Astronomy UN2002 (Introduction to Astrophysics II) Spring 2020
Due Wed. April 1 Prof. J. Halpern

Homework #7

1. Consider the expression for apparent tangential velocity (derived in Appendix of Chapter 21 and graphed in the accompanying Figure on the web page):

$$\beta_{\rm app} = \frac{\beta \sin\theta}{1 - \beta\cos\theta}$$

Here $\beta = v/c$ and θ is the angle of the direction of motion with respect to the line of sight.

(a) Prove that $\beta_{\rm app}$ is a maximum (for a given β) when $\cos\theta = \beta$. [For example, if $\beta = 0.99$, $\beta_{\rm app,max} = 7.0$ when the angle $\theta = 8.^\circ 1$].

(b) Show that there can be superluminal motion ($\beta_{\rm app} > 1$) only if $\beta > 0.707$.

2. Assume that a black hole accreting mass at a rate dM/dt converts that mass to radiation of luminosity

$$L = \eta \frac{dM}{dt} c^2$$

with efficiency factor $\eta \approx 0.1$ (to be justified in class).

(a) If enough gas is available in the vicinity of the black hole such that the accretion luminosity is always equal to the Eddington limit

$$L_{\rm Edd} = \frac{4\pi c\, G m_p\, M_{\rm BH}}{\sigma_e}, \qquad (21.13)$$

show that the $M_{\rm BH}$ grows exponentially with time. That is, show that

$$M_{\rm BH}(t) = M_{\rm BH}(0)\, e^{t/\tau}$$

and evaluate the time-scale τ. [In this formula $\sigma_e = 6.65 \times 10^{-29}$ m^2 is the electron-scattering (Thompson) cross section.]

(b) How long would it take for a "seed" black hole of $M_{\rm BH}(0) = 10\, M_\odot$ to grow to $10^9\, M_\odot$ by this accretion process?

3. This problem is about microlensing, as illustrated on page 5 of "Lecture notes on gravitational lensing," course web page, March 9. [Also see page 443 of the textbook.] Consider a stationary star at a distance of 2 kpc (the source), and a $0.03\, M_\odot$ brown dwarf at a distance of 1 kpc (the lens), which has a tangential velocity of 30 km s^{-1}. The source will be significantly magnified during the time it is within the Einstein ring.

(a) What is the angular radius in arcseconds of the Einstein ring?

(b) How long (in days) will it take for the source to traverse the diameter of the Einstein ring?