1. In Problem 3(b) of Homework #7, electrons have $10^2 - 8 	imes 10^5$. They see photons of energy 28 eV in the frame of the jet, and scatter them to energies

$$h\nu' = \frac{4}{3} \nu^2 \nu$$

where $h\nu' = 28$ eV

Therefore $3.7 \times 10^5 \leq h\nu \leq 3.7 \times 10^6$ eV. But the highest energy of the electrons is $\gamma mc^2 = 10^5 \times 5 \times 10^4 = 5 \times 10^{10}$ eV. The scattered photon can't have energy greater than that of the electron. The flaw is in the formula $\nu = \frac{\sqrt{4\pi h\nu}}{3 \hbar}$ which is only valid in the limit where the incident photon in the rest frame of the electron has energy $\leq mc^2$.

2. (a) $h\nu_3 = \frac{3 \hbar^2 \omega_B}{4\pi} \frac{1}{mc}$

where $\beta = 10^{-7}$, $\nu_3 = \frac{10^9 \times 6.6 \times 10^{-27}}{6.6 \times 10^{-27}}$

$$\gamma = \frac{h\nu_3}{mc}$$

$$\gamma mc^2 = \frac{\sqrt{3 \hbar \nu_3 \omega_B}}{\hbar} = 62 \text{ ergs} = 3.9 \times 10^{13} \text{ eV}$$

(b) $r_s = \frac{\gamma mc^2}{\omega_B} = \frac{62}{1.3 \times 10^{15} \frac{10}{4.8 \times 10^{-15}}} = \frac{1.3 \times 10^{15}}{4.8 \times 10^{-15}}$

which is smaller than the thickness.

(c) The energy gained in one round trip is $E = E_0 (1 + \frac{1}{2} \beta)$ where $\beta$ is the shock velocity. Assume an average velocity over the age of the remnant $V = V_3/t = 2 \times 10^8 / 3 \times 10^{10} = 6300 \text{ km s}^{-1}$. Then $\beta = 0.021$. The number of round trips for the particle energy to increase from $10^4$ eV to $3.9 \times 10^{13}$ eV is $k$, where

$$1 + \frac{1}{2} \beta = \frac{3.9 \times 10^{13}}{10^4}$$

$$k = \log \left( \frac{3.9 \times 10^9}{1.021} \right) = 1062$$
10^3 round trips means an average of 1 per year, during which a particle travels 10^{18} cm at the speed c, but staying within 10^{17} cm of the shock front. This is possible for a diffusion process, in which there are many scatterings per round trip.

Extra details - not required for this problem, but very interesting

**Diffusion and Random Walks** - how far does a particle travel after scattering N times? Let the mean free path be \( l \).

The total distance travelled is \( R \), where

\[
R = (\vec{r}_1 + \vec{r}_2 + \cdots + \vec{r}_N)
\]

where \( \langle r_i \rangle = l \)

\[
R^2 = (\vec{r}_1 + \vec{r}_2 + \cdots + \vec{r}_N) \cdot (\vec{r}_1 + \vec{r}_2 + \cdots + \vec{r}_N)
\]

dot-product

\[
R^2 = r_1^2 + r_2^2 + r_3^2 + \cdots + r_N^2
\]

cross-terms like \( \vec{r}_1 \cdot \vec{r}_2 \) cancel on average, being both positive and negative. This is the key to random walk diffusion.

\[
R = \sqrt{N} l
\]

The number of scatterings in time \( t \) is

\[
N = \frac{ct}{l}
\]

for a relativistic particle with velocity \( c \).

so \( R = \sqrt{ct} l \)

In a dilute plasma, the magnetic field dominates the mean-free path, and the fastest diffusion is when \( l \approx r_0/3 \). This is called Bohm diffusion.

In Problem 2 we have \( t = 10^3 \) year, \( r_0 = 1.3 \times 10^{15} \) cm, so

\[
R = \sqrt{3 \times 10^{10} \times 10^3 \times 3 \times 10^7 \times 1.3 \times 10^{15} / 3} = 6 \times 10^{17} \) cm

This is the maximum distance a particle can diffuse. It is about 3% of the radius of the SNR.
Another consideration is how fast the electron can radiate. The synchrotron lifetime is

$$\tau_{\text{synch}} = \frac{E}{dE/d\tau} = \frac{\gamma m_e c^2}{\frac{4}{3} \sqrt{\pi} \gamma^2 B^2} = \frac{7.7 \times 10^8}{8 \times 10^2} \text{ s}$$

In this case $\gamma = \frac{3.9 \times 10^{13} \text{ eV}}{5 \times 10^3 \text{ eV}} = 7.6 \times 10^7$, and $B = 10^{-4} \text{ G}$, so $\tau_{\text{synch}} = 10^9 \text{ s} = 30 \text{ yr}$.

Radiation will limit the highest energy that can be given to the electron, but not to the protons. The maximum energy to which protons can be accelerated in a SNR is $\approx 10^{15} \text{ eV}$, because at this energy the gyroradius is $3 \times 10^{16} \text{ cm}$ and the diffusion length in $10^3$ years is $R \approx 3 \times 10^{11} \text{ cm}$, comparable to the radius of the SNR, and they escape.

3 (a) Estimate the explosion energy from the mass defect of $^{56}\text{Ni}$ compared to $^{12}\text{C}$, starting with $1\text{ MD} = 2 \times 10^{33} \text{ g}$ of $^{12}\text{C}$.

$$E = 2 \times 10^{33} \left( \frac{56 - 55.942134}{56} \right) (3 \times 10^{10})^2 = 1.85 \times 10^{51} \text{ ergs}$$

This energy must overcome the binding energy of the white dwarf, which is

$$U = - \frac{3}{10} \frac{GM^2}{R}$$

with $M = 2 \times 10^{33} \text{ g}$, $R = 2 \times 10^8 \text{ cm}$.

$$U = - 4 \times 10^{50} \text{ ergs}$$

This means that $E - 4 \times 10^{50} \text{ ergs}$ is available for kinetic energy of the explosion, or $K = 1.45 \times 10^{51} \text{ ergs}$.

$$v = \sqrt{\frac{2K}{M}} = \sqrt{2 \times 1.45 \times 10^{51}} = 1.2 \times 10^9 \text{ cm/s} = 12,000 \text{ km/s}$$
\[ \begin{align*}
\text{Co} & \rightarrow \text{Fe} \\
E &= 2 \times 10^{33} \left( \frac{55.942134 - 55.939841}{56} \right) \times 9.4 \times 10^{20} \\
&= 7 \times 10^{49} \text{ eV} \\
E &= 1.6 \times 10^{50} \text{ eV}
\end{align*} \]

To be more accurate (for neutrino losses), you can use 1.7 MeV per \( {}^{56}\text{Ni} \) decay and 3.7 MeV per \( {}^{56}\text{Co} \) decay, with the number of nuclei being

\[ \frac{2 \times 10^{33}}{56 \times 1.66 \times 10^{-24}} = 2.15 \times 10^{55} \]

Then \( E \left( {}^{56}\text{Ni} \rightarrow {}^{56}\text{Co} \right) = 2.15 \times 10^{55} \times 1.7 \times 10^{6} = 3.66 \times 10^{61} \text{ eV} \)

\[ \approx 5.9 \times 10^{49} \text{ eV} \]

\( E \left( {}^{56}\text{Co} \rightarrow {}^{56}\text{Fe} \right) = 2.15 \times 10^{55} \times 3.7 \times 10^{6} = 8.0 \times 10^{61} \text{ eV} \)

\[ \approx 1.3 \times 10^{50} \text{ eV} \]