High-Energy Radiation Mechanisms (Continuum)

Cyclotron (non-relativistic)  Bremsstrahlung (breaking radiation)
Synchrotron (relativistic)  

Inverse Compton Scattering ($v_1 > v_0$)  Pair annihilation  
$hv \geq 511 \text{ keV}$

Blackbody Spectrum  
$E_{\text{max}} = 2.82 \text{ keV}$  
$k = 1.38 \times 10^{-16} \text{ erg/K}$

Particle Energy in Relativity  
$\beta = \frac{v}{c}$,  $\gamma = \frac{1}{\sqrt{1 - \beta^2}}$

$E^2 = p^2c^2 + m_0^2c^4$  
$E = \gamma m_0 c^2$  $p = \gamma m_0 v$  $K.E_1 = (\gamma - 1)m_0 c^2$  

$E = \gamma p c$  $\gamma = \frac{1}{\sqrt{1 - \beta^2}}$
A photon has zero rest mass, so $E=pc$ for a photon. A non-relativistic particle has $\beta \ll 1$, or $\gamma \\approx 1$. A highly relativistic particle has $\gamma \gg 1$, $\beta \approx 1$, $\gamma \gg 1$.

**Cyclotron Radiation (non-rel.)**

\[ F = q \left( \vec{v} \times \vec{B} \right) \text{ (SI)} \]

\[ F = \frac{q \left( \vec{v} \times \vec{B} \right)}{c} \text{ (cgs)} \]

\[ \alpha = \frac{q \gamma B}{mc} = \frac{\gamma^2}{v_B} \]

\[ \omega = \frac{v}{B} = \frac{qB}{mc} \] \text{ angular frequency (gyro-frequency)}

\[ r_g = \frac{mc}{qB} \] \text{ gyroradius}

Classically, the frequency of radiation is equal to $\frac{\omega_B \sqrt{c}}{2\pi}$, and the power emitted is $P = \frac{2q^2a^2}{3c^3}$ [erg/s].

This assumes that the charge radiates only a small fraction of its energy during one orbit, and that the photons radiated have much less energy than the kinetic energy of the charge ($\hbar \omega_B \ll \frac{1}{2}mv_B^2$).

Consider an electron of charge $e$. In cgs units:

\[ e = 4.8 \times 10^{-10} \text{ esu} \]

\[ m_e = 9.11 \times 10^{-28} \text{ g} \]

\[ h = 6.63 \times 10^{-27} \text{ erg \cdot s} \]

\[ \hbar = 1.05 \times 10^{-27} \text{ erg \cdot s} \]

(remember 1 eV = 1.602 \times 10^{-12} \text{ erg})
\[ E_c = \frac{\hbar \omega_B}{mc} = \frac{\hbar eB}{mc} = 1.84 \times 10^{-8} \text{erg} \quad \text{if } B = 10^{12} \text{ G} \]

or
\[ E_c = 11.6 \left( \frac{B}{10^{12}} \right) \text{ keV} \]

Taking into account quantum mechanics, remember that angular momentum is quantized in units of \( \hbar \).

\[ l = m_e v r g = n \hbar \]

or
\[ m_e v^2 = n \hbar \omega_B \]

Therefore
\[ m_e v \left( \frac{m_e c v}{eB} \right) > \hbar \]

Since \( v < c \),
\[ B < \frac{m_e^2 c^3}{e \hbar} = 4.4 \times 10^{13} \text{ G} \]

for electron cyclotron radiation to occur.

Conclusions: Majestic fields \( > 10^{10} \text{ G} \) are needed to have cyclotron radiation in the X-ray band, and the energy levels of the electron are quantized.

But if \( B > 4.4 \times 10^{13} \text{ G} \), the electron must be relativistic \((E > m_e c^2)\) to radiate.

Synchrotron Radiation - occurs when the electron is relativistic \((\gamma > 1)\). This modifies the gyrofrequency, emitted spectrum and the power. (Details to come later).

\[ \omega_B = \frac{eB}{\gamma mc}, \quad a = \frac{euvB}{\gamma mc}, \quad P = \frac{2e^2 q^2 \gamma^4}{3 c^3} \]

\[ v_s \approx \gamma^3 \omega_B \quad \text{if } v = c \]

\[ v_s = \frac{\gamma^2 e B}{mc} \]

or
\[ P = \frac{2e^4 \gamma^2 B^2}{3 c^3 m^2} \]
Cyclotron harmonics and quantized energy levels (non-rel.)

\[
\begin{align*}
3E_c & \quad \frac{3}{2}kW \\
2E_c & \quad \frac{5}{2}kW \\
E_c & \quad \frac{3}{2}kW \\
\frac{1}{2} & \quad \frac{1}{2}kW
\end{align*}
\]

\(kW = E_c\) is the fundamental, \(nE_c\) are the harmonics, energy levels are \((n+\frac{1}{2})kW\)

G.F. Bignami et al.: Out of the chorus line

Data and best fit continuum model

Fig. 2. The spectrum of 1E 1207.4-5209 as seen by the pn camera. The upper panel shows the data points together with the best fitting continuum model folded with the instrumental response. The lower panel shows the residuals in units of standard deviations from the best fitting continuum. The presence of four absorption features at \(\sim 0.7, \sim 1.4, \sim 2.1, \sim 2.8\) keV is evident.

Spectrum of a neutron star showing cyclotron absorption at fundamental energy \(E_c = 0.7\) keV, and harmonics at \(2E_c, 3E_c, 4E_c\).

What is the magnetic field strength?
Compton Scattering Cross Section

\[ \sigma_T = 6.65 \times 10^{-25} \text{ cm}^2 \]

Fig. 12-16. Graph of total scattering cross section for free electrons [Eq. 12-3(18)]. Extensive graphs relating to Compton scattering can be found in *Nat. Bur. Standards Circ.* 542, 1953.

any of the effects of coherence in the scattering of the electrons in a given atom, since all electrons are supposed to be free, independent, and distributed at random.

EXERCISE

12-22. Show that, as \( \epsilon \to 0 \), \( \sigma_\ast \to \sigma_0 \). Show also that, as \( \epsilon \to \infty \), \( \sigma_\ast \to \frac{3}{8} \sigma_0 \left( \frac{1}{\epsilon} \right) \left( \frac{1}{2} + \ln 2\epsilon \right) \).

In the observations of X-ray scattering at a given angle \( \theta' \), each nonochromatic "line" is observed to be scattered into two lines, one unmodified in frequency and the other modified according to the Compton formula (15). These are called unmodified scattering and modified, or Compton, scattering and are attributable in an approximate way to those electrons in the atom whose binding energies are respectively greater than or less than that energy represented by the change in frequency of the modified line. Thus, if the energy
Processes Contributing to the Absorption of High-Energy Photons

Figure 4.16. The total mass absorption coefficient for high energy photons in lead, indicating the contributions associated with the photoelectric absorption, Compton scattering and electron–positron pair production. (From H. A. Enge (1966). *Introduction to nuclear physics*, page 193, London: Addison-Wesley Publishing Co.)

![Graph showing absorption coefficients](image)

Figure 7.17. The relative importance of different forms of energy loss mechanisms for γ-rays as a function of photon energy and the atomic number of the material. (From R. Hiller (1984). *Gamma-ray astronomy*, page 52, Oxford: Clarendon Press.)

![Graph showing relative importance](image)
4. Interactions of high energy photons

$E_0$  
$E_0/2$  
$E_0/4$  
$E_0/8$  
$E_0/16$  

Figure 4.17. A simple model for an electromagnetic shower.

$R$ is the "radiation length," or average distance a particle or photon travels before interacting. The incident X-ray has $E = 10^{12}$ eV. After $10^4$ particles are created in the shower, those particles still have $\sim 10^5$ eV of energy each, and will be moving faster than the speed of light in air.

Speed of light in a medium is $c/n$, where $n$ is the index of refraction. For blue light in air, $n = 1.000296$, or $(n-1) = 3 \times 10^{-4}$. 

$$c/n = \left(1 - 3 \times 10^{-4}\right)c$$

An electron of $E = 10^5$ eV has $E = x m_e c^2$.

Since $m_e c^2 = 5 \times 10^{5}$ eV, $x = 200$, $\beta = \sqrt{1 - \frac{1}{x^2}}$, so $v = \left(1 - 1.2 \times 10^{-6}\right)c$. This is essentially $c$.

The speed of the electron $v$ is greater than $c/n$.

Note: I have used the binomial expansions for $E \ll 1$,

$$\frac{1}{1 + E} \approx 1 - E$$

and

$$\sqrt{1 - E} \approx 1 - \frac{E}{2}.$$
When a particle moves faster than the speed of light in a medium, it emits Čerenkov radiation, in a cone whose angle is determined by the speed of propagation of light relative to the speed of the particle.

**Example**

\[ \cos \theta = \frac{c}{c/n} \]

\[ n = 2, \quad v = c \]

\[ \theta = 60^\circ \]

![Diagram](image)

**Figure 4.19.** Illustrating Huygens' construction for determining the direction of propagation of the wavefront of Čerenkov radiation.

In our case,

\[ \frac{c}{c/n} = 1 - \frac{3 \times 10^{-4}}{1 - 1.2 \times 10^{-6}} \]

\[ \cos \theta = 0.9997, \quad \theta = 1.4^\circ \]

The "cone" is almost flat, a "pancake".

Its thickness is determined by the difference in speeds of the photons and the particles.

The pancake travels \( \approx 10 \) km

in a time \( t = \frac{z}{c} = \frac{10 \text{ km}}{3 \times 10^5 \text{ km/s}} = 3 \times 10^{-5} \text{s} \).

The photons travel at a speed \( (1 - 3 \times 10^{-4}) c = v \).

The particles travel at speed \( c \).

So the difference in their travelled distance is

\[ \Delta z = (c - v) t = \Delta v t. \]

The particles and photons are separated by at most \( \Delta z = \Delta v t \) or \( \Delta z = 3 \text{ m} \) when the shower is finished.

The photons arrive at the ground within \( 10^{-8} \) s.
Pair Production in Atmosphere

\[ \sigma_{\text{pair}} \approx 4 \times 10^{-25} \text{ cm}^2 \]

Since the atmosphere is mostly nitrogen, the mass absorption coefficient for pair production is estimated as

\[ \lambda_{\text{pair}} = \frac{\sigma_{\text{pair}}}{M \rho} = \frac{4 \times 10^{-25}}{14 \times 1.61 \times 10^{-24}} = 0.017 \text{ cm}^2 / \text{g} \]

The radiation length is then \( \frac{1}{\lambda_{\text{pair}}} = 58 \frac{\text{g}}{\text{cm}^2} \), and the total atmospheric depth is 1000 \( \frac{\text{g}}{\text{cm}^2} \).

The atmosphere's density is approximately exponential as a function of height

\[ \rho(z) = \rho_0 e^{-z/z_h} \quad \text{with} \quad \rho_0 = 1.2 \times 10^3 \frac{\text{g}}{\text{cm}^3}, \quad z_h = 8.6 \times 10^5 \text{ cm} \]

The height at which the incident x-ray converts to a pair is then approximated as \( h \), where

\[ \rho_0 \int_h^\infty e^{-z/z_h} \, dz = 58 \frac{\text{g}}{\text{cm}^2} = \frac{1}{\lambda_{\text{pair}}} \]

\[ \rho_0 z_h e^{-h/z_h} = \frac{1}{\lambda_{\text{pair}}} \]

\[ h = z_h \ln \left( \frac{1}{\lambda_{\text{pair}}} \rho_0 z_h \right) = 2.5 \times 10^6 \text{ cm} = 25 \text{ km} \]

The extensive air shower ends when the average energy per particle drops below 100 MeV. For an incident x-ray of energy 10^{12} eV, this corresponds to a maximum number of particles in the shower of 10^4. The number of radiation lengths \( m \) is given by the process in Figure 4.17 (page 10)

\[ 2^{(m-1)} = 10^4, \quad \text{or} \quad m = 14.3 \]

\[ m \lambda_{\text{pair}} = 14.3 \times 58 = 829 \frac{\text{g}}{\text{cm}^2} \]
The shower reaches a maximum number of particles at height \( h \) such that

\[
h = h_0 \ln \left( \frac{X_{\text{pair}} f_0 Z_h}{14.3} \right) = 1.9 \times 10^5 \text{ cm} = 1.9 \text{ km}
\]

Minimum Energy for Cherenkov Radiation

If the velocity of light in air is \( c/\text{m sec} \), an electron must have velocity \( v_{\text{min}} = c/\text{m sec} \) to radiate,

or

\[
X_{\text{min}} = \frac{1}{\sqrt{1 - (v_{\text{min}}/c)^2}} = \frac{1}{\sqrt{1 - (1/1.0003)^2}}
\]

where \( N_a = 1.0003 \), therefore, \( X_{\text{min}} = 41 \).

This corresponds to \( E_{\text{min}} = X_{\text{min}} M_e c^2 = 21 \text{ MeV} \).

The flash of Cherenkov photons arrives in a burst only 10 nanoseconds long, as derived on page 11, and spread over a radius of \( \sim 100 \text{ m} \). Several million photons can be produced, and because the burst is so short it can be discriminated from the background light.

VERITAS - Four 12 m telescopes - Mt Hopkins, AZ