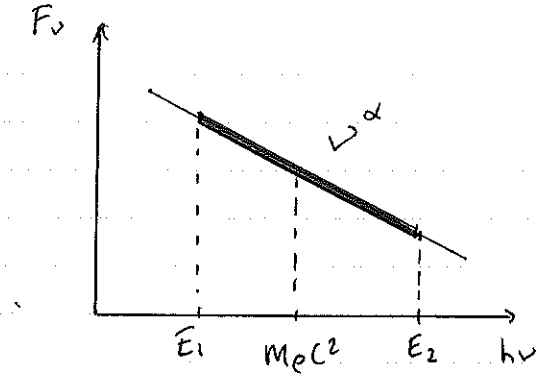
Accounting for the spectral index α twice, we have

$$\frac{L}{L'} = \delta^{3-2\alpha}$$



The compactness parameter in the jet frame is

$$\tau' = \frac{L' \sigma}{4\pi m_e c^4 \Delta t'} \qquad \text{where } \Delta t' = \delta \Delta t$$

$$\text{then } \tau' = \frac{L \sigma}{4\pi m_e c^4 \Delta t} \frac{1}{\delta^{4-2\alpha}} = \frac{\tau}{\delta^{4-2\alpha}}$$

In the case of Mrk 421, $\alpha = -0.63$, $\tau = 10^5$, therefore

$$\delta > (10^5)^{\frac{1}{4-2\alpha}} = (10^5)^{0.19} = 9$$

Extensive Air Showers and Atmospheric Cherenkov Telescopes

120 4 *Interactions of high energy photons*

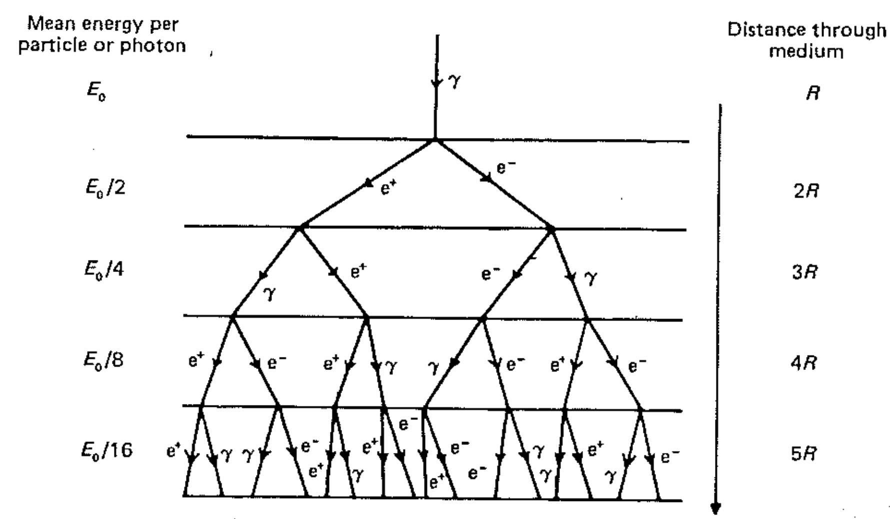


Figure 4.17. A simple model for an electromagnetic shower.

Nucleonic (Hadronic) Air Showers

5.5 *Cosmic rays in the atmosphere* 149

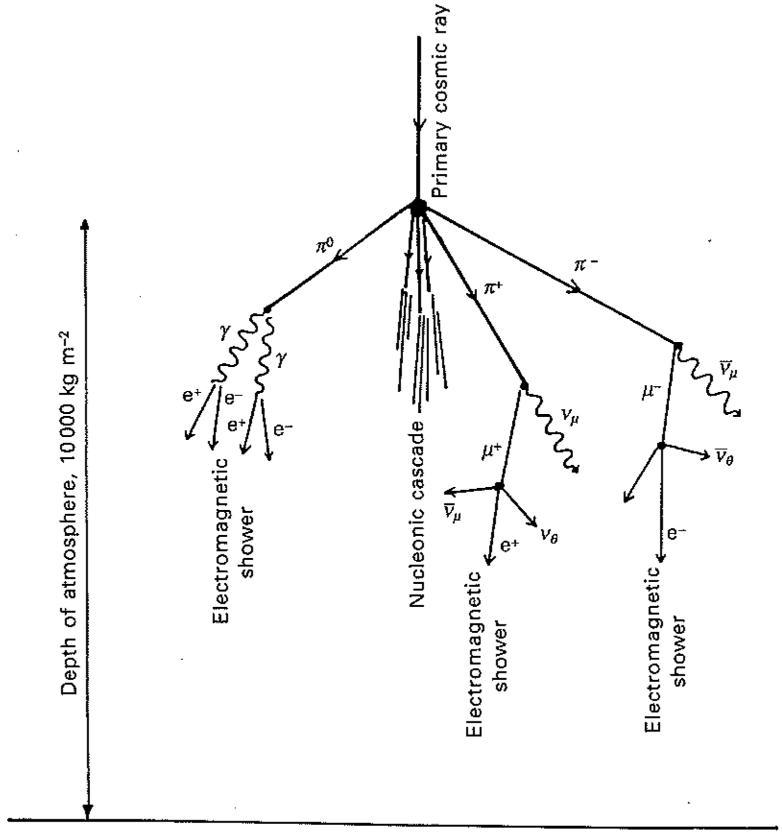


Figure 5.11. A schematic diagram showing the development of a nucleonic cascade in the atmosphere. Such cascades initiated by high energy particles develop in exactly the same fashion inside cosmic ray telescopes.

These are much more numerous than electromagnetic (leptonic) air showers, Nucleonic air showers are caused by cosmic ray particles, and are a large source of background for ACTs (atmospheric Cherenkov telescopes) that observe gamma-rays, which were discussed on page 10 of lecture notes. Hadronic showers can be distinguished on the basis of their shape, which is "lumpier" than electromagnetic showers.

Pair Production in the Atmosphere and EAS

The atmosphere is mostly nitrogen, for which 10^{12} eV γ-rays have a pair-production cross section $\sigma_{pair} \approx 4\times10^{-25}$ cm²

$$\gamma + {}^{14}N \rightarrow {}^{14}N + e^- + e^+$$

which initiates the extensive air shower shown on the previous page. Each charge radiates a new γ-ray by bremsstrahlung, with approximately the same cross section, doubling the number of particles/photons after each radiation length, R. The radiation length is

$$R = \frac{1}{\kappa_{pair}} = \frac{m_N}{\sigma_{pair}} = \frac{14\times1.66\times10^{-24}}{4\times10^{-25}} = 58 \ \mathrm{g/cm^2}$$

The atmosphere's density profile is approximately exponential with height:

$$\rho(z) = \rho_0 \, e^{-z/z_h} \qquad \text{with } \rho_0 = 1.2\times10^{-3} \ \mathrm{g\,cm^{-3}}, \quad z_h = 8.6\times10^{5} \ \mathrm{cm}$$

The height at which the primary γ-ray converts is h, where

$$\rho_0 \int_h^\infty e^{-z/z_h}\, dz = \frac{1}{\kappa_{pair}}$$

$$\rho_0 z_h e^{-h/z_h} = \frac{1}{\kappa_{pair}} \qquad \text{with } \frac{1}{\kappa_{pair}} = 58 \ \mathrm{g/cm^2}$$

$$h = z_h \ln(\kappa_{pair}\rho_0 z_h) = 2.5\times10^{6} \ \mathrm{cm} = 25 \ \mathrm{km}$$

The extensive air shower ends when the average energy per particle drops to 100 MeV. For a primary γ-ray of energy 10^{12} eV, this corresponds to a maximum number of particles in the shower of 10^4. The number of radiation lengths m is then

$$2^{(m-1)} = 10^4 \quad \text{or} \quad m = 14.3 \quad \text{and} \quad \frac{m}{\kappa_{pair}} = 14.3\times58 = 829 \ \frac{\mathrm{g}}{\mathrm{cm^2}}$$

The shower ends at a height $h = z_h \ln\left(\frac{\kappa_{pair}\rho_0 z_h}{m}\right)$

$$= 1.9\times10^{5} \ \mathrm{cm} = 1.9 \ \mathrm{km}$$

Cherenkov Radiation from EAS

We do not detect the particles or gamma-rays directly, but a particle moving faster than the speed of light in a medium emits Cherenkov radiation, blue light which is imaged by the atmospheric Cherenkov telescopes.

The speed of light in a medium is c/n, where n is the index of refraction.

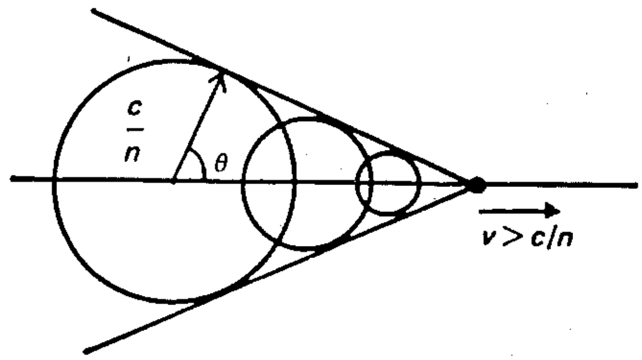


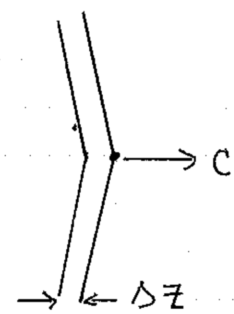
Figure 4.19. Illustrating Huygens' construction for determining the direction of propagation of the wavefront of Cherenkov radiation.

For blue light in air $n = 1.000296$.

$$\cos\theta = \frac{c}{nv}$$

Since the electron has $v \approx c$, $\cos\theta = 0.9997$ and $\theta = 1.^\circ 4$. The Cherenkov "cone" is almost flat, like a "pancake".

The thickness Δz of the pancake of light is determined by the difference in speeds of the photons and particles.



$$\Delta z = (c - c/n)\,\Delta t$$

where $\Delta t \approx 3 \times 10^{-5}$ s, the time to travel ≈ 10 km

$$\Delta z \approx c\Delta t \times 0.000296 \simeq 10 \text{ km} \times 3\times10^{-4} = 3\times10^{-3} \text{ km} = 3 \text{ m}.$$

All of the photons arrive at the ground in a time

$$\frac{\Delta z}{c} = 10^{-8} \text{ s}$$

The flash of photons arrives in a burst only 10 nanoseconds long and spread over a radius of ~100 m. Several million photons can be produced, and because the burst is so short, it can be discriminated from the background light.