Accretion-Disk Luminosity and Temperature

Elaborates on Section 21.2.3 Accretion Disks

Side View

Top View

Mechanical Energy in a ring of mass $dm$

\[ E = \frac{1}{2} dm \cdot v^2 - \frac{GM_{BH}}{r} dm = K + U \]

\[ E = \frac{1}{2} U = -\frac{1}{2} \frac{GM_{BH}}{r} dm = -2K \]

At the inner radius

\[ E = -\frac{1}{2} \frac{GM_{BH}}{r_{in}} dm \]

Total Luminosity radiated

\[ \frac{dE}{dt} = \frac{1}{2} \frac{GM_{BH} \cdot dm}{r_{in}} \cdot \frac{dt}{dt} = \frac{1}{2} \frac{GM_{BH} \cdot \dot{M}}{r_{in}} \]

Let $r_{in} = \frac{6GM_{BH}}{C^2}$

\[ \frac{dE}{dt} = \frac{1}{12} \frac{\dot{M}C^2}{} \]

Efficiency $\eta = \frac{1}{12}$
What is the flux per unit area emitted by the disk?

Energy in a ring

$$E = -\frac{1}{2} \frac{G M_{BH}}{r} \frac{d m}{d r}$$

Energy emitted when $d m$ moves

$$d E = \frac{1}{2} \frac{G M_{BH}}{r^2} d m \frac{d r}{r^2}$$

Flux $F(r) = \frac{d E}{d t \cdot d A} = \frac{1}{2} \frac{G M_{BH} d m}{r^2} \frac{d r}{d t} (2 \times 2 \pi r d r)$

$$F(r) = \frac{G M_{BH} M}{8 \pi r^3}$$

This is an approximation.

A more accurate expression is

$$F(r) = \frac{3}{8 \pi} \frac{G M_{BH} \dot{M}}{r^3} \left[ \frac{1 - \left( \frac{r_{in}}{r} \right)^{1/2}}{\left( \frac{r}{r} \right)} \right]$$

If the disk emits as a blackbody, $F(r) = \sigma T^4$

$$T(r) = \left\{ \frac{3}{8 \pi \sigma} \frac{G M_{BH} \dot{M}}{r^3} \left[ 1 - \left( \frac{r_{in}}{r} \right)^{1/2} \right] \right\}^{1/4}$$

Compare with Equation (21.23) of text...