Astronomy UN2002 (Introduction to Astrophysics II) Spring 2020
Mon. Feb. 10 Prof. J. Halpern

Rotation Curve of the Galaxy and the Oort Constants

The following is a summary of the derivations and key results in Sections 19.5 and 19.6 of Ryden & Peterson, for the February 10 class.

Rotation Curve

Using the notation in Figure 1, the *observed* radial velocity v_r of a star or gas cloud is

$$v_r = \Theta \cos\alpha - \Theta_0 \cos(90^\circ - \ell) = \Theta \cos\alpha - \Theta_0 \sin\ell \tag{1}$$

where ℓ is Galactic longitude. Use the law of sines,

$$\frac{\sin(90^\circ + \alpha)}{R_0} = \frac{\sin\ell}{R},$$

which becomes

$$\cos\alpha = \frac{R_0}{R}\sin\ell. \tag{2}$$

Substitute Equation (2) into Equation (1), giving

$$v_r = R_0\left(\frac{\Theta}{R} - \frac{\Theta_0}{R_0}\right)\sin\ell \tag{3}$$

or

$$v_r = R_0(\omega - \omega_0)\sin\ell. \tag{4}$$

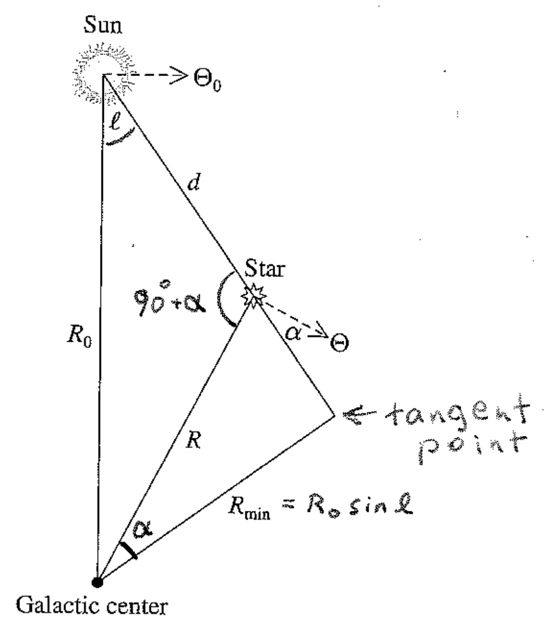

Figure 1

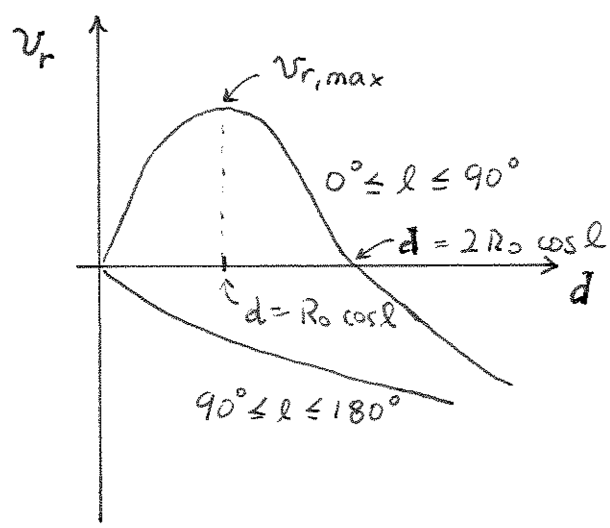

Figure 2

If ω increases with decreasing Galactocentric radius R, then for any ℓ in the first quadrant ($0^\circ \leq \ell \leq 90^\circ$), v_r reaches a maximum at the **tangent point** in Figure 1. A graph of v_r versus distance d is shown in Figure 2. Thus, we can measure the rotation curve $\Theta(R)$ by measuring the maximum value of v_r at each longitude, e.g., from the 21 cm line of H I, assuming that there is gas at the tangent point. Since $\omega = \Theta/R$ and $R = R_0 \sin\ell$ at the tangent point, Equation (4) becomes

$$\Theta(R) = v_{r,\max} + R_0\,\omega_0 \sin\ell. \tag{5}$$

In the fourth quadrant ($270^\circ \leq \ell \leq 360^\circ$), $\sin\ell$ is negative, and the sign of v_r is reversed from its values in the first quadrant, reaching a minimum at the tangent point. In the second and third quadrants (the outer Galaxy: $90^\circ \leq \ell \leq 270^\circ$), there is no $v_{r,\max}$ or $v_{r,\min}$ (see example in Figure 2), so to convert v_r into a rotation curve there, the distance to the star or cloud has to be measured directly. For example, the spectroscopic parallax of a star gives its distance, which, in combination with velocity from a molecular cloud associated with the star, can be used to solve for Θ in Equation (3).

Tangential velocity v_t (of stars only) can be measured via proper motion μ, where $v_t = \mu\, d$. Referring to Figure 1 it can be seen that

$$v_t = \Theta \sin\alpha - \Theta_0 \cos\ell = \omega\, R \sin\alpha - \omega_0\, R_0 \cos\ell.$$

But $R \sin\alpha = R_0 \cos\ell - d$, so

$$v_t = (\omega - \omega_0)\, R_0 \cos\ell - \omega\, d. \tag{6}$$

Equations (4) and (6) are the **Oort Equations**.

Application of the Oort Equations to stars in the solar neighborhood can be simplified in the approximation $d << R_0$, i.e, for stars within a few hundred parsecs of the Sun. Expand $\omega(R)$ in a Taylor series around $R = R_0$,

$$\omega(R) = \omega_0 + \left(\frac{d\omega}{dR}\right)_{R=R_0} (R - R_0) + \frac{1}{2}\left(\frac{d^2\omega}{dR^2}\right)_{R=R_0} (R - R_0)^2 + \ \ldots$$

Keeping terms only up to the first derivative, rewrite Equation (4) as

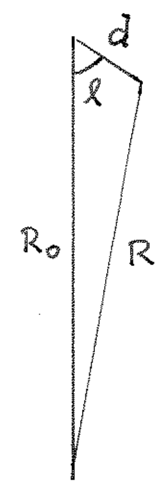

Figure 3

$$v_r \approx R_0 \left(\frac{d\omega}{dR}\right)_{R=R_0} (R - R_0) \sin\ell.$$

Now defining the **Oort constant** A as

$$A \equiv \frac{-R_0}{2}\left(\frac{d\omega}{dR}\right)_{R=R_0},$$

$$v_r \approx -2A\,(R - R_0) \sin\ell.$$

For stars at small d (see Figure 3), $-(R - R_0) \approx d \cos\ell$, so

$$v_r \approx 2A\, d \cos\ell \sin\ell.$$

Using the trigonometric identity $2\cos\ell \sin\ell = \sin 2\ell$,

$$v_r \approx A\, d \sin 2\ell. \tag{7}$$

Using a similar analysis, Equation (6) can be approximated in the limit $d << R_0$ as

$$v_t \approx 2A\,d\cos^2\ell - \omega_0\,d = A\,d\,(1+\cos 2\ell) - \omega_0\,d.$$

Now define B, the second Oort constant, as $B \equiv A - \omega_0$. Then

$$v_t \approx d\,(A\cos 2\ell + B). \tag{8}$$

Oort Constants

Equations (7) and (8) can be fitted to the velocity measurements of stars of known distance in the solar neighborhood (e.g., Figures 19.16 and 19.17 of Ryden & Peterson), which results in fitted values of the Oort constants A and B:

$$A = 14.8 \pm 0.8 \;\; \mathrm{km\ s^{-1}\ kpc^{-1}}$$

$$B = -12.4 \pm 0.6 \;\; \mathrm{km\ s^{-1}\ kpc^{-1}}$$

This directly yields the local angular velocity

$$\omega_0 = A - B = 27.2\ \mathrm{km\ s^{-1}\ kpc^{-1}} \tag{9}$$

Later, we will see how $R_0 \approx 8.0$ kpc is measured, most accurately from the orbits of stars near the Galactic center. Using this, we find $\Theta_0 = \omega_0\,R_0 = 220$ km s^{-1}.

Another way to write A and B in a more symmetric way is in terms of V instead of Ω:

$$A = \frac{-R_0}{2}\left(\frac{d\omega}{dR}\right)_{R=R_0} = \frac{-R_0}{2}\left[\left(\frac{d\Theta}{dR}\frac{1}{R}\right)_{R=R_0} - \frac{\Theta_0}{R_0^2}\right]$$

$$A = \frac{-1}{2}\left[\left(\frac{d\Theta}{dR}\right)_{R=R_0} - \frac{\Theta_0}{R_0}\right]$$

$$B = \frac{-1}{2}\left[\left(\frac{d\Theta}{dR}\right)_{R=R_0} + \frac{\Theta_0}{R_0}\right]$$

which shows that

$$\left(\frac{d\Theta}{dR}\right)_{R=R_0} = -(A+B) \tag{10}$$

Thus, the Oort constants tell us what the angular velocity (Equation 9) and gradient of Θ (Equation 10) are in the solar neighborhood. According to Equation (10), a flat rotation curve would result in $A + B = 0$.

Summary

Measurement of radial *and* tangential velocities of stars over a range of Galactic longitudes can be used to determine the Oort constants A and B, which are the amplitude and offset of the sinusoidal functions in Equations (7) and (8), which are graphed in the attached Figure 24.23. The Oort constants tell us ω_0 and $(d\Theta/dR)_{R=R_0}$, as shown in Equations (9) and (10). The rotation curve $\Theta(R)$ of the Galaxy can be measured from H I and CO radial velocity, using Equation (5), but its absolute normalization also depends on R_0, which has to be measured independently.

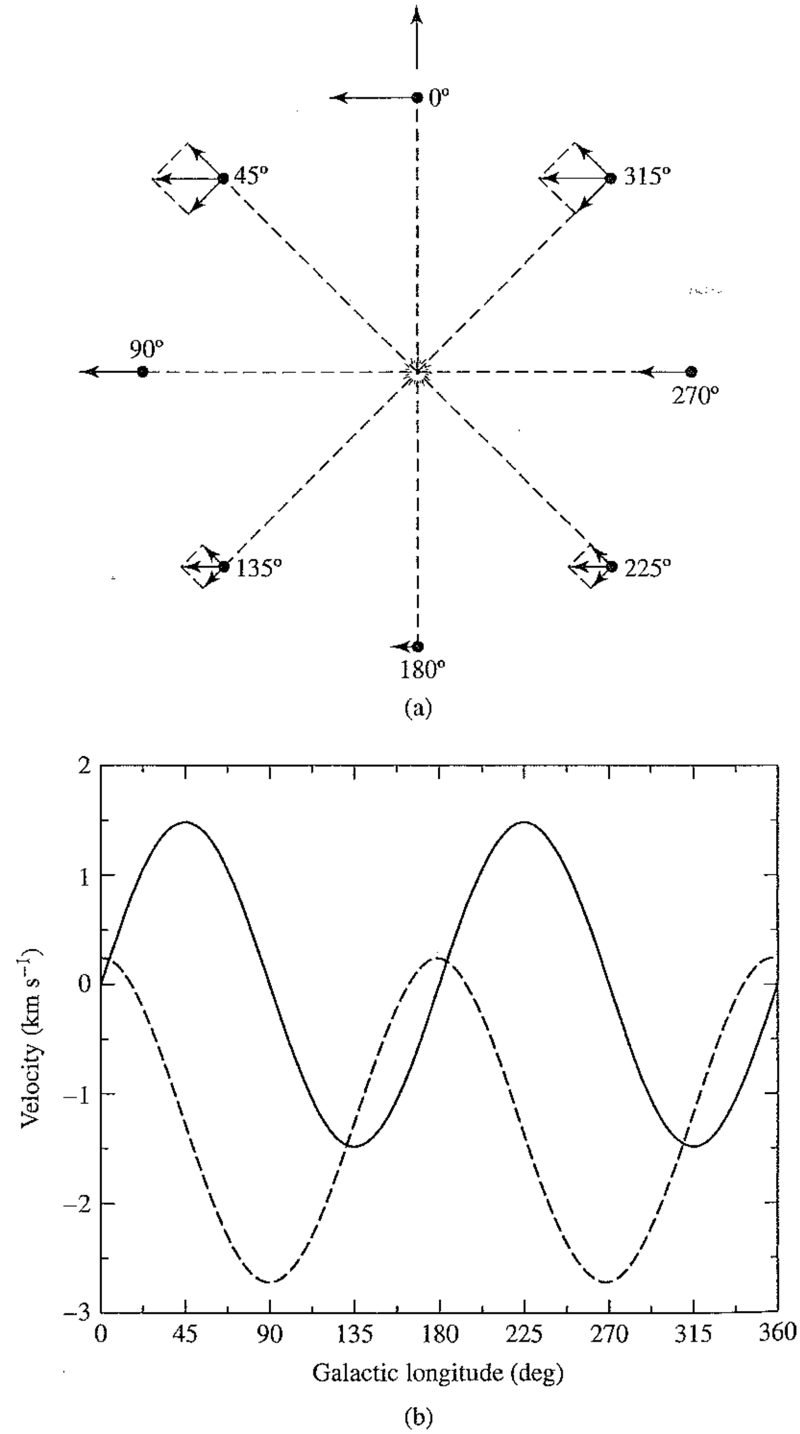

FIGURE 24.23 (a) The differential rotation of stars near the Sun is revealed through the dependence of radial and transverse velocities on Galactic longitude. (b) Radial velocity is proportional to $\sin 2\ell$ (solid line), and transverse velocity is a function of $\cos 2\ell$ (dashed line). The curves depict stars located 100 pc from the Sun with $A = 14.8\ \mathrm{km\ s^{-1}\ kpc^{-1}}$ and $B = -12.4\ \mathrm{km\ s^{-1}\ kpc^{-1}}$.