Astronomy UN2002 (Introduction to Astrophysics II) Spring 2020
Due Tues. May 12 Prof. J. Halpern

Homework #12

1. The thin crust of the Earth comprises $\sim 1\%$ of the mass of the planet. Potassium is the seventh most abundant element in the Earth's crust, accounting for 2.6% of its mass. Only 0.012% of potassium is the radioactive isotope ^{40}K, which has a half-life of 1.25×10^9 yr. The average energy released per decay of ^{40}K is 1.33 MeV. Estimate the heating rate of the crust, in W m^{-2}, due to the decay of ^{40}K. How does this compare to the flux of solar radiation on the Earth?*

2. The nucleus of a comet rotates rapidly and has an albedo $A = 0.05$. When its surface temperature reaches 150 K, its ice starts to sublime and forms a gaseous coma. How far is the comet from the Sun when this happens?

3. We didn't talk about possible dynamical effects of the solar wind (Chapter 7.1) on small bodies in the solar system. The Sun emits $\sim 10^9$ kg s^{-1} of mostly electrons, protons, and helium nuclei, with a velocity of 400 km s^{-1} measured at $r = 1$ AU. Compare the momentum flux (momentum per unit area per unit time) of the solar wind to the momentum flux of solar radiation at 1 AU.

4. A pebble on a circular orbit around the Sun loses angular momentum due to Poynting-Robertson drag at a rate
$$mr\frac{dr}{dt} = -\frac{L_\odot}{2c^2}R^2$$
where the spherical pebble's radius is R and its mass is m.

   (a) Estimate how long it will take a pebble of $R = 0.01$ m and density 3000 kg m^{-3} to spiral in to the Sun from $r = 5$ AU.

   (b) Over the age of the solar system, the Poynting-Robertson effect would clear the main asteroid belt of rocks smaller than what size? (In actuality, small rocks are replenished by comets and fragments of asteroids that collide.)

* Optional: 11% of the ^{40}K decays produce ^{40}Ar. The rest produce ^{40}Ca. If you have extra time, it would be interesting to estimate the amount of ^{40}Ar produced over the age of the Earth, and see how your estimate compares to the amount of argon in the atmosphere now.