Small Bodies in the Solar System

As shown in Figure 8.1, those bodies with $m < 0.003\,M_{\oplus}$

(1) Main asteroid belt (between the orbits of Mars and Jupiter).

Total mass is estimated as $0.001\,M_{\oplus}$. Largest is Ceres, with $m = 1.6 \times 10^{-4}\,M_{\oplus}$, $R = 0.075\,R_{\oplus}$. Discovered in 1801 its magnitude is in the range 6.7–9.3. Estimated that 100,000 have diameter > 1 km. Close to 1 million are cataloged. The LSST (Vera Rubin Telescope) is expected to discover 5 million.

Most have semi-major axes between 1.8 and 3.3 AU, eccentricities $e = 0.05 - 0.3$, inclination angle to the ecliptic $i = 0° - 30°$.

Moons of Jupiter or Mars could be capture asteroids. Jupiter could have prevented another planet from forming in the asteroid belt, and could have limited the size of Mars by creating a gap in the pebble disk that would have drifted in to Mars. This could be why Mars is smaller than Earth rather than larger

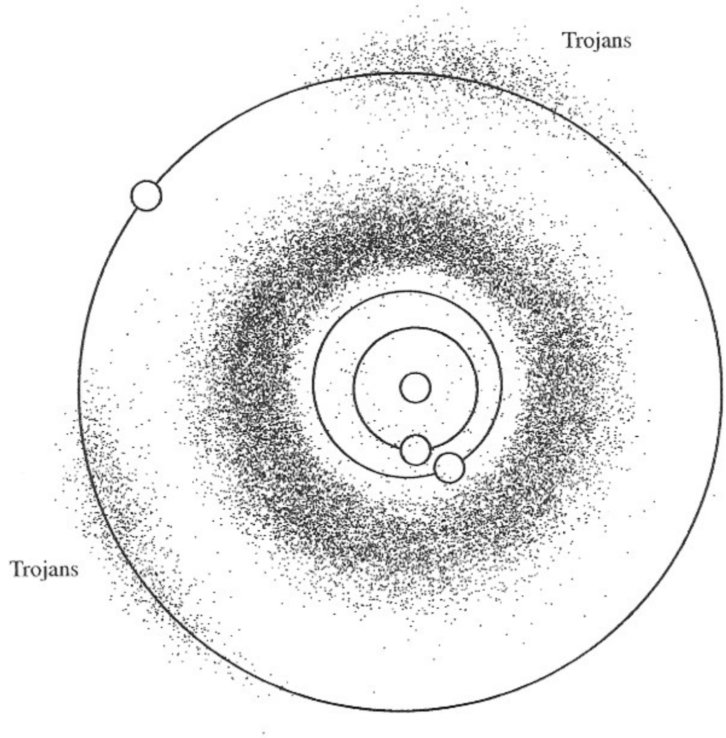

FIGURE 11.1 Location of asteroids with well-determined orbits, at a particular instant in time (which happens to be the summer solstice of the year 2003). The orbits and positions of Earth, Mars, and Jupiter are also shown. Note that the Trojan asteroids are in approximately the same orbit as Jupiter, but leading or following Jupiter by ~ 60°.

the **Trojan asteroids**.[4] The Trojan asteroids reside near two of the five **Lagrangian points** in the Sun–Jupiter system. In the eighteenth century, the mathematician Joseph Lagrange demonstrated that if two bodies of dissimilar mass (the Sun and Jupiter, for instance) are on circular orbits about their center of mass, there exist five points where the combined gravitational acceleration of the Sun and Jupiter is exactly that required to keep a small test body on a circular orbit co-rotating with the Sun–Jupiter line. As shown in Figure 11.3, three of these Lagrangian points, called L_1, L_2, and L_3, are colinear with the Sun and Jupiter. The L_1, L_2, and L_3 points are unstable equilibrium points. For instance, if we take a particle at the L_1 point (between the Sun and Jupiter) and move it slightly, it

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[4] The first Trojan asteroid to be discovered was named Achilles; when further asteroids on similar orbits were discovered, they were given names taken from the *Iliad*. This explains the otherwise puzzling name "Trojan" asteroid.

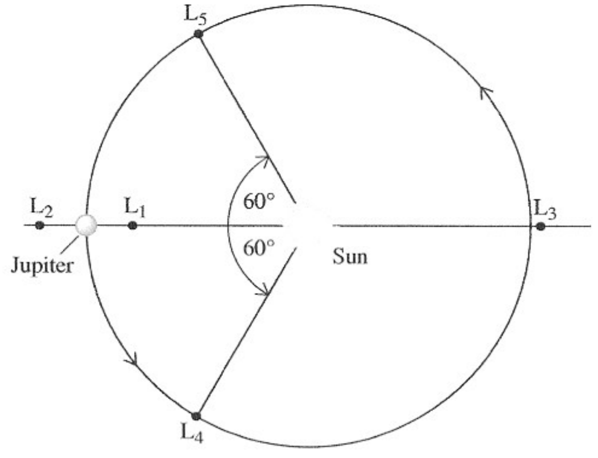


FIGURE 11.3 The Lagrangian points of the Sun–Jupiter system.

L_4 and L_5 are stable equilibrium points because

$$\frac{M_\odot}{M_J} > 24.96$$

Kirkwood Gaps

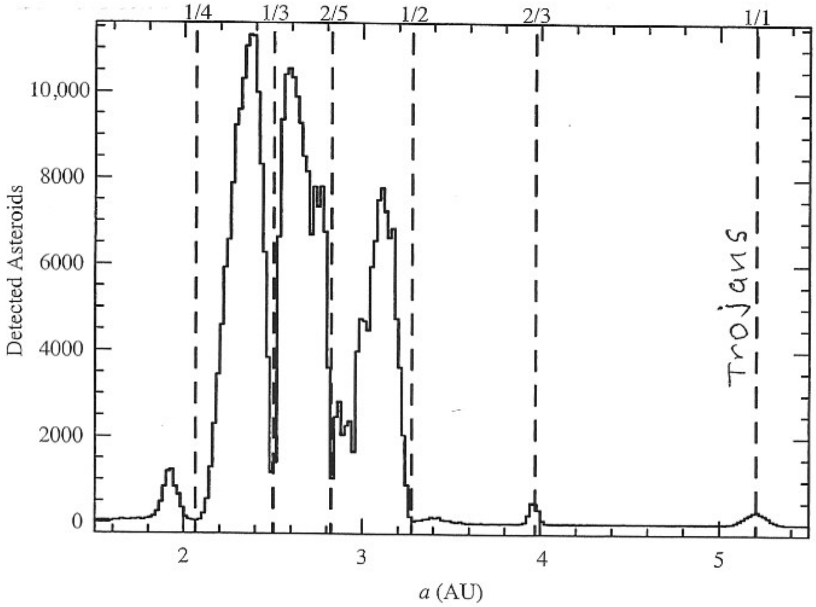


FIGURE 11.2 The number of detected asteroids as a function of semimajor axis. The vertical dashed lines are labeled with the orbital period, expressed as a fraction of Jupiter's orbital period.

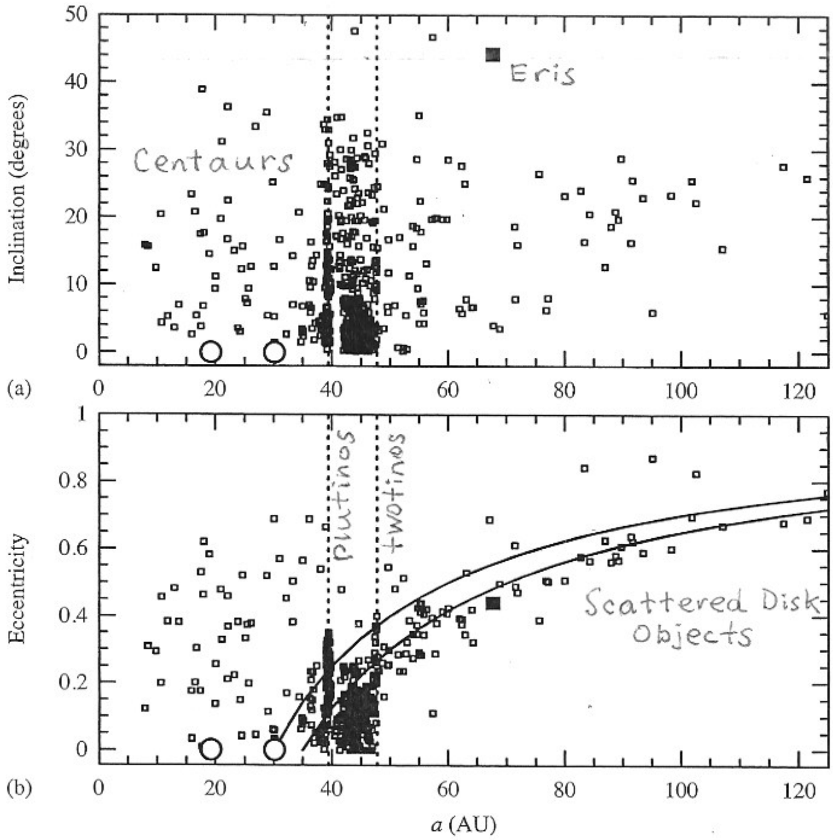
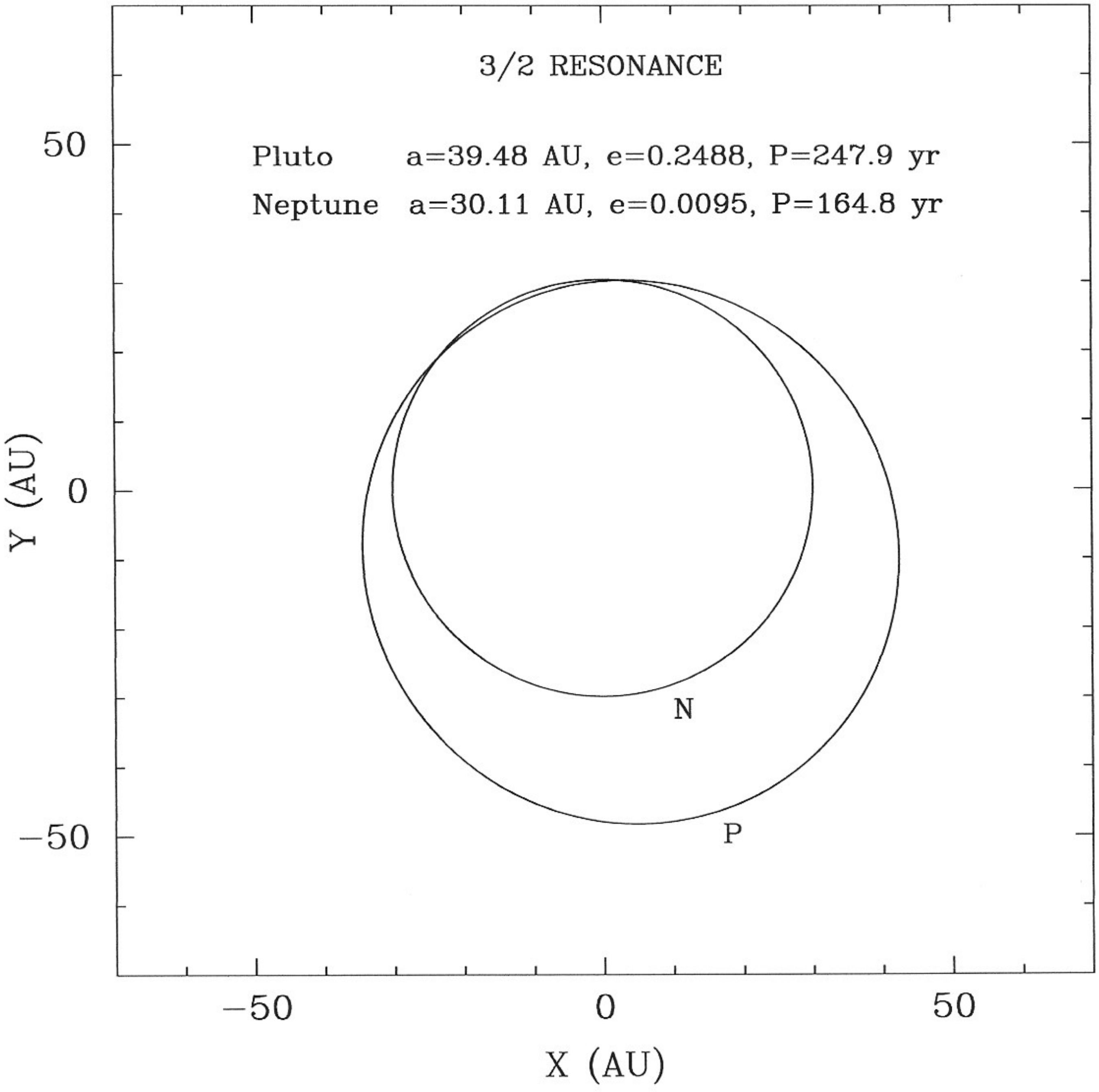


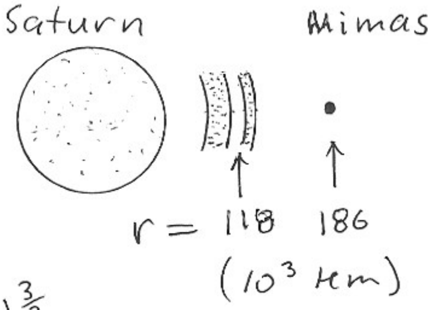
FIGURE 11.5 (a) Orbital inclination as a function of semimajor axis for all known TNOs. Vertical dotted lines are drawn at the 3:2 and 2:1 orbital resonance with Neptune. (b) Orbital eccentricity as a function of semimajor axis. Solid curved lines are drawn for orbits with perihelia of $q = 30$ AU and $q = 35$ AU.



Cassini Division has 2/1 orbital resonace with mimas.

Mimas period 22.6 hr
Cassini period 11.3 hr

$P^2 \propto r^3$

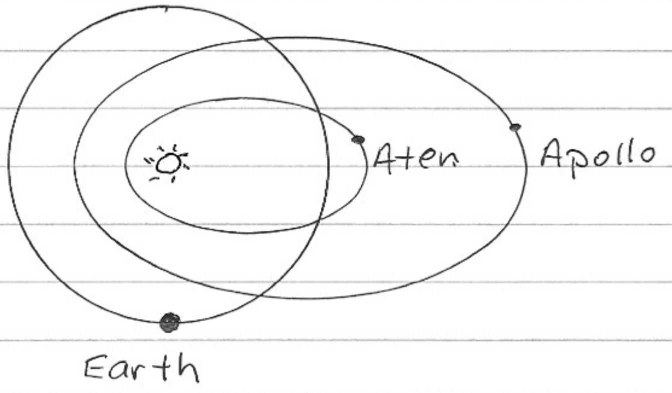


$$\left(\frac{186}{118}\right)^{\frac{3}{2}} = 2.0$$

(1a) Earth-crossing asteroids have orbits that take them inside 1 AU. There are two classes:

a) Apollo ~10,500 that have $a > 1$ AU

b) Aten ~1500 that have $a < 1$ AU



(2) Kuiper Belt Objects (KBOs) lie outside the orbit of Neptune, in the range $30 < a < 50$ AU. The majority of the trans-Neptunian objects are KBOs. The first KBO was 1992 QB_1. Thousands are known now, but there are estimated $>100{,}000$ larger than 100 km in diameter. These are icy objects.

Pluto, with $m = 2.2\times10^{-3} M_\oplus$ and $R = 0.19 R_\oplus$ is the largest KBO. It has moons Charon, discovered in 1979, Nix and Hydra in 2005, which enabled the mass of Pluto and Charon to be measured accurately.

(3) <u>Scattered disk objects</u> are also trans-Neptunian objects, but they have larger semi-major axis a and eccentricity e. Eris is one, and it is slightly heavier than Pluto, with $m = 2.8 \times 10^{-3} M_\oplus$.

The total mass of the Kuiper belt is estimated as $0.02\ M_\oplus$, much greater than $0.001\ M_\oplus$ of the main asteroid belt.

What are now called dwarf planets, like Ceres, Pluto, and Eris, are the largest of their respective classes of asteroids, with masses large enough that gravity makes them spherical.

(4) <u>Comets</u> are thought to be small, 1-10 km KBOs that are loosely packed balls of ice and rocks (dirty snowballs). Their tails are created when close enough to the Sun for the surface to sublime, releasing rocks and dust

~ 200 <u>short-period comets</u> are known, which have $P < 200$ yr, $a < 34$ AU, and $e \sim 0.8$. Inclination angles are moderate, $i < 30°$, although a few are retrograde ($i > 90°$). Comet Halley with $P = 76$ yr is retrograde.

Comets won't last more than a few hundred orbits, so they have to be replenished from a reservoir in the outer solar system.

~ 800 <u>long-period comets</u> are known, which have $P > 200$ yr and $a > 34$ AU. The longest have $P \sim 3 \times 10^7$ yr, $a \sim 10^5$ AU. (Note that the nearest star is at $d = 2.7 \times 10^5$ AU.) They have random inclination angles and $e \sim 1$. Jan Oort proposed that they reside in a spherical cloud (<u>Oort cloud</u>), and their orbits are perturbed by passing stars, sometimes sending them to the inner solar system. It is estimated that there are 10^{12-13} comets having total mass 1-10 $M_\oplus$, much more than the Kuiper Belt or main belt.

The chemistry of long-period comets, particulary water, indicates that they were formed around 20-30 AU where $T \approx 100$ K (between Uranus and Neptune), and must have been kicked out by giant planets. Some escape the solar system, some made the Oort cloud, some ended up in the Kuiper belt, and some became the short-period comets.

The <u>Centaurs</u>, which are asteroids between the orbits of Jupiter and Neptune, may become Jupiter-family comets.

(5) Meteoroids and Dust

Small particles left over from the formation of the solar system can be removed by radiation pressure from the Sun.

* $\mathbb{P}_r = \frac{L_\odot}{4\pi r^2 c}$ radiation pressure
r = distance from Sun

$F_r = \mathbb{P}_r \sigma_p$ radiation force on particle of cross section σ_p

$\sigma_p = Q \pi R^2$ R = radius of particle

$Q = 1$ for $R \gg \lambda$ (wavelength)
$Q \ll 1$ for $R \ll \lambda$ and $Q \propto R^2$

$$\Rightarrow F_r = \frac{L_\odot}{4\pi r^2 c} Q \pi R^2$$

Compare with the force of gravity on the particle of mass $m = \frac{4\pi}{3} R^3 \rho$

$$F_g = -\frac{G M_\odot}{r^2} \frac{4\pi}{3} R^3 \rho$$

$$\frac{F_r}{|F_g|} = \frac{3}{16\pi} \frac{L_\odot}{G M_\odot c} \frac{Q}{\rho R}$$

For $R \ll \lambda$, $Q \propto R^2$, so the ratio increases $\propto R$
For $R \gg \lambda$, $Q = 1$, so the ratio decreases $\propto 1/R$

The ratio $F_r/|F_g|$ is maximized when $R \approx \lambda$.

Grains will be forced out of the solar system if $F_r/|F_g| > 1$, thus if

$$R < \frac{3}{16\pi} \frac{L_\odot}{G M_\odot c} \frac{Q}{\rho} = 5.7 \times 10^{-4} \frac{Q}{\rho} \qquad [m]$$

Typically, $1000 < \rho < 3000$ kg m^{-3}

Grains with $R \sim 5.7 \times 10^{-7}$ m $= 5700$ Å will be removed. Larger ones are too heavy. But smaller ones will not be removed either, because their Q is too small, $\ll 1$.

Conclusion: only grains whose size is comparable to the wavelength of sunlight will be removed by radiation pressure.

However, there is a more effective mechanism for clearing particles from the solar system, called the Poynting - Robertson effect.

* Pressure is momentum flux:

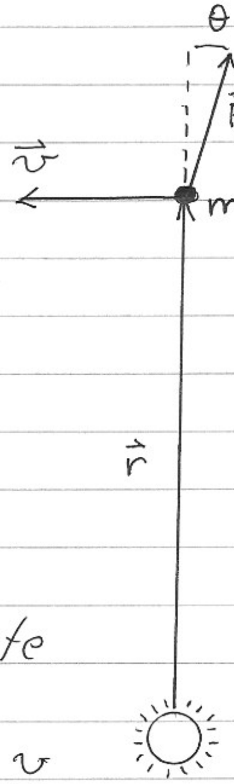
$$\mathbb{P} = \frac{1}{A} \frac{dp}{dt} \quad \text{where } p = \frac{E}{c} \text{ for a photon, and } A = 4\pi r^2$$

$$\Downarrow$$

$$\frac{dp}{dt} = \frac{L}{c} \quad \text{and} \quad \mathbb{P}_r = \frac{L}{4\pi r^2 c}$$

Poynting - Robertson Effect

Sunlight exerts a drag force on small particles, causing them to spiral into the Sun.



For $v \ll c$, aberration of light as seen from the moving particle gives it an angle θ, where

$\tan\theta \approx v/c$ $\qquad$ $\tan\vartheta \approx \sin\vartheta \approx \vartheta$

At Earth $v/c \approx 1\times10^{-4}$ $\qquad$ $\theta = 20.''6$

A torque results, which decreases the angular momentum of the particle at a rate

$$\frac{d\vec{L}}{dt} = \vec{r}\times\vec{F}_r = -rF_r\sin\vartheta = -\frac{rF_r v}{c}$$

$$\frac{d(mvr)}{dt} = -r\,\frac{L_\odot}{4\pi r^2 c}\,\pi R^2\,\frac{v}{c}$$

Since $v = \sqrt{\dfrac{GM_\odot}{r}}$

m = mass of grain
R = radius of grain
assume Q = 1

$$\frac{d}{dt}\left[m\sqrt{GM_\odot r}\right] = \frac{-L_\odot R^2\sqrt{GM_\odot}}{4r^{3/2}c^2}$$

$$\frac{m}{2}\frac{dr}{r^{1/2}} = \frac{-L_\odot}{4r^{3/2}c^2}\,dt$$

$$r\,dr = \frac{-L_\odot R^2}{2c^2 m}\,dt$$

Integrating r from a to zero gives

$$\boxed{t = \frac{a^2 c^2 m}{L_\odot R^2}}$$

For a spherical grain, let $m = \frac{4\pi}{3} \rho R^3$.

Then $$\boxed{t = \frac{a^2 c^2}{L_\odot} \frac{4\pi}{3} \rho R}$$

Example: $a = 1\,AU = 1.5 \times 10^{11}\,m$
$\rho = 3000\,kg\,m^{-3}$
$R = 1\,m$

$t = 6.6 \times 10^{16}\,s = 2.1 \times 10^{9}\,yr$

Therefore, rocks with radii less than 1 m will be cleared out to a distance $a \simeq 1.5$ 1AU from the Sun.

The fact that we see such particles via the <u>zodiacal light</u> means that they are replenished by comets as their surfaces sublime, and when asteroids collide and release small fragments.

Comets are the source of most <u>meteors</u>, while <u>meteorites</u> come from asteroids.

The (arbitrary) boundary between meteoroids and asteroids is conventionally around $d = 300\,m$.