Astronomy UN2002 | Spring 2020
Due Wed. Feb. 5 | Prof. J. Halpern

Homework #2

1. The interstellar medium of the Milky Way Galaxy contains approximately $5 \times 10^9\ M_\odot$ of atomic hydrogen. Assume that 3/4 of the H atoms are in the upper level of the hyperfine (spin-flip) transition, which decays at the rate $A_{21} = 2.85 \times 10^{-15}\ \mathrm{s}^{-1}$. That is, the mean lifetime of the upper level is $A_{21}^{-1} = 11$ million years.

   (a) What is the total rate [photons s^{-1}] of the 21 cm line emitted by the Galactic hydrogen, and the luminosity in the 21 cm line?

   (b) If there were a Milky Way twin at a distance of 1 Mpc, what flux of 21 cm line photons would arrive from it, in units of [photons $\mathrm{m}^{-2}\ \mathrm{s}^{-1}$]?

2. The wavelengths λ of the hydrogen atom are given by

$$\frac{1}{\lambda} = R_H \left(\frac{1}{n^2} - \frac{1}{m^2} \right)$$

   where n and m are integers,

$$R_H = \frac{\mu\, e^4}{8\, \epsilon_0^2\, c\, h^3} = 10967760\ \mathrm{m}^{-1}$$

   is the Rydberg constant, and

$$\mu = \frac{m_e m_p}{(m_e + m_p)}$$

   is the reduced mass, so-called because it is less than the mass of the electron.

   (a) Calculate the wavelength of H109α (the m=110 $\rightarrow$ n=109 transition of hydrogen). Convert it to frequency.

   (b) The so-called Rydberg atoms have one very highly excited electron and behave like hydrogen atoms, in that the excited electron orbits very far from a "nucleus" of effectively +1 positive charge. That is, for an atom of atomic number Z, the inner $Z-1$ electrons cancel the charge of all but one of the protons. This means that you can use the hydrogenic formula above to calculate the wavelengths of radio recombination lines from Rydberg atoms. Calculate the frequencies of He109α and C109α, the analogous transitions of helium and carbon, by replacing the mass of the proton in the reduced mass μ with the mass of the "nucleus" in each case. Check your results against the figure "Radio Recombination Lines" on the course web page. Hint: It's easier *not* to recompute the Rydberg constant from scratch, but just correct it for the change in reduced mass μ. Also, note that the formulas on the web page use cgs units for the Rydberg constant, whereas the above formula is in SI units. This can be confusing because formulas involving electromagnetic forces, as well as the value of charge of the electron are, unfortunately, *different* between the two systems of units.

3. Consider the H II region surrounding an O6 V star, as described at the end of Section 16.3.1 in Ryden & Peterson.

   (a) Estimate the recombination time in the H II region (the average time it takes a proton to capture a free electron).

   (b) Estimate the total thermal energy (in J) in the H II region, that is, the kinetic energy of all of the electrons and protons. (For this purpose you may neglect the elements other than hydrogen.)

   (c) Estimate the heating rate of the H II region by the central star (in J s^{-1}) using the approximation presented in section 16.3.2 of the text, concluding in Equation 16.29.

   (d) Divide the result of (b) by the result of (a), and compare that to the result of (c). Discuss the meaning of this comparison. Are they equal? If not, why not?