Astronomy UN2002 (Introduction to Astrophysics II) Spring 2020
Mon. April 6 Prof. J. Halpern

Refresher on Radians and the Small-Angle Approximation

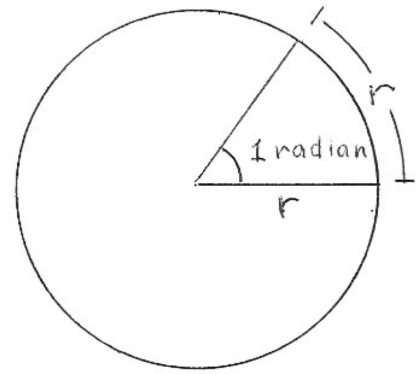

Figure 1

There are 2π radians in a circle (Figure 1).
There are $360 \times 60 \times 60$ arcseconds in a circle.
Therefore, there are

$$\frac{360 \times 60 \times 60}{2\pi} = 206,265 \text{ arcseconds radian}^{-1}.$$

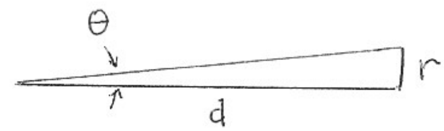

Figure 2

In the small-angle approximation illustrated in Figure 2, $\theta \ll 1$ radian and $r \ll d$. In this approximation, $\sin\theta \approx \tan\theta \approx \theta$. Think of r as a straight line which is almost the same length as the small curved segment of a circle whose radius is d. Therefore

$$\theta = \frac{r}{d} \text{ radians} = 206,265 \left(\frac{r}{d}\right) \text{ arcseconds}$$

The small-angle approximation applies to almost all of the examples and problems in this class.

Example: In the Notes on Gravitational Lensing (March 30) the small-angle approximation can be used to derive Equation (3), the angular radius θ_E of the Einstein ring, from Equation (1), the deflection angle ϕ.

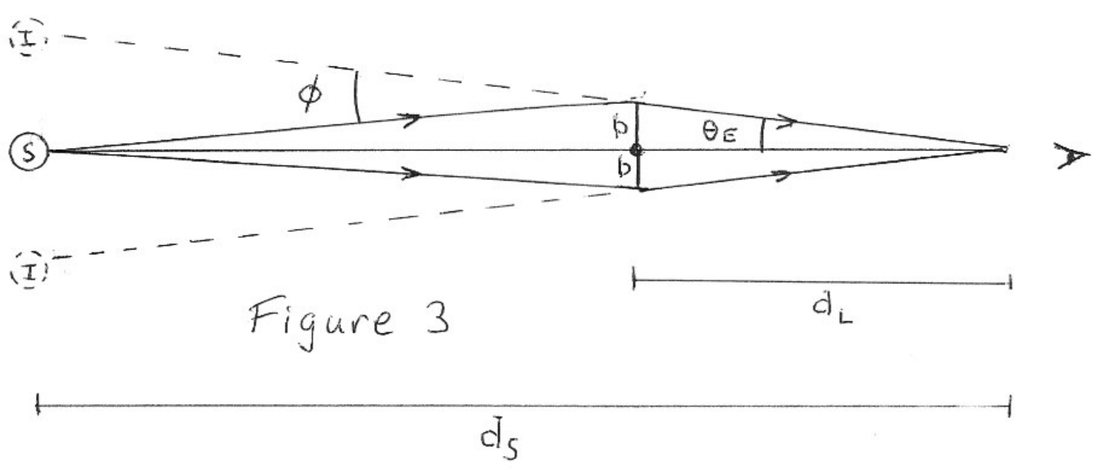

Figure 3

Since the (exaggerated) angles in Figure 3 are actually very small, you can use the approximations

$$b = \theta_E d_L$$

$$\phi(d_S - d_L) = \theta_E d_S$$

Using these two relations with

$$\phi = \frac{4GM}{c^2 b} \tag{1}$$

to eliminate ϕ and b in Equation (1) gives us

$$\theta_E = \sqrt{\frac{4GM}{c^2}\left(\frac{d_S - d_L}{d_S\, d_L}\right)} \tag{3}$$

If you were wondering why there is a square root in Equation (3) but not in Equation (1), now you know.