Big Bang Nucleosynthesis

The density parameter $\Omega_{m,0} \approx 0.3$ is due mostly to dark matter, but the baryonic (ordinary) matter comprises only $\Omega_{b,0} = 0.044$. This result was known originally from calculations of nucleosynthesis in the early universe which matched the observed abundances of the light elements ^4He, ^2H, ^7Li, and is a cornerstone of the Big Bang theory.

When the temperature was high enough, the only nucleons were free protons and neutrons which reacted quickly in statistical equilibrium via the reactions:

$$n \rightarrow p + e^- + \bar{\nu} \qquad T_{1/2} = 611 \text{ seconds}$$
$$e^+ + n \leftrightarrow p + \bar{\nu}$$
$$\nu + n \leftrightarrow p + e^-$$

In statistical equilibrium the neutron-to-proton ratio is

$$\frac{n_n}{n_p} = e^{-\Delta E/kT}$$

where $\Delta E = (m_n - m_p)c^2 = 1.293$ MeV

That is because $m_n c^2 = 939.565$ MeV
$m_p c^2 = 938.272$ MeV

The temperature required to maintain a substantial fraction of free neutrons can be expressed as $kT = \Delta E$, or

$$T = \frac{\Delta E}{k} = \frac{1.293\ \text{MeV}}{1.38 \times 10^{-23}\ \text{J/K}} \times \frac{1.602 \times 10^{-13}\ \text{J}}{\text{MeV}} = 1.5 \times 10^{10}\ \text{K}$$

Protons and neutrons were kept from combining to become deuterium by blackbody photons (γ) that were in thermal equilibrium with matter

$$p + n \leftrightarrow {}^{2}H + \gamma \qquad \gamma = 2.22\ \text{MeV}$$

The peak frequency of the blackbody spectrum corresponds to $h\nu = 2.82\ kT$, or

$$T = \frac{h\nu}{2.82\ k} = \frac{2.22\ \text{MeV}}{2.82 \times 1.38 \times 10^{-23}\ \text{J/K}} \times \frac{1.602 \times 10^{-13}\ \text{J}}{\text{MeV}}$$

$$T = 9 \times 10^{9}\ \text{K}$$

At lower temperatures than this, nuclei started to form, beginning with deuterium (^2H). Also, the positrons annihilated with electrons to create 0.511 MeV photons as T decreased

$$e^+ + e^- \leftrightarrow \gamma + \gamma \qquad \gamma = 0.511\ \text{MeV}$$

Nucleosynthesis proceeds with a series of reactions:

$$p + n \to {}^2H + \gamma$$

$$^2H + n \to {}^3H + \gamma$$

$$^3H + p \to {}^4He + \gamma$$

$$^2H + p \to {}^3He + \gamma$$

$$^3He + n \to {}^4He + \gamma$$

$$^2H + {}^2H \to {}^3He + n$$

$$^2H + {}^2H \to {}^3H + {}^1H$$

$$^3H + {}^2H \to {}^4He + n$$

$$^3He + {}^2H \to {}^4He + {}^1H$$

The net result is that almost all of the neutrons end up in ^4He, with a tiny fraction in ^2H and ^3He. Tritium (^3H) is produced, but it has a half-life of 12.5 yr, decaying into ^3He (a neutron decays to a proton)

There is no stable nucleus with a mass of 5, so only trace amounts of ^6Li, ^7Li, and ^7Be can be produced via

$$^4He + {}^2H \rightarrow {}^6Li + \gamma$$

$$^4He + {}^3H \rightarrow {}^7Li + \gamma$$

$$^4He + {}^3He \rightarrow {}^7Be$$

There is again no stable nucleus with mass 8. This is why the triple-alpha process in stars:

$$^4He + \underbrace{{}^4He + {}^4He}_{^8Be} \rightarrow {}^{12}C$$

requires high density, because unstable ^8Be only lasts for a short time, during which it has to be hit by a ^4He.

<u>Compare BBN with stellar nucleosynthesis (PPI)</u>

$$p + p \rightarrow {}^2H + e^+ + \nu \quad \leftarrow \text{no free neutrons, very slow, } \tau \simeq 10^{10}\,\text{yr}$$

$$^2H + p \rightarrow {}^3He + \gamma \quad \text{twice}$$

$$^3He + {}^3He \rightarrow {}^4He + p + p$$

Figure 26-1 on the April 27 class notes for "Big Bang Nucleosynthesis" is the standard model time line for the formation of the light nuclei:

1. Before $t = 2$ seconds, the temperature was $> 10^{10}$ K, and the reactions that create and destroy neutrons and protons (see page 1) were fast, and in thermal equilibrium, with protons more numerous than neutrons.

2. After $t = 2$ seconds, the reactions began to take longer than the age of the universe (because of the decreasing density and temperature) and the neutron-to-proton ratio became "fixed" at $\approx \frac{1}{6}$.

3. At around 220 seconds the n/p ratio had declined to $\simeq \frac{1}{7}$ because the neutrons were decaying with their half-life of 611 seconds, (still longer than the age of the universe!) Since the temperature is now 10^9 K, there are no longer photons that can destroy deuterium, which is the first nucleus that forms

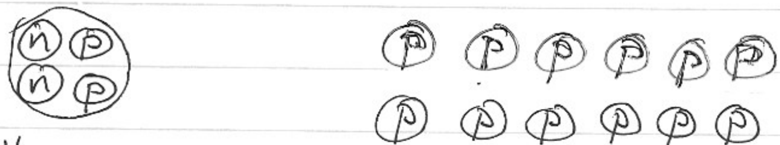
$$p + n \rightarrow {}^{2}H + \gamma \qquad \gamma = 2.22 \text{ MeV}$$

4. Once deuterium was made, it reacted to form other species, namely ^3He, ^4He, ^6Li, ^7Li, ^7Be.

Almost all of the neutrons that didn't decay ended up in ^4He. This is why the primordial mass fractions are

H	75%
He	25%

There were 2 neutrons for 14 protons. Of the 16 nucleons, ^4He has 4 and H has 12.



^4He 25%   H 75%

After $\approx \frac{1}{2}$ hour, primordial nucleosynthesis was complete.

The results shown in Figure 26-1 are very sensitive to the assumed density of baryons, which was chosen to best match the observed abundances of the light elements. Figure 29.14 shows what the final abundances of the light elements are in models that assume different values of $\Omega_{b,0}$. The dark, vertical shade line corresponds to $\Omega_{b,0} = 0.044$, which agrees with the observed abundances of the elements.

But how was the temperature and density modelled at such an early time? Start with the result of applying the 1st law of thermodynamics,

$$\frac{d\rho}{dt} = -3\left(\rho + \frac{P_r}{c^2}\right)\frac{1}{a}\frac{da}{dt}$$

BBN is well inside the radiation-dominated era, so $\rho = \rho_r$ and $P_r = (1/3)\rho c^2$. Then

$$\frac{d\rho}{dt} = -4\rho H \qquad (1)$$

Since $\Omega = 1$ at early times, ρ must be equal to the critical density.

$$\rho_c = \frac{3H^2}{8\pi G} \qquad (2)$$

Combing Equation (1) and (2) to eliminate H,

$$\frac{d\rho}{dt} = -\left(\frac{128\pi G}{3}\right)^{1/2}\rho^{3/2}$$

$$\int_{\infty}^{\rho} d\rho\, \rho^{-3/2} = -\left(\frac{128\pi G}{3}\right)^{1/2}\int_0^t dt$$

which has the solution

$$\rho = \frac{3}{32\pi G t^2}$$

Since $\rho = \rho_r = \frac{u_r}{c^2} = \frac{4\sigma T^4}{c^3}$

$$T^4 = \frac{3c^3}{128\pi\sigma G t^2}$$

$$T = \frac{1.52\times10^{10}}{t^{1/2}} \text{ K}$$

This shows that the temperature was 10^{10} K at a time of 2 seconds, or 10^9 K at a time of 200 s, as Figure 26-1 indicates.

Remember that ρ is the density of radiation, not matter. But you can use the temperature as a measure of redshift, $T = (1+z)T_0$, where $T_0 = 2.725$ K, and then use $(1+z)^3 = \rho_b/\rho_{b,0}$ to find ρ_b, the actual density of baryons.
Remember that

$$\Omega_{b,0} \equiv \frac{\rho_{b,0}}{\rho_{c,0}} = 0.044 \qquad (3)$$

The reason that Figure 29.14 graphs $\Omega_{b,0} H_0^2$ instead of just $\Omega_{b,0}$ is because the nuclear calculations are sensitive to ρ_b, which is related to $\rho_{b,0}$, which is in turn, from Equation (3)

$$\rho_{b,0} = \Omega_{b,0} \, \rho_{c,0}$$

$$= \Omega_{b,0} \, \frac{3 H_0^2}{8 \pi G}$$

Before H_0 was known accurately, BBN calculations could only tell you the value of the combination

$$\Omega_{b,0} H_0^2 ,$$

not $\Omega_{b,0}$ itself.

In the Figure, lower case "h" refers to

$$h = \frac{H_0}{100 \text{ km s}^{-1} \text{ Mpc}^{-1}}$$

So we would now say $h = 0.71$, $h^2 = 0.50$.