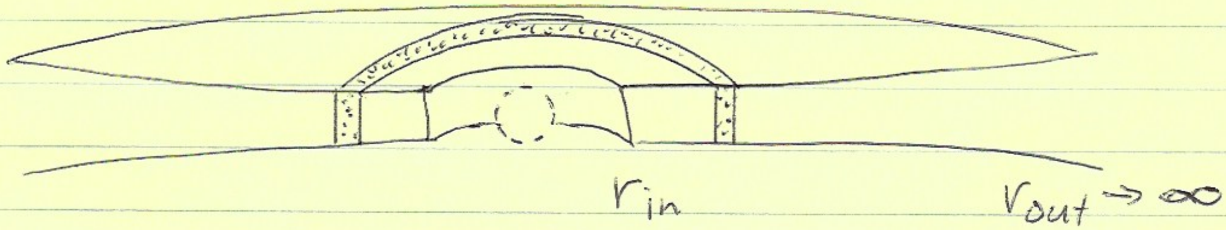
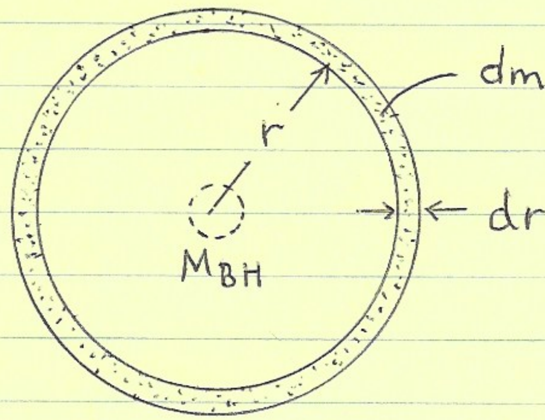
Accretion-Disk Luminosity and Temperature

Elaborates on Section <u>21.2.3 Accretion Disks</u>

Side view



Top View



$\leftarrow \dot{M} \equiv \dfrac{dm}{dt}$

Mechanical Energy in a ring of mass dm

$$E = \frac{1}{2} dm\, v^2 - \frac{G M_{BH}\, dm}{r} = K + U$$

$K = -\frac{1}{2}U$, $U = -2K$

$$E = \frac{1}{2} U = -\frac{1}{2} \frac{G M_{BH}\, dm}{r}$$

At the inner radius

$$E = -\frac{1}{2} \frac{G M_{BH}\, dm}{r_{in}} \qquad (E = 0 \text{ at } r = \infty)$$

Total Luminosity radiated

$$\frac{dE}{dt} = \frac{1}{2} \frac{G M_{BH}}{r_{in}} \frac{dm}{dt} = \boxed{\frac{1}{2} \frac{G M_{BH} \dot{M}}{r_{in}}}$$

Let $r_{in} = \dfrac{6 G M_{BH}}{c^2} \Rightarrow \boxed{\dfrac{dE}{dt} = \dfrac{1}{12} \dot{M} c^2}$ efficiency $\eta = \dfrac{1}{12}$

What is the flux per unit area emitted by the disk?

Energy in a ring $\quad E = -\frac{1}{2} \frac{G M_{BH}\, dm}{r}$

Energy emitted when dm moves $\quad dE = \frac{1}{2} \frac{G M_{BH}\, dm\, dr}{r^2}$ $\quad$ when dm moves by dr

Flux $\quad F(r) = \frac{dE}{dt\, dA} = \frac{1}{2} \frac{G M_{BH}\, dm\, dr}{r^2\, dt\, (2 \times 2\pi r\, dr)}$

← 2 faces of disk

$$F(r) = \frac{G M_{BH} \dot{m}}{8\pi r^3}$$

This is an approximation

A more accurate expression is

$$\boxed{F(r) = \frac{3}{8\pi} \frac{G M_{BH} \dot{m}}{r^3} \left[1 - \left(\frac{r_{in}}{r} \right)^{1/2} \right]}$$

If the disk emits as a blackbody, $F(r) = \sigma T^4$

$$T(r) = \left\{ \frac{3}{8\pi\sigma} \frac{G M_{BH} \dot{m}}{r^3} \left[1 - \left(\frac{r_{in}}{r} \right)^{1/2} \right] \right\}^{1/4}$$

Compare with Equation (21.23) of text...