Astronomy UN2002 | Spring 2020
Due Wed. Feb. 26 | Prof. J. Halpern

Homework #5

1. The star S2 orbits the Galactic center on an elliptical orbit with semimajor axis $a = 970$ AU, eccentricity $e = 0.885$, and orbital period $P = 16.05$ yr. Evaluate

   (a) the mass of the central black hole,

   (b) the pericenter distance in AU,

   (c) the orbital velocity at pericenter, in km s^{-1}, and as a fraction of the speed of light.

2. The disk of a spiral galaxy typically has an exponential luminosity and surface mass density profile:

$$I(r) = I_0\, e^{-r/h_r} \quad \mathrm{L}_\odot\ \mathrm{pc}^{-2}$$
$$\Sigma(r) = \Sigma_0\, e^{-r/h_r} \quad \mathrm{M}_\odot\ \mathrm{pc}^{-2}$$

   where r is the physical radius and h_r is the radial scale length.

   (a) Show that the apparent surface brightness of the disk as a function of radius is

$$\mu(r) = \mu_0 + 1.086\,(r/h_r) \quad \text{mag arcsec}^{-2}$$

   where μ_0 is the central surface brightness, which is related to I_0. Also, explain why $\mu(r)$ is independent of the distance to a galaxy.

   (b) Galaxies of a given spiral type tend to have the same I_0 and Σ_0 regardless of their h_r, resulting in $\mu_0 \approx 21.5$ mag arcsec^{-2} in the B band. Show that fixing Σ_0 and I_0 implies that the total luminosities of these galaxies should be proportional to the fourth power of their rotation velocities.

3. Elliptical galaxies usually have a surface brightness profile

$$\log_{10}\left[\frac{I(r)}{I_h}\right] = -3.3307\left[\left(\frac{r}{r_h}\right)^{1/4} - 1\right],$$

   the de Vaucouleurs "law," where r_h is the "effective," or half-light radius, which contains half of the total luminosity of the galaxy, and I_h is the surface brightness at r_h.

   (a) Show that this is equivalent to

$$\mu(r) = \mu_h + 8.3268\left[\left(\frac{r}{r_h}\right)^{1/4} - 1\right] \quad \text{mag arcsec}^{-2}$$

   (b) The elliptical galaxy NCG 3379 has $\mu_B(0) \approx 14$ mag arcsec^{-2} and $r_h = 3$ kpc. At what radius r does its surface brightness drop to $\mu_B(r) = 21.5$ mag arcsec^{-2} ?