Astronomy UN2002 (Introduction to Astrophysics II) Mon. March 11

Spring 2019 Prof. J. Halpern

Notes on Gravitational Lensing

Gravitational lensing is the bending of the light rays from a background souce, and the consequent magnification of its image by the gravitational field of an intervening object (the lens). It can only be understood in the context of general relativity, in which space is curved in the neighborhood of mass. It is analogous to optical lensing due to the index of refraction of a medium, but unlike optical refraction, the gravitational effect is independent of the wavelength of the light. There are four regimes in which gravitational lensing is observed in astronomy:

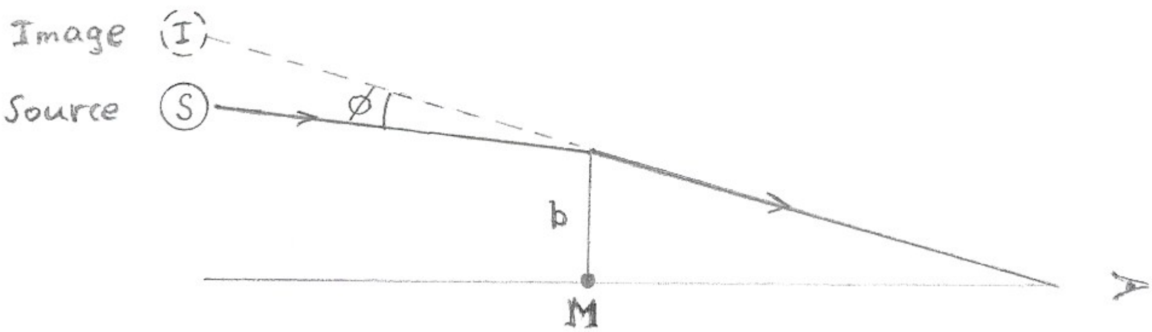
1. Compact objects such as black holes and neutron stars produce the strongest of all gravitational lensing of emission from material in their vicinity. This phenomenon is what the Event Horizon Telescope will use to study strong gravity around the Galactic center supermassive black hole, and SMBHs in nearby AGNs.

2. Strong cosmological lensing, in which multiple images of a quasar are projected within a few arcseconds of the line of sight of an intervening galaxy. Included in this category is the phenomenon of the Einstein ring, which occurs when the background source is directly behind the lens, and is also approximated by the arc-like images of background galaxies seen in projection behind clusters of galaxies.

3. Weak cosmological lensing. All images are slightly distorted by the irregular distribution of intervening mass in the Universe. The subtle effects on the apparent shapes of the many faint galaxies in a field can be used to make a map of the intervening mass distribution.

4. Microlensing. Stellar mass objects can also act as lenses. Due to relative motion of stars in the Galaxy, this effect produces a characteristic light curve (time dependence) of the magnification of the source that lasts a few weeks as the lens passes in front of the source. It is even possible to infer from the light curve whether the lens has a binary companion or a planet. Microlensing by stars in the lensing galaxy of a QSO can also produce additional magnification and time dependence of already strongly lensed QSO images.

All of these effects are illustrated in the accompanying images of gravitational lenses on the web page for this lecture, which you should examine. The bending effect on the images of stars near the Sun, first observed during the solar eclipse of 1919, was the earliest observation of a prediction of the general theory of relativity. So we begin with the basics of light bending in this context. In the following Figure, the distance of closest approach of a light ray to a mass M is called the **impact parameter**, denoted b. The deflection angle ϕ is the deviation of the image of the source from its true position.



As long as the angle ϕ is small (the weak-field approximation) and the source and observer are both far from the lens (like the thin-lens approximation in optics), general relativity gives

$$\phi = \frac{4GM}{c^2 b}. \tag{1}$$

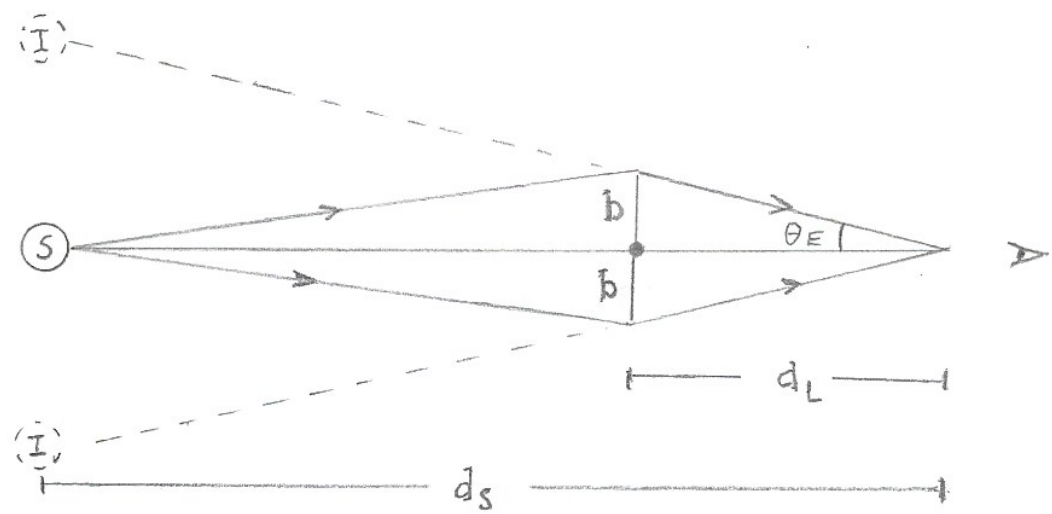
In the context of the solar eclipse, $M = 1\,M_\odot$, while the minimum possible impact parameter is $b = 1\,R_\odot$. You should verify that this gives a maximum deflection angle of $\phi = 1''.75$. It is also interesting to note the similarity of this expression to the Schwarzschild radius of a black hole if we replace b with $R_{\rm Sch}$ and M with $M_{\rm BH}$, since

$$R_{\rm Sch} = \frac{2GM_{\rm BH}}{c^2}. \tag{2}$$

When light passes very close to a black hole, the bending angle can be of order unity, but in this strong gravity regime, Equation (1) is no longer valid. It is also interesting that a naive calculation, in which we assume that a photon acts like a Newtonian test mass moving at speed c, produces a deflection angle just a factor of two less than Equation (1) in the small-angle regime. See the **Appendix** for that derivation.

Einstein Ring

If a source is directly behind a lens then, by symmetry, its image must be projected into a ring.



The angular radius of the ring is

$$\theta_E = \sqrt{\frac{4GM}{c^2}\left(\frac{d_S - d_L}{d_S\,d_L}\right)}, \tag{3}$$

which can be proven from Equation (1) and the small angle assumption. Here d_S is the distance from the observer to the source, and d_L is the distance from the observer to the lens. Note that for a source at $d_S = \infty$,

$$\theta_E = \sqrt{\frac{4GM}{c^2 d_L}}. \tag{4}$$

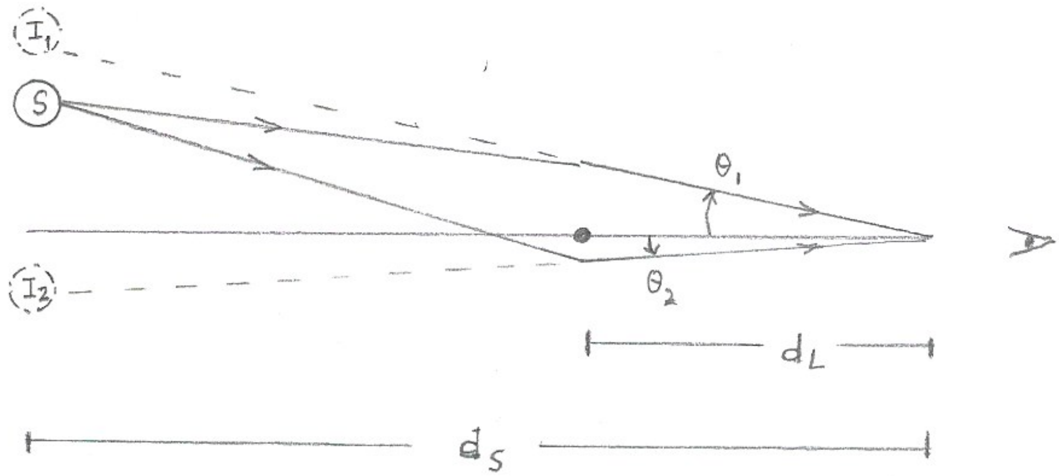
Examples of Einstein rings, and nearly complete rings, are shown in the accompanying images on the web page.

Multiple Images

In the case that the source is not directly behind the lens, if the lens is a point mass there will be two images on opposite sides of the lens. In this case, there is a relation for the two angles θ_1 and θ_2 between the images and the lens,

$$\theta_1 \theta_2 = -\frac{4GM}{c^2} \left(\frac{d_S - d_L}{d_S \, d_L} \right) . \tag{5}$$

The minus sign is because θ_2 is negative.



The separation of the images of a lensed QSO can be estimated for a typical case in which the source is twice as far as the lens. Then Equation (3) reduces to

$$\theta_E = \sqrt{\frac{4GM}{c^2 d_S}} . \tag{6}$$

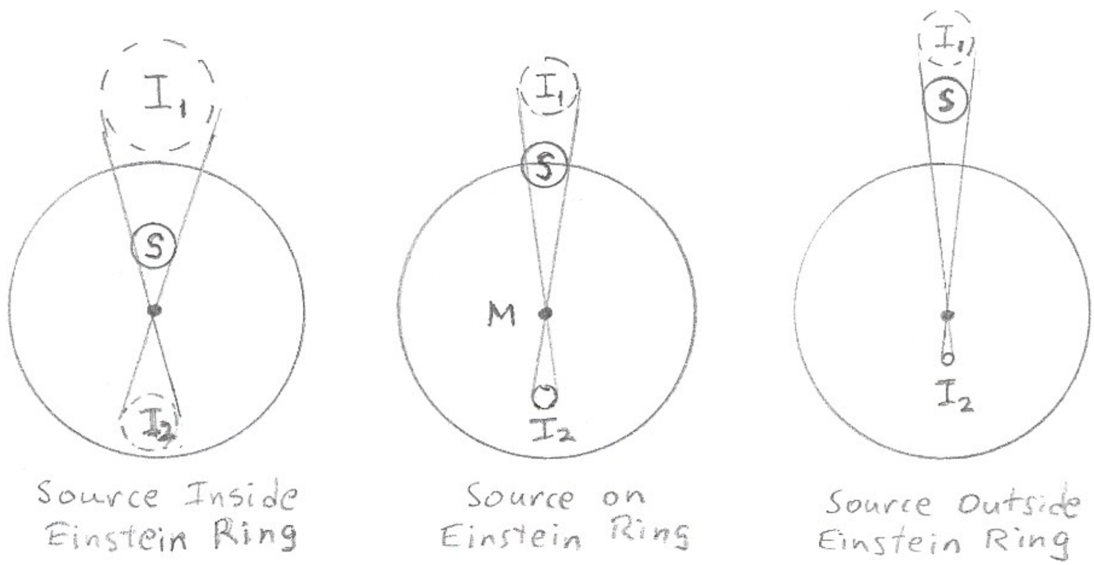
If the mass of the lensing galaxy is $\sim 10^{12}\, M_\odot$ and the QSO is at $d_s \sim 3$ Gpc, then $\theta_E \sim 1''\!.7$. The impact parameter is $b \approx \theta_E d_L = 12$ kpc, which is within the outskirts of the lensing galaxy.

An example of such an image is the original double QSO 0957+561 shown on the web page. There are two images of the QSO separated by $6''$. The reason that the lensing galaxy is not exactly on the line between the images of the QSO is that the lensing mass, actually a cluster of galaxies, is not a point. Evidence that the QSO images are really of one QSO, and not a physical pair, comes from the spectra of the two images (also on the web page) which are identical for all practical purposes, including having no detectable difference in radial velocity. One would expect that two AGNs in a bound pair or in members of a cluster of galaxies would have some relative velocity.

Magnification in Gravitational Lensing

Lensing also changes the magnitudes of the images, leading to an increase or decrease in the flux compared to the intrinsic flux of the source in the absence of a lens. This can be understood as the lens changing the solid angle (angular area, e.g., in square arcseconds) of an image without changing its surface brightness (magnitude per square arcsecond), the same way optical lenses behave. So the magnification of the total flux of an image is proportional to the magnification of its angular area. For a QSO, we can't usually resolve the image, whether it is lensed or not, so all we see as a result of the magnification is a brighter point source. In the case of a point lens and a point source, the combined flux of both images increases the closer the lens is projected to the line of sight to the source, and is always larger than the intrinsic flux of the source.

The following Figure indicates schematically how one would calculate the magnification of images in the presence of a point lens for different projection angles. The ratio of the area of the image to the true area of the source gives its magnification factor in flux. As you can see, when the source is very far from the Einstein ring that would occur if the source were on axis, the displacement and magnification of its primary image are negligible, and the secondary image is faint and close to the lens. The magnification becomes more significant when the source is projected close to the radius θ_E of the Einstein ring. For example, when the source is at an angle equal to θ_E, the total magnification is $3/\sqrt{5} = 1.34$. The magnification increases further when a source is inside Einstein ring, and becomes formally infinite for a point source on axis. Of course, no source or lens is truly a point, so a real Einstein ring has a finite magnification.



Many lensed QSOs have more than two images. This is because the lens is not a point and is not spherically symmetric. A striking example is the "Einstein Cross," also known as Huchra's lens, shown on the web page. It has four images of a QSO projected on the center of a nearby intervening galaxy. Another property of gravitational lensing is that the different images are seen at different times because the gravitational field delays the light as it passes closer to the lens. Similar to Fermat's principle in optical lensing, an image is the locus of points corresponding to an extremum in the light travel time or rays (either a maximum or a minimum). But each image can correspond to a different light travel time. Therefore, since QSOs are intrinsically variable, the light curves of lensed QSO images are seen to be identical, but delayed by months or years. This, by the way, is additional evidence that the images are from a single lensed QSO and not a physical pair.

Microlensing

Originally conceived as a way to discover Massive Compact Halo Objects (MACHOs) if stellar-mass black holes or other massive dark objects comprise the dark matter in galaxies, microlensing is so named because of the small size of the resulting Einstein rings. Such microlensing of background stars is detected in wide-area, high-cadence surveys toward the bulge of the Galaxy and the Magellanic Clouds, but it is inferred that the lensing object is also a normal star (or a brown dwarf). Because of the relative motion between the lens and the source, a microlensing event has a characteristic and symmetric curve of magnification that is easy to recognize and discriminate from those of other (intrinsic) variable stars. An example is shown in the following Figure, which also reveals that the lensing star has a planet. It is left to a homework problem to work out the typical duration of a microlensing event.

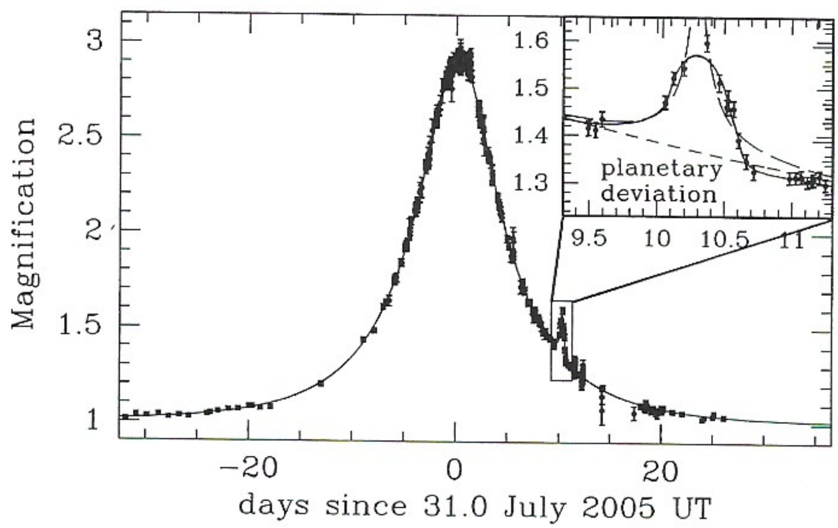
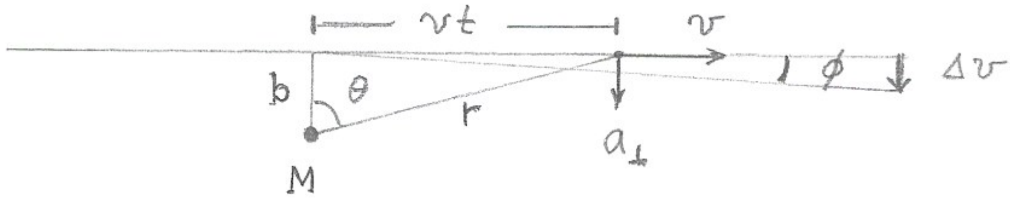


Figure 6.10 Example of an observed microlensing event involving a foreground star that temporarily magnifies the light from a background star projected behind it (both in the general direction of the center of our Galaxy, the Milky Way). Note the excellent fit (solid curve) to the data provided by Eqs. 6.23 and 6.28 during most of the event. However, the small, day-long perturbation in the curve of magnification vs. time on its falling side (enlarged in the inset) reveals the presence of a planet around the lens star. A detailed model (solid curve in the inset) indicates an approximately 5-Earth-mass planet at an orbital radius of several AU from the main lens star. The single-lens model (short-dashed curve) and a model assuming a binary source star (long-dashed curve) are both ruled out by the data. Figure credit: PLANET, OGLE, and MOA collaborations, see J.-P. Beaulieu et al. 2006, *Nature*, 439, 437.

Microlensing can also be detected as rapid events in the light curves of already strongly lensed QSOs due to individual stars in the lensing galaxy. Since the source is the accretion disk of the QSO, this is a way of getting unique information about the size and structure of the disk, which is normally unresolved optically.

APPENDIX

You may find a "classical" derivation of light bending to be interesting, even though it is wrong. It gives a result that is only a factor of 2 smaller than Equation (1). Consider a test particle moving with velocity v with impact parameter b past a large mass M. We can consider that, as long as v and b are sufficiently large, the magnitude of v will not change during the passage, but the path of the particle will be bent by a small angle $\phi = \Delta v/v$, where Δv is perpendicular to v.



Then, we can calculate Δv from the perpendicular component of acceleration, $a_\perp$, that is, from the component of a perpendicular to v.

$$\frac{dv}{dt} = a_\perp \tag{7}$$

so

$$\Delta v = \int_{-\infty}^{\infty} a_\perp \, dt. \tag{8}$$

Here we define $t = 0$ at closest approach. From the Figure, we can see that

$$a_\perp = \frac{GM}{r^2}\cos\theta = \frac{GM}{r^2}\frac{b}{r} = \frac{GMb}{(b^2 + v^2t^2)^{3/2}}. \tag{9}$$

Substituting $a_\perp$ from Equation (9) into the integral of Equation (8) gives

$$\Delta v = GMb \int_{-\infty}^{\infty} \frac{dt}{(b^2 + v^2t^2)^{3/2}}. \tag{10}$$

This integral is easily solved with a change of the variable of integration from t to θ. Notice from the Figure that

$$\frac{v\,t}{b} = \tan\theta, \tag{11}$$

which also gives

$$dt = \frac{b}{v}\frac{d\theta}{\cos^2\theta}. \tag{12}$$

Then Equation (10) can be written

$$\Delta v = \frac{GM}{vb}\int_{-\pi/2}^{\pi/2} \cos\theta \, d\theta = \frac{2GM}{vb}. \tag{13}$$

Here we have also made use of the identity

$$1 + \tan^2\theta \;=\; \frac{1}{\cos^2\theta}. \tag{14}$$

Finally, as advertised,

$$\phi = \frac{\Delta v}{v} = \frac{2GM}{v^2 b} = \frac{2GM}{c^2 b} \tag{15}$$

when we choose for the particle velocity $v = c$.