Homework #6

1. This problem is to investigate whether the Milky Way Galaxy falls on the Tully-Fisher relation for absolute blue magnitude \(M_B\) as a function of maximum rotation velocity \(V_{\text{max}}\) (km s\(^{-1}\)), usually stated as:

\[
M_B = -10.2 \log V_{\text{max}} + 2.7
\]

Use \(L_B = (2.3 \pm 0.6) \times 10^{10} L_\odot\) as the total blue luminosity of the Galaxy, and convert it to \(M_B\) using \(M_{B,\odot}\). Plot the point for the Milky Way on the Tully-Fisher relation figure, which you can find on the course web page for February 19. Include an estimate of the uncertainty in \(V_{\text{max}}\) drawn from previous discussions in class.

2. The sphere of influence of a supermassive black hole is the region within which it affects the dynamics of the bulge stars around out, because the black hole mass is comparable to the mass of stars within this sphere. The radius of influence is usually defined as

\[
r_{\text{BH}} = \frac{GM_{\text{BH}}}{\sigma^2}
\]

where \(\sigma\) is the radial velocity dispersion of the stars in the bulge. Calculate the radius (in arcseconds) of the sphere of influence of the supermassive black holes in the following galaxies. What does this tell you about which black hole is easiest to detect?

(a) Milky Way: \(M_{\text{BH}} = 4.3 \times 10^6 M_\odot\), \(\sigma_{\text{bulge}} = 105 \text{ km s}^{-1}\), \(d = 8.3 \text{ kpc}\)

(b) M31: \(M_{\text{BH}} = 1.4 \times 10^8 M_\odot\), \(\sigma_{\text{bulge}} = 160 \text{ km s}^{-1}\), \(d = 780 \text{ kpc}\)

(c) M87: \(M_{\text{BH}} = 6.5 \times 10^9 M_\odot\), \(\sigma_{\text{bulge}} = 375 \text{ km s}^{-1}\), \(d = 16.8 \text{ Mpc}\)

3. Calculate the apparent size of the photon capture radius in arcseconds for each of the black holes in Problem 2. What does this tell you about which black hole is easiest to image directly?