Homework #6

1. Consider steady accretion through a thin disk with inner radius $r_{\text{in}}$ and outer radius $r_{\text{out}} >> r_{\text{in}}$.

   (a) Use the expression for surface flux as a function of radius

   \[ F(r) = \frac{3}{8\pi} \frac{GM\dot{M}}{r^3} \left[ 1 - \left( \frac{r_{\text{in}}}{r} \right)^{3/2} \right] \]

   to derive an expression for the total luminosity emitted by the disk.

   (b) Prove that the disk is brightest at $r = (49/36)r_{\text{in}}$.

2. (a) Calculate the maximum temperature in a blackbody accretion disk for a non-rotating (Schwarzschild) black hole of $7 \, M_\odot$, and for a $1.4 \, M_\odot$ neutron star, each accreting at its Eddington limit. Assume in the case of the neutron star that the disk also extends to its ISCO, $r_{\text{in}} = 6GM/c^2$.

   (b) What is interesting about the result of this problem?

3. Using the expression for apparent superluminal velocity,

   \[ \beta_{\text{app,} \perp} = \frac{\beta \sin \theta}{\Gamma - \beta \cos \theta}, \]

   show that the maximum possible value of $\beta_{\text{app,} \perp}$ is $\beta \Gamma$, where

   \[ \Gamma = \frac{1}{\sqrt{1 - \beta^2}}. \]

4. Show that as $\beta \to 1$ the maximum possible Doppler shift is

   \[ \left( \frac{\nu}{\nu_0} \right)_{\text{max}} = 2 \Gamma. \]

5. In the article “Sources of Relativistic Jets in the Galaxy” by Mirabel and Rodriguez, derive equations (3)-(5) from equations (1) and (2). (These are also in the Lecture notes for March 23: Kinematics of Relativistic Jets). Hint: multiply and divide equations (1) and (2).