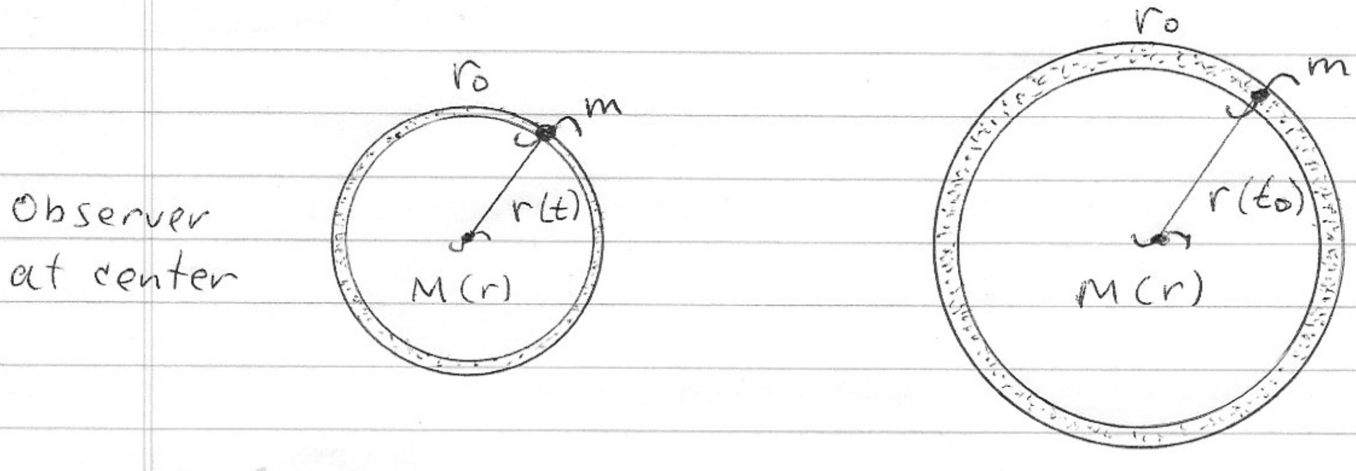
"Newtonian" Cosmology



Earlier time (t) Present time (t_0)

$$r(t) = a(t)\, r_0$$

coordinate distance (increasing)

comoving coordinate (constant)

The dimensionless scale factor (increasing) but is the same everywhere.

Let $a(t_0) = 1$, or $a_0 \equiv a(t_0) = 1$

Enclosed mass $M(r)$ is constant, but its average density decreases such that

$$M(r) = \frac{4\pi}{3} r^3(t)\, \rho(t) = \text{constant}$$

A galaxy of mass m has velocity $v(t)$ with respect to the observer.

Energy of galaxy is constant

$$\frac{1}{2}\cancel{m}v^2(t) - \frac{GM(r)\cancel{m}}{r(t)} = \underbrace{-k\frac{\cancel{m}c^2 r_0^2}{2}}_{\text{constant}}$$

Hubble law

$$v(t) = H(t)\,r(t)$$

$$\Rightarrow \frac{dr}{dt} = H(t)\,r$$

$$H(t) = \frac{1}{r}\frac{dr}{dt} = \frac{1}{a}\frac{da}{dt}$$

$$\frac{1}{2}H^2a^2\cancel{r_0^2} - \frac{4\pi}{3}Ga^2\cancel{r_0^2}\rho = -k\frac{c^2\cancel{r_0^2}}{2}$$

Friedmann Equation

$$\boxed{H^2a^2 - \frac{8\pi G a^2}{3}\rho = -kc^2} \qquad (1)$$

Conservation of Mass

$$\rho a^3 = \rho_0 a_0^3 \qquad (a_0 = 1)$$

$$\left(\frac{da}{dt}\right)^2 - \frac{8\pi G\rho_0}{3a} = -kc^2 \qquad (1a)$$

If $k > 0$ total energy is negative $\Rightarrow$ bound
$k < 0$ " " " positive $\Rightarrow$ unbound
$k = 0$ " " " zero $\Rightarrow$ "

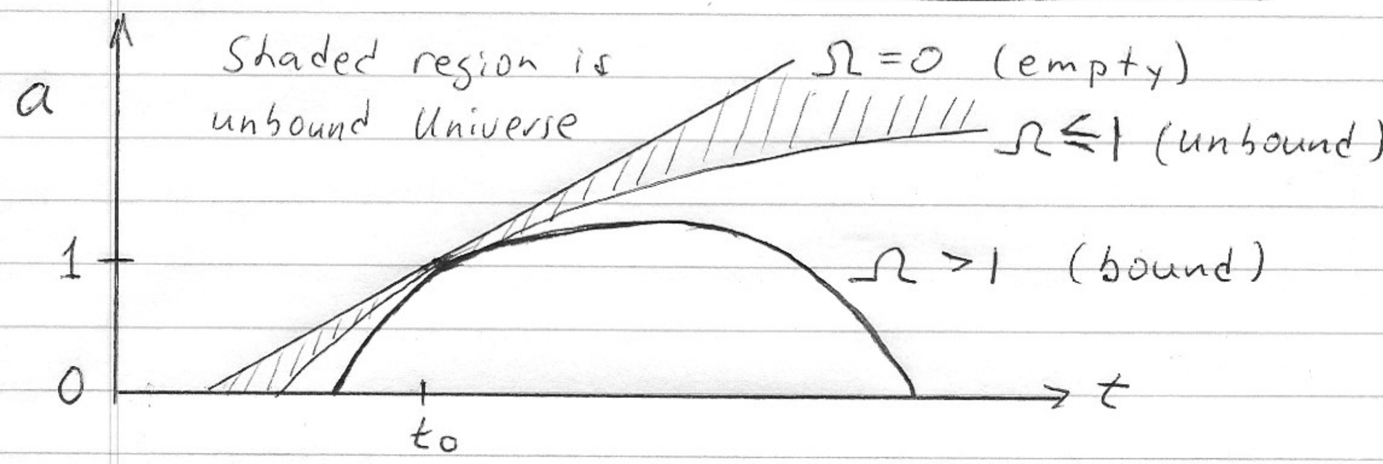
For $k = 0$ $\quad \rho = \rho_c \equiv \dfrac{3H^2(t)}{8\pi G}$ "critical density"

At the present time, when $H_0 = 71$ km s^{-1} Mpc^{-1}

$$\rho_{c,0} = 9.47 \times 10^{-27} \text{ kg m}^{-3}$$

$\Omega(t)$, the <u>density parameter</u>, is defined as the ratio of the actual density to the critical density

$$\boxed{\Omega(t) \equiv \frac{\rho(t)}{\rho_c(t)} = \frac{8\pi G \rho(t)}{3H^2(t)}} \qquad (2)$$



Equation (1) can be written as

$$H^2(1-\Omega)a^2 = -kc^2 \qquad (3)$$

Since k is a constant the sign of $1-\Omega$ doesn't change at any time.
Next time we will show that redshift z comes from a

$$1 + z = \frac{\lambda_{\text{observed}}}{\lambda_{\text{emitted}}} = \frac{1}{a}$$

Question: How old was the Universe at redshift z?
It depends on both H_0 and Ω_0.
Use the Friedmann Equation in the form of Equation (3):

$$H^2(1-\Omega)\,a^2 = -kc^2 \qquad (3)$$

$$\Rightarrow \quad H_0^2\,(1-\Omega_0) = -kc^2 \qquad \text{at the present time}$$

Remember that $H = \frac{1}{a}\frac{da}{dt}$

Example: The "empty" universe ($\Omega = 0$)

$$\left(\frac{da}{dt}\right)^2 = H_0^2$$

$$\int_0^a da = H_0 \int_0^t dt$$

Answer: $\boxed{t = \frac{a}{H_0} = \frac{1}{H_0(1+z)}}$

The present age t_0 (at $z=0$) $= 1/H_0$

Example: The marginally bound Universe ($\Omega = 1$)

In this case $k = 0$

[Note that if $\Omega = 1$, it must always be 1.
If $\Omega = 0$, it must always be 0.
For all other values, Ω is not constant with time.]

case $k=0$

$$H^2 a^2 = H^2 \Omega a^2$$ (from Equation 3)

$$\left(\frac{da}{dt}\right)^2 = H^2 \frac{8\pi G\rho}{3H^2} a^2 = \frac{8\pi G\rho}{3} a^2$$

But $\rho = \rho_0 / a^3$

$$= \frac{8\pi G \rho_0}{3a}$$

$$\frac{da}{dt} = \sqrt{\frac{8\pi G\rho_0}{3a}}$$ (Same as Equation 1a)

$$\int_0^a a^{1/2}\, da = \sqrt{\frac{8\pi G\rho_0}{3}} \int_0^t dt$$

$$\frac{2}{3} a^{3/2} = \sqrt{\frac{8\pi G\rho_0}{3}}\, t$$

$$\frac{2}{3} = \sqrt{\frac{8\pi G\rho_0}{3}}\, t_0$$ (the present time)

$$\Rightarrow \quad a = \left(\frac{t}{t_0}\right)^{2/3}$$

But $a = \frac{1}{1+z}$

so $$t = \frac{t_0}{(1+z)^{3/2}}$$

$$\frac{da}{dt} = \frac{2}{3}\frac{t^{-1/3}}{t_0^{2/3}}$$

$$\Downarrow$$

$$\frac{da}{dt} = \frac{2}{3}\frac{a}{t}$$

$$\Downarrow$$

$$t_0 = \frac{2}{3H_0}$$

$$\boxed{t = \frac{2}{3H_0}\frac{1}{(1+z)^{3/2}}}$$