Puzzles in Cosmology

1) Flatness Problem (Why is Ω so close to 1)

$$\Omega = 1 + \frac{(\Omega_0 - 1)}{1 + \Omega_0 z} \quad (8)$$

matter only

This formula is for a matter-only universe. More generally

$$\Omega = \frac{\rho}{\rho_c} = \frac{8\pi G}{3H^2}\rho \quad (2)$$

$$H^2(1-\Omega)\frac{1}{(1+z)^2} = H_0^2(1-\Omega_0) \quad (6)$$

Friedmann Eq.

$$\Omega H^2 = (1+z)^3 \Omega_0 H_0^2 \quad (7)$$

matter only

$$\Omega H^2 = (1+z)^4 \Omega_0 H_0^2 \quad (79)$$

radiation only

The equivalent of Equation (8) for a radiation dominated universe, like our early universe, is

$$\Omega = 1 + \frac{\Omega_0 - 1}{1 + 2\Omega_0 z + \Omega_0 z^2} \quad (89)$$

In general, the Friedmann Equation (6) tells us that

$$|\Omega - 1| \propto \frac{1}{[a(t)\ H(t)]^2} \propto \left(\frac{da}{dt}\right)^{-2}$$

In the matter-dominated era

$$\left.\begin{array}{l} a(t) \propto t^{2/3} \\ \dfrac{da}{dt} \propto t^{-1/3} \end{array}\right\} \quad |\Omega - 1| \propto t^{2/3} \propto a(t)$$

In the radiation-dominated era

$$\left.\begin{array}{l} a(t) \propto t^{1/2} \\ \dfrac{da}{dt} \propto t^{-1/2} \end{array}\right\} \quad |\Omega - 1| \propto t \propto a(t)^2$$

Example: at $t = 200$ s, $T = 1 \times 10^9$ K (BBN)

$$(1+z) = T/T_0 = 10^9/2.73 = 3.7 \times 10^8$$

Assuming that $|\Omega_0 - 1| < 0.1$, and extrapolating over the matter dominated and radiation dominated eras to $t = 200$ s

$$|\Omega - 1| < 10^{-14}$$

2) Horizon Problem (Why is the CMB so smooth)

The distance a photon travels is

$$r = \frac{2c}{H_0}\left[1 - \frac{1}{\sqrt{1+z}}\right] \quad \text{(matter only)}$$

$$r = \frac{c}{H_0}\left[1 - \frac{1}{1+z}\right] \quad \text{(radiation only)}$$

The horizon distance is

$$l_h = \frac{2c}{H_0} \text{ (matter)} \qquad l_h = \frac{c}{H_0} \text{ (radiation)}$$

How did we compute these?

$$a(t)\, dr_0 = c\, dt$$

$$\int_r^0 dr_0 = \int_t^{t_0} \frac{c\,dt}{a(t)}$$

$$r = \int_t^{t_0} \frac{c\,dt}{(t/t_0)^{1/2}} \quad \text{for radiation only}$$

$$r = 2ct_0^{1/2}\left(t_0^{1/2} - t^{1/2}\right) = 2ct_0\left[1 - \left(\frac{t}{t_0}\right)^{1/2}\right]$$

$$r = 2ct_0(1-a)$$

But $\frac{da}{dt} = \frac{1}{2} \frac{t^{-1/2}}{t_0^{1/2}} = \frac{1}{2} \frac{a}{t}$

$$t_0^{-1} = \frac{2}{a} \frac{da}{dt} = 2 H_0$$

$$t_0 = \frac{1}{2 H_0}$$

$$r = \frac{c}{H_0} \left[1 - \frac{1}{1+z} \right] \qquad \ell_h = \frac{c}{H_0}$$

In general, for $a(t) = (t/t_0)^n$ $\quad 0 < n < 1$

(decelerating)

$$\ell_h = \frac{n}{1-n} \frac{c}{H_0}$$

(see page 546 of text)

In the Consensus Model

$$\ell_h = 3.24 \frac{c}{H_0} \approx 14{,}000 \text{ Mpc}$$

At $z = 1100$, the horizon distance corresponds to an angular size of only $\sim 1.7^\circ$ on the sky.

Yet, the CMB temperature is the same everywhere to a fraction 10^{-5}.

Solution: Inflation, caused by a very early epoch of Λ (much larger than the present Λ) in which the universe expanded exponentially.

Note: In a $\underline{\Lambda \text{ only}}$ universe

(25) $$\frac{da}{dt} = H_0\left(\cancel{\frac{\Omega_{r,0}}{a^2}} + \cancel{\frac{\Omega_{m,0}}{a}} + \Omega_{\Lambda,0}\, a^2\right)^{1/2}$$

$$\frac{1}{a}\frac{da}{dt} = H_0$$

$$\int_a^1 \frac{da}{a} = \int_t^{t_0} H_0\, dt$$

$$-\ln a = H_0(t_0 - t) = -\ln\left(\frac{1}{1+z}\right)$$

$$a = e^{H_0(t-t_0)} = \frac{1}{1+z}$$

$$r = \int_t^{t_0} \frac{c\,dt}{a(t)} = c\int_t^{t_0} e^{H_0(t_0-t)}\, dt$$

$$r = \frac{c}{H_0}\left[e^{H_0(t_0-t)} - 1\right] = \frac{c}{H_0}\left[1+z-1\right]$$

$$r = \frac{cz}{H_0} \quad \text{and} \quad l_h = \infty$$

Solves

Flatness Problem!

Suppose that rapid inflation occurred between $t_1 = 10^{-35}$ s and $t_2 = 10^{-33}$ s, such that

$$H_1 \simeq t_1^{-1} , \quad \text{and} \quad H_1 (t_2 - t_1) = 100$$

$$\text{Then} \quad \frac{a(t_2)}{a(t_1)} = e^{100} = 10^{43}$$

$$\text{and} \quad |\Omega - 1| \propto \frac{1}{[a(t)\, H(t)]^2} \propto \frac{1}{e^{2H_1 t}} \propto e^{-2H_1 t}$$

$$\text{Then} \quad \frac{|\Omega_2 - 1|}{|\Omega_1 - 1|} = e^{-2H_1 t} \simeq e^{-200} \simeq 10^{-87}$$

and Ω after inflation can be very close to 1.

Horizon Problem:

$\boxed{\text{At } t_2 = 10^{-33}\text{ s, the end of inflation}}$ the scale factor $a \approx 8 \times 10^{-27}$

So the size of the currently visible universe was then $l_2 = a\, l_{h,0}$

$$l_2 = 8 \times 10^{-27} \times 14{,}000 \text{ Mpc} = 4 \text{ m}$$

$\boxed{\text{At } t_1 = 10^{-35}\text{ s, the beginning of inflation}}$ $l_{h,1} = 2ct_1 = 6 \times 10^{-27}$ m

$l_{h,1}$ = horizon size at t_1

$$l_1 = e^{-100}\, l_2 \simeq 10^{-43} \text{ m} \ll l_{h,1}$$

So everything we see now was in communication with itself.