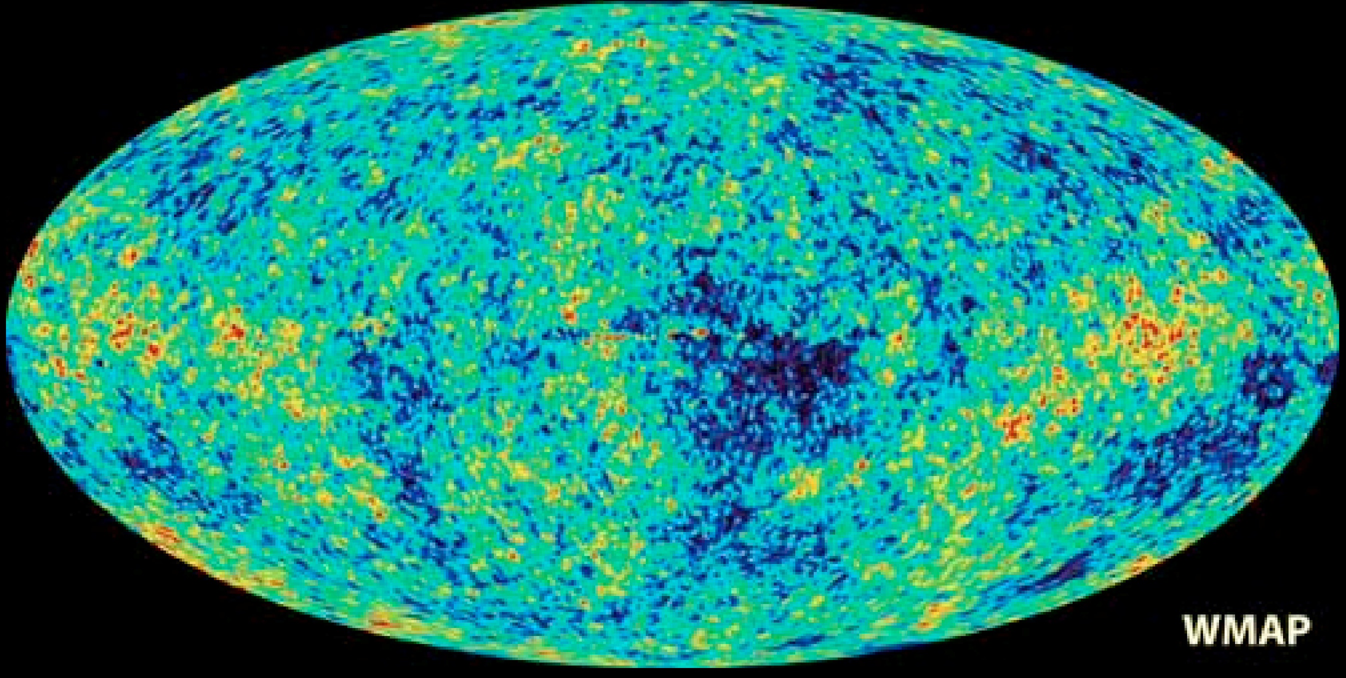


Horizon problem: 2 sides of the same coin

- On the scale of typical galaxy separations: how did the early universe know the density should be fairly similar?
- On the scale of typical galaxy separations: how did the early universe know the density should differ by a small amount?

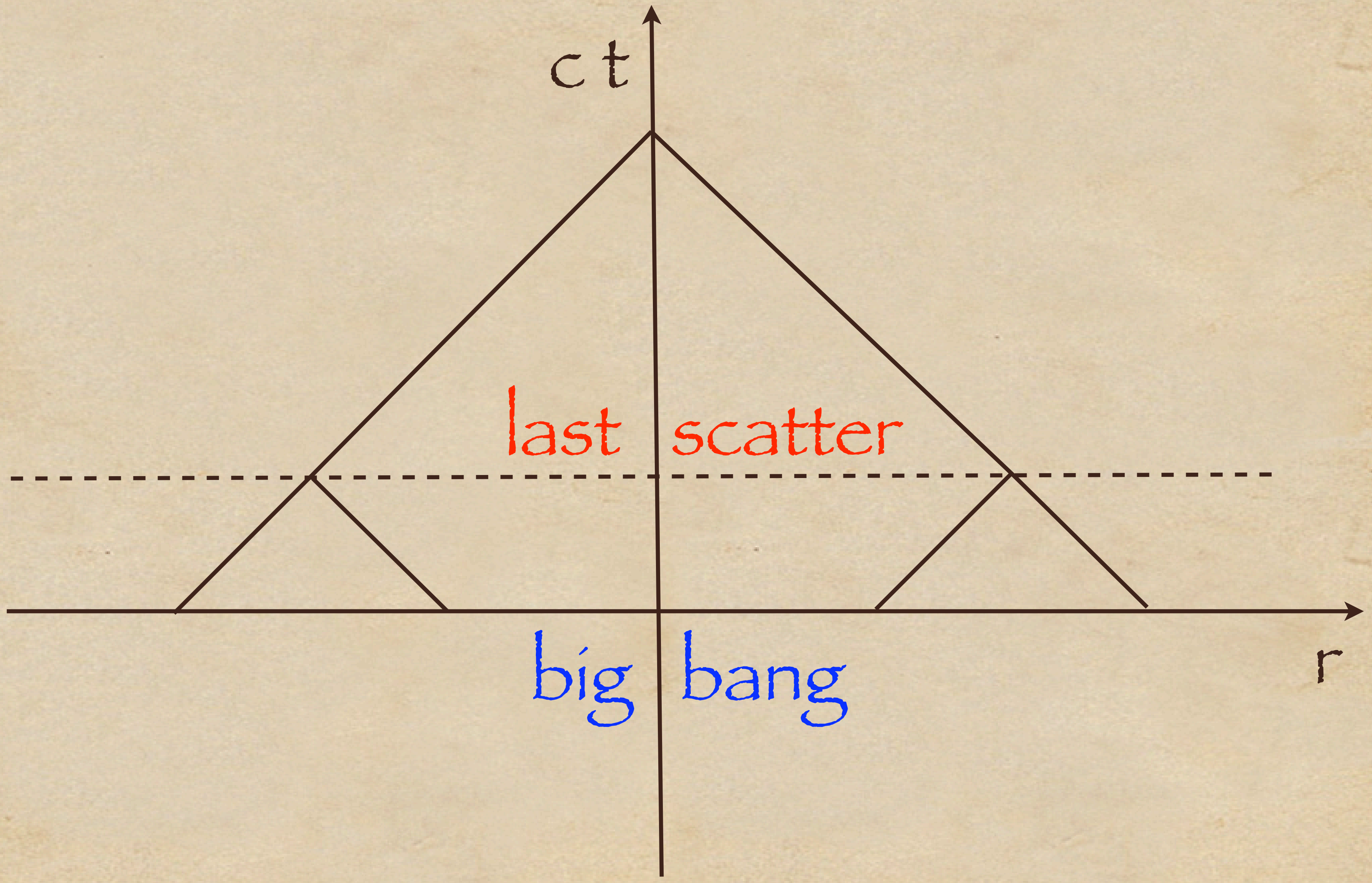


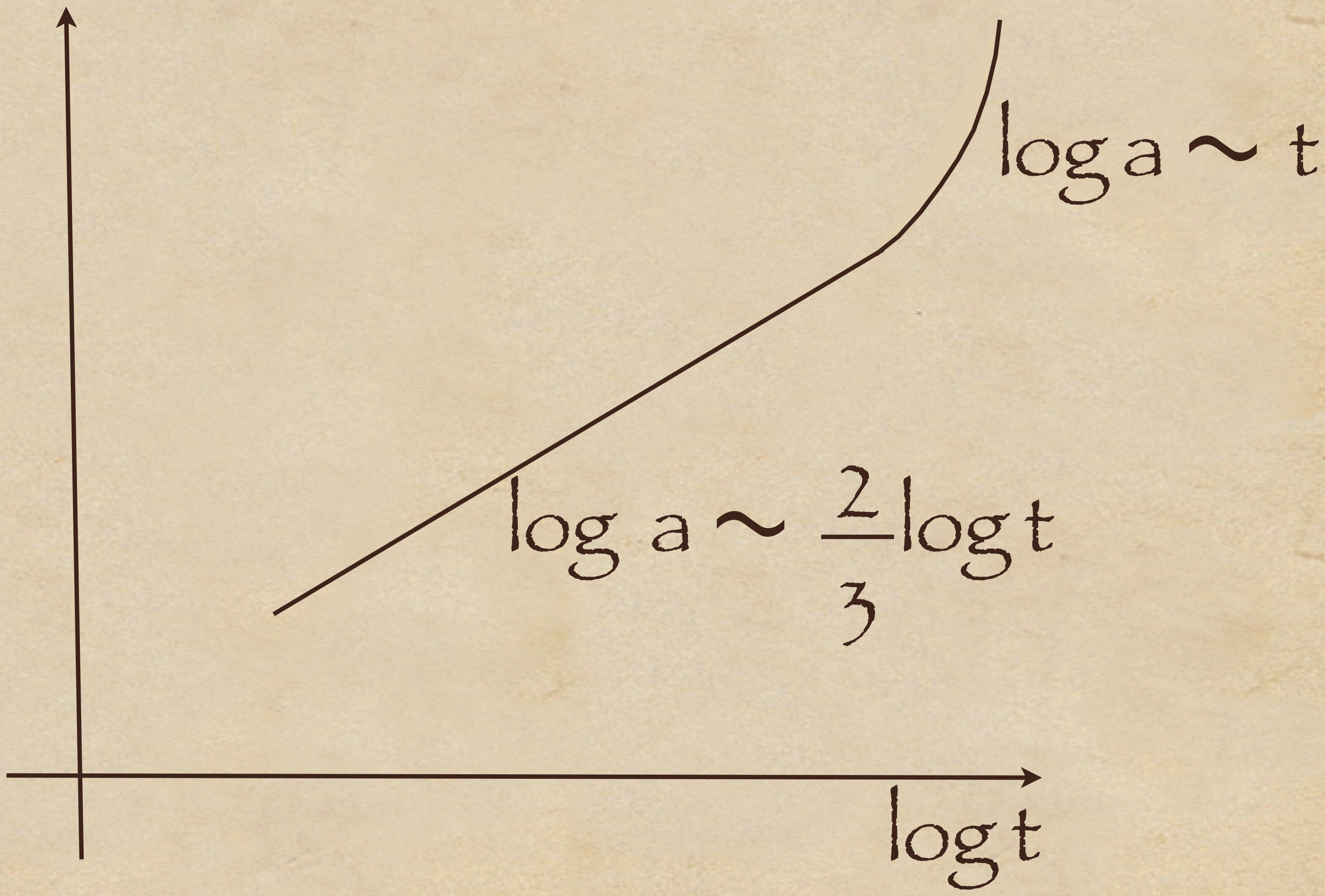
COBE

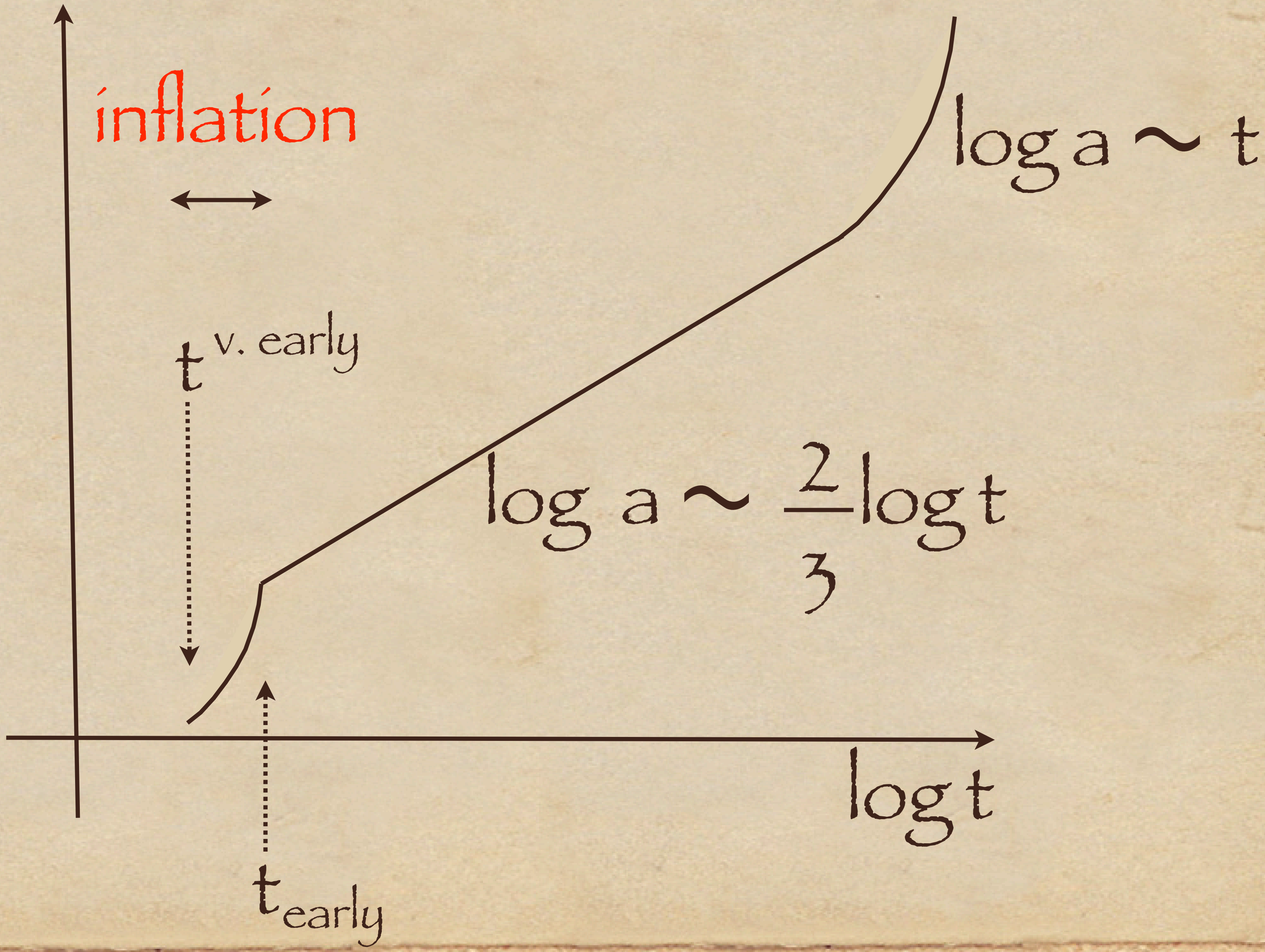


WMAP

Another way to view the horizon problem:




$\log a \sim t$
$\log a \sim \frac{2}{3} \log t$
$\log t$


inflation
$\log a \sim t$
$t^{\text{v. early}}$
$\log a \sim \frac{2}{3} \log t$
$\log t$
t_{early}

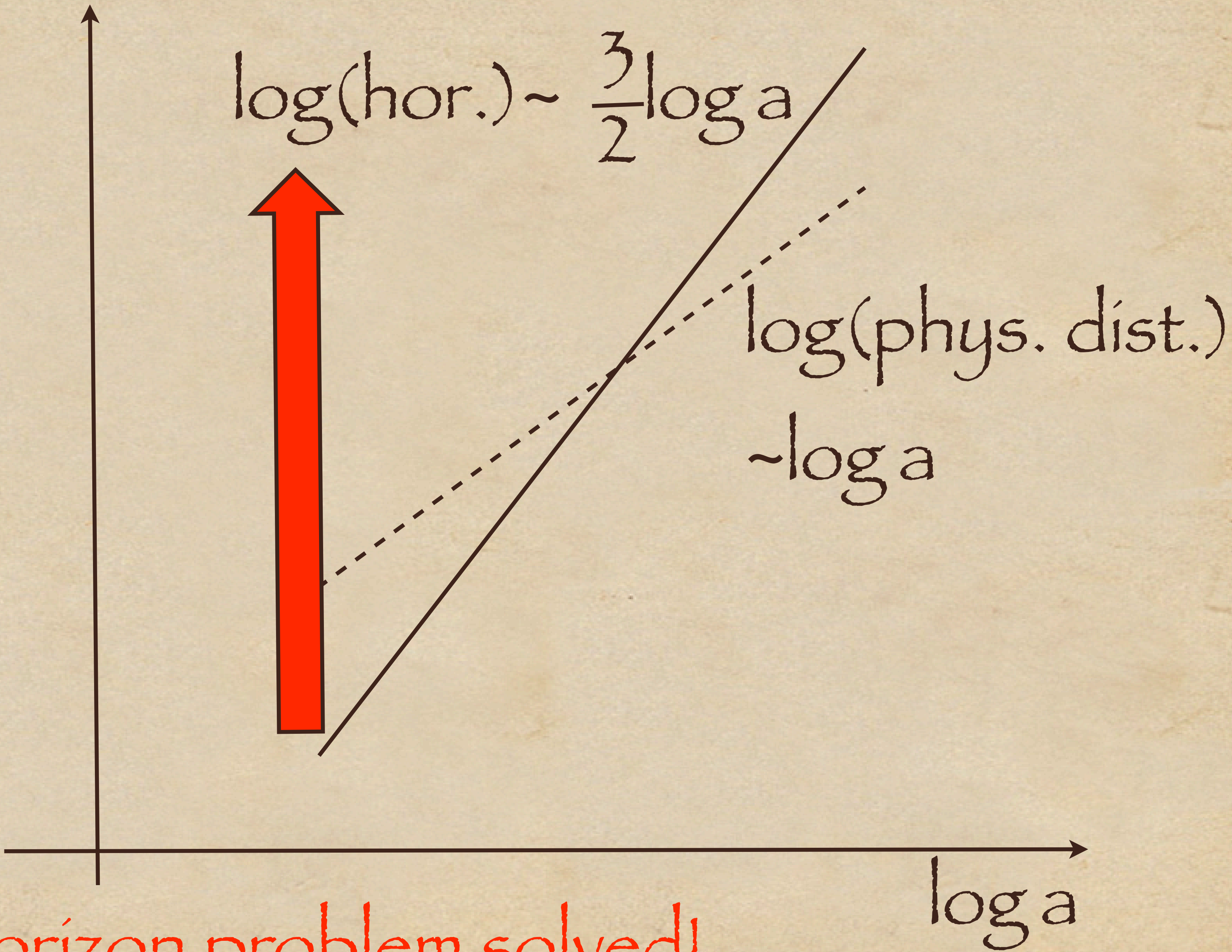
Horizon $\approx a(t)\int dx \approx a(t)\int_{t_{\text{v. early}}}^{t} c\, dt'/a(t')$

$$\approx a(t)\int_{t_{\text{v. early}}}^{t_{\text{early}}} c\, dt'/a(t') + a(t)\int_{t_{\text{early}}}^{t} c\, dt'/a(t')$$

$$\approx \frac{a(t)}{a(t_{\text{v. early}})}\,\frac{c}{H} + 3ct$$

Note: $a(t') \propto e^{Ht'}$ $\qquad$ $a(t') \propto t'^{2/3}$

$t_{\text{v. early}} < t' < t_{\text{early}}$ $\qquad$ $t_{\text{early}} < t'$



$\log(\text{hor.}) \sim \frac{3}{2}\log a$

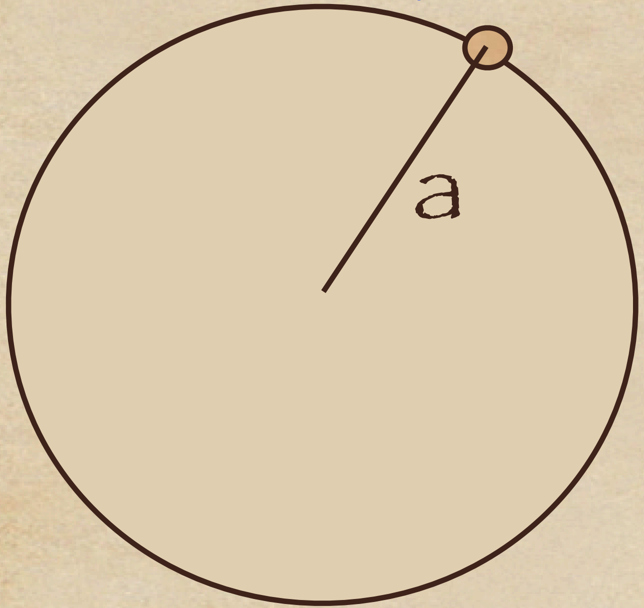
$\log(\text{phys. dist.})$

$\sim \log a$

$\log a$

Horizon problem solved!

Inflation's prediction for flatness



Energy conservation

$$\frac{1}{2}\dot{a}^2 - \frac{GM}{a} = E$$

$$M = 4\pi\rho a^3/3$$

$$1 - \frac{2GM}{a\dot{a}^2} = \frac{2E}{\dot{a}^2} \longrightarrow 0$$

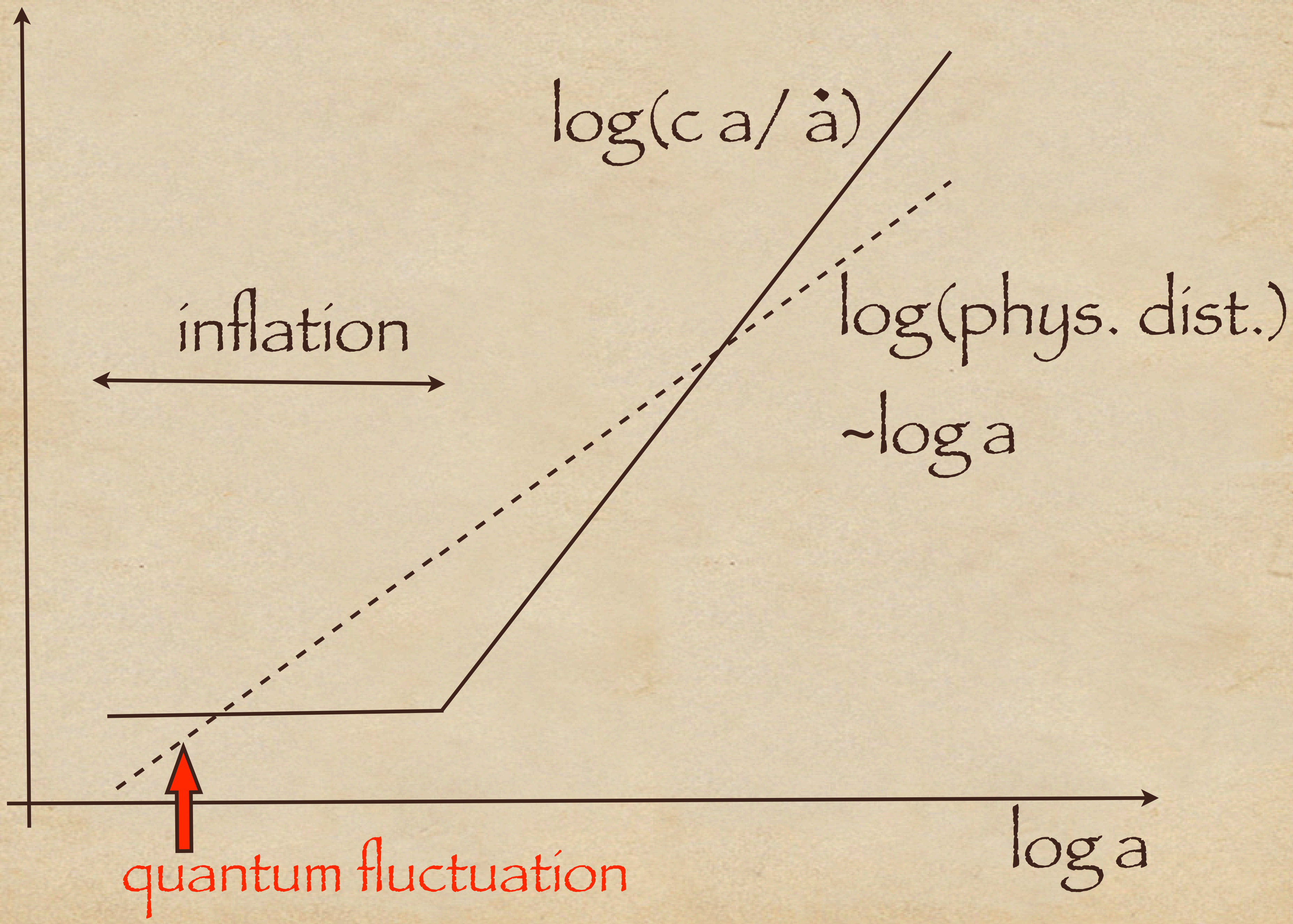
Data tell us $|2E/\dot{a}^2| < 0.03$.

Inflation's prediction for large scale structure

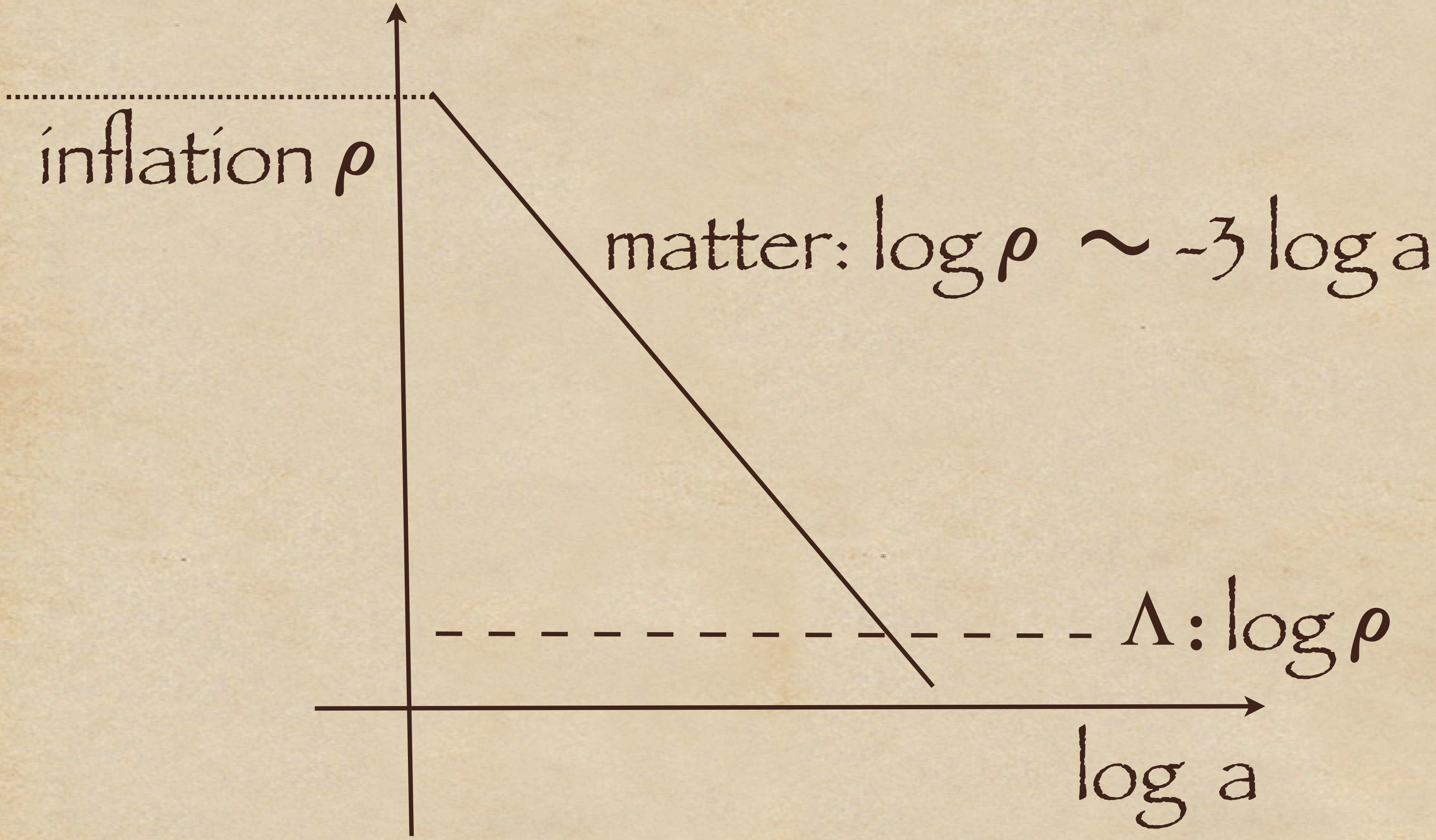
'Hawking' radiation from inflation seeds structure formation.

Inflation predicts nearly equal power on all scales: amplitude of fluctuations ∝ scale^{n-1}.

Data tell us n ~ 0.95 ± 0.02.


log(c a/ ȧ)
inflation
log(phys. dist.)
~log a
quantum fluctuation
log a

Inflation: problems

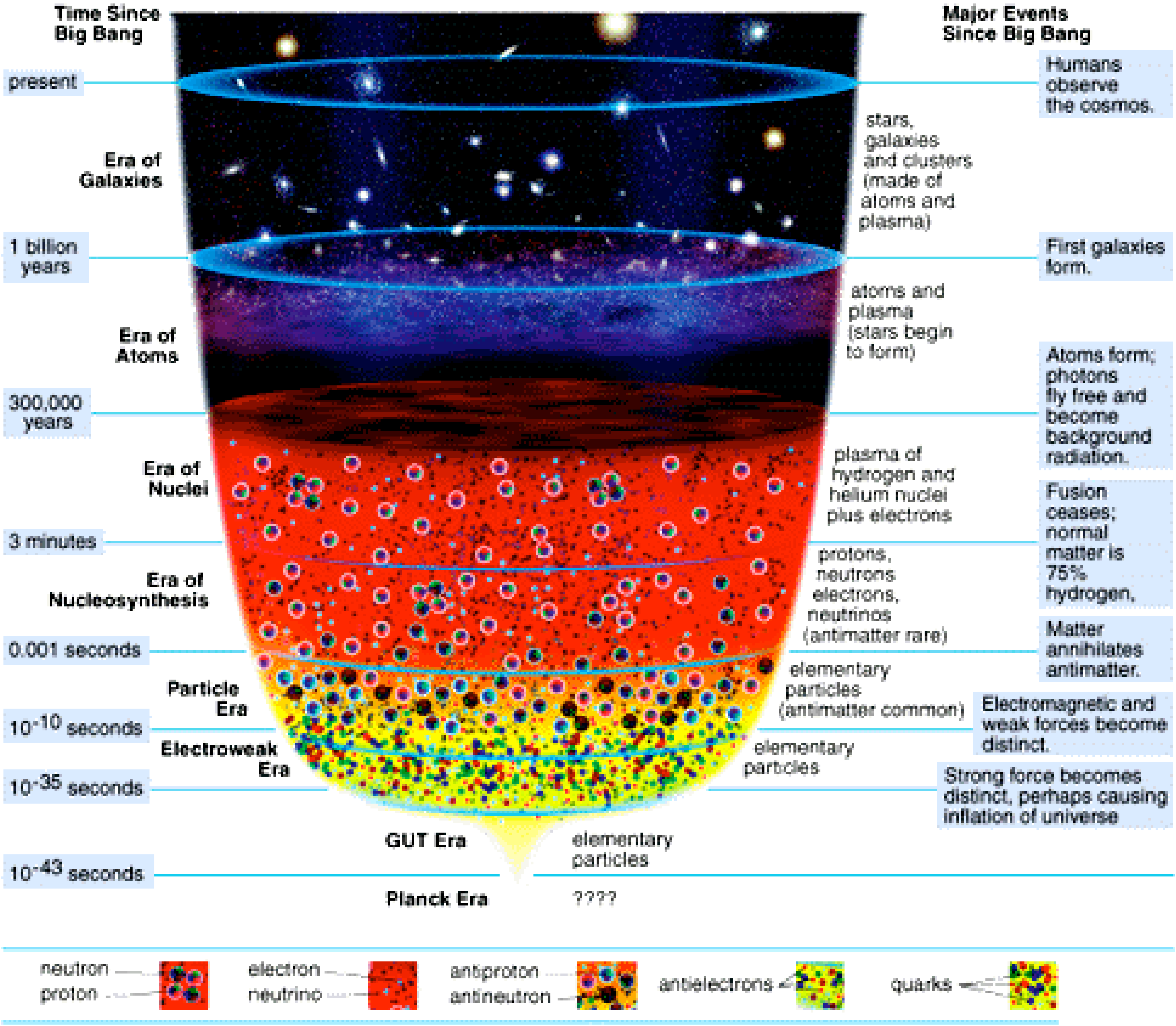


Problems of inflation:

Why is the ρ associated with inflation so constant?

Why did inflation stop?

Why did inflation start?



Time Since Big Bang

Major Events Since Big Bang

present — Humans observe the cosmos.

Era of Galaxies

stars, galaxies and clusters (made of atoms and plasma)

1 billion years — First galaxies form.

Era of Atoms

atoms and plasma (stars begin to form)

300,000 years — Atoms form; photons fly free and become background radiation.

Era of Nuclei

plasma of hydrogen and helium nuclei plus electrons

3 minutes — Fusion ceases; normal matter is 75% hydrogen.

Era of Nucleosynthesis

protons, neutrons electrons, neutrinos (antimatter rare)

0.001 seconds — Matter annihilates antimatter.

Particle Era

elementary particles (antimatter common)

10^{-10} seconds — Electromagnetic and weak forces become distinct.

Electroweak Era

elementary particles

10^{-35} seconds — Strong force becomes distinct, perhaps causing inflation of universe

GUT Era

elementary particles

10^{-43} seconds

Planck Era

????

neutron

proton

electron

neutrino

antiproton

antineutron

antielectrons

quarks