(1998)

—accelerating

—$\Omega_M = 1$

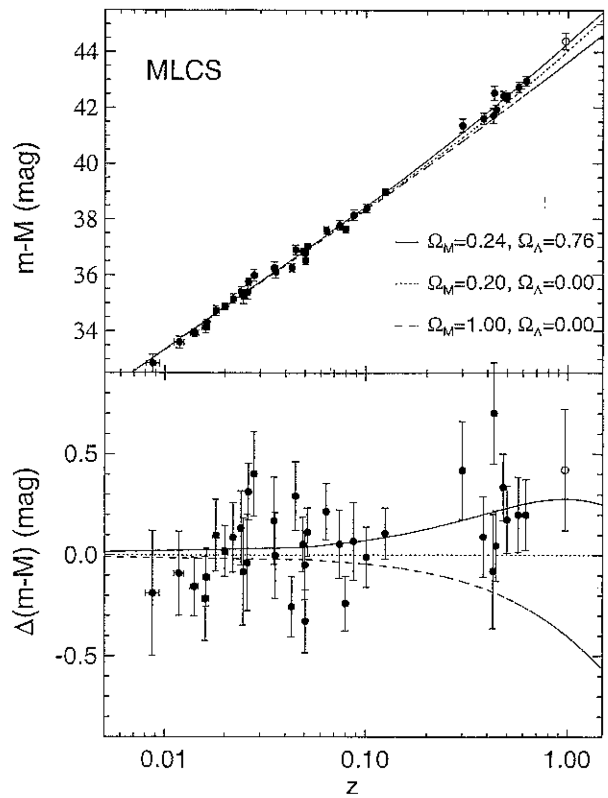

FIG. 4.—MLCS SNe Ia Hubble diagram. The upper panel shows the Hubble diagram for the low-redshift and high-redshift SNe Ia samples with distances measured from the MLCS method (Riess et al. 1995, 1996a; Appendix of this paper). Overplotted are three cosmologies: "low" and "high" Ω_M with $\Omega_\Lambda = 0$ and the best fit for a flat cosmology, $\Omega_M = 0.24$, $\Omega_\Lambda = 0.76$. The bottom panel shows the difference between data and models with $\Omega_M = 0.20$, $\Omega_\Lambda = 0$. The open symbol is SN 1997ck ($z = 0.97$), which lacks spectroscopic classification and a color measurement. The average difference between the data and the $\Omega_M = 0.20$, $\Omega_\Lambda = 0$ prediction is 0.25 mag.

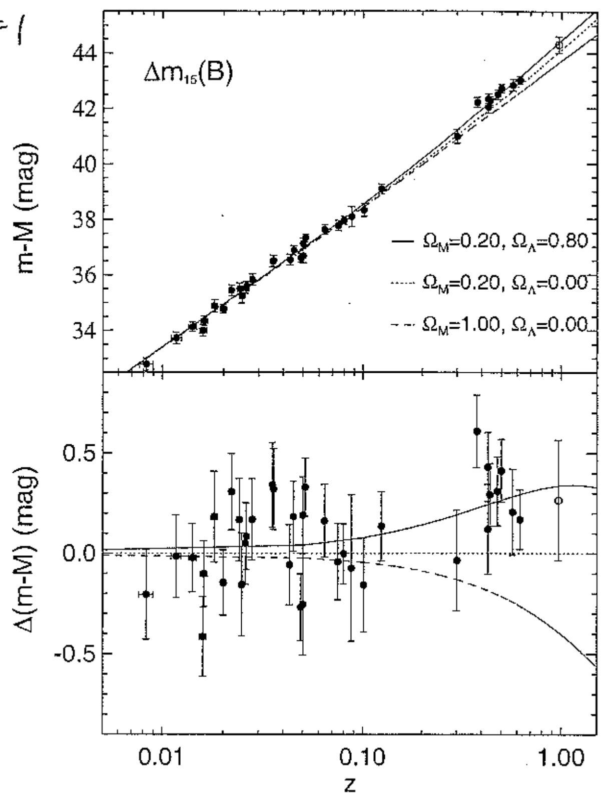

FIG. 5.—$\Delta m_{15}(B)$ SN Ia Hubble diagram. The upper panel shows the Hubble diagram for the low-redshift and high-redshift SNe Ia samples with distances measured from the template-fitting method parameterized by $\Delta m_{15}(B)$ (Hamuy et al. 1995, 1996d). Overplotted are three cosmologies: "low" and "high" Ω_M with $\Omega_\Lambda = 0$ and the best fit for a flat cosmology, $\Omega_M = 0.20$, $\Omega_\Lambda = 0.80$. The bottom panel shows the difference between data and models from the $\Omega_M = 0.20$, $\Omega_\Lambda = 0$ prediction. The open symbol is SN 1997ck ($z = 0.97$), which lacks spectroscopic classification and a color measurement. The average difference between the data and the $\Omega_M = 0.20$, $\Omega_\Lambda = 0$ prediction is 0.28 mag.

The present data set has only a modest range of redshifts, so we can only constrain specific cosmological models or regions of $(\Omega_M, \Omega_\Lambda)$ parameter space to useful precision.

The χ^2 statistic of equation (4) is well suited for determining the most likely values for the cosmological parameters H_0, Ω_M, and Ω_Λ as well as the confidence intervals surrounding them. For constraining regions of parameter space not bounded by contours of uniform confidence (i.e., constant χ^2), we need to define the probability density function (PDF) for the cosmological parameters. The PDF (p) of these parameters given our distance moduli is derived from the PDF of the distance moduli given our data from Bayes's theorem,

$$p(H_0, \Omega_m, \Omega_\Lambda \mid \mu_0) = \frac{p(\mu_0 \mid H_0, \Omega_m, \Omega_\Lambda) p(H_0, \Omega_m, \Omega_\Lambda)}{p(\mu_0)} , \tag{5}$$

where μ_0 is our set of distance moduli (Lupton 1993). Since we have no prior constraints on the cosmological parameters (besides the excluded regions) or on the data, we take $p(H_0, \Omega_m, \Omega_\Lambda)$ and $p(\mu_0)$ to be constants. Thus, we have for the allowed region of $(H_0, \Omega_m, \Omega_\Lambda)$

$$p(H_0, \Omega_m, \Omega_\Lambda \mid \mu_0) \propto p(\mu_0 \mid H_0, \Omega_m, \Omega_\Lambda) . \tag{6}$$

We assume each distance modulus is independent (aside from systematic errors discussed in § 5) and normally distributed, so the PDF for the set of distance moduli given the parameters is a product of Gaussians:

$$p(\mu_0 \mid H_0, \Omega_m, \Omega_\Lambda) = \prod_i \frac{1}{\sqrt{2\pi(\sigma^2_{\mu_{0,i}} + \sigma^2_v)}} \times \exp\left\{ - \frac{[\mu_{p,i}(z_i; H_0, \Omega_m, \Omega_\Lambda) - \mu_{0,i}]^2}{2(\sigma^2_{\mu_{0,i}} + \sigma^2_v)} \right\} . \tag{7}$$

Rewriting the product as a summation of the exponents and combining with equation (4), we have

$$p(\mu_0 \mid H_0, \Omega_m, \Omega_\Lambda) = \left[\prod_i \frac{1}{\sqrt{2\pi(\sigma^2_{\mu_{0,i}} + \sigma^2_v)}} \right] \exp\left(- \frac{\chi^2}{2} \right) . \tag{8}$$

The product in front is a constant, so combining with equation (6) the PDF for the cosmological parameters yields the standard expression (Lupton 1993)

$$p(H_0, \Omega_m, \Omega_\Lambda \mid \mu_0) \propto \exp\left(- \frac{\chi^2}{2} \right) . \tag{9}$$