1. In Homework #7 we calculated the time it would take for a $10 M_\odot$ black hole to grow to $10^{9} M_\odot$. High-redshift quasars must have such a mass to be luminous enough to be detected. In a matter-dominated $\Omega = 1$ universe having the present value of the Hubble parameter, $H_0 = 71 \, \text{km s}^{-1} \, \text{Mpc}^{-1}$, estimate the highest redshift at which it is possible for a $10^{9} M_\odot$ black hole to be present if it grew by this process. What is implied about either the initial mass, or the growth process of black holes, by the recent discovery of a quasar at $z = 7.61$?

2. Carry out the derivation of Equation (8) for a matter-dominated universe in the “Accompanying notes” from April 8

$$\Omega = 1 + \left( \frac{\Omega_0 - 1}{1 + \Omega_0 z} \right)$$

(8)

This shows that $\Omega$ was extremely close to 1 at early times, which begs for an explanation. Stay tuned . . .

3. Observations of nearby galaxies show that the average luminosity density of the universe in starlight is $\approx 2.3 \times 10^{8} L_\odot \, \text{Mpc}^{-3}$. If this luminosity has persisted, in comoving coordinates, for the age of the universe (not really a good assumption, but use it), estimate the energy density of starlight and compare it to the present energy density of the cosmic microwave background (CMB). Was the energy density of starlight ever a significant dynamical component of the Universe?