

## Overview of Radiative Transitions

The probability per unit time for an induced transition (absorption or stimulated emission) is

$$\langle W_{fi} \rangle = \frac{4\pi^2}{3\hbar^2 c} |d_{fi}|^2 I(\omega_{fi}) \quad [s^{-1}]$$

where  $I(\omega_{fi}) = 4\pi J_\omega = 2J_\nu$

$$|d_{fi}|^2 = \left| \int \phi_f^* \vec{\ell} \cdot \vec{d} \phi_i d^3x \right|^2$$

$$\vec{d} = e \sum_j \vec{r}_j \quad \text{is the electric dipole operator (sum over all electrons)}$$

$\vec{\ell}$  is a unit vector in the direction of polarization of the incident wave

Einstein coefficients are related to  $W_{fi}$

$$\langle W_{fi} \rangle = B_{fi} J_\nu$$

$$g_1 B_{12} = g_2 B_{21}$$

$$B_{12} = \frac{8\pi^2}{3\hbar^2 c} |d_{12}|^2$$

$$A_{21} = \frac{2h\nu^3}{c^2} B_{21}$$

Oscillator strength  $f_{12}$  is defined as

$$\int_0^\infty \sigma(\nu) d\nu = \frac{\pi e^2}{mc} f_{12} = B_{12} \frac{h\nu_{12}}{4\pi}$$

Units of  $B_{12}$  are  $cm^2 s^{-1} erg^{-1}$ , and

$$B_{12} = \frac{4\pi^2 e^2}{mc h \nu_{12}} f_{12}$$

$$f_{12} = \frac{2}{3} \frac{m}{e^2} \frac{h \nu_{12}}{\hbar^2} |d_{12}|^2$$

If there is more than one lower level state, with degeneracy  $g_1$ , then

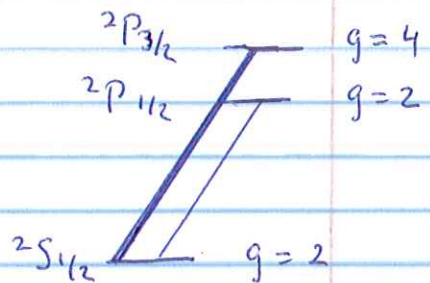
$$g_1 f_{12} = \frac{2}{3} \frac{m}{e^2} \frac{h\nu_{12}}{h^2} \sum_i |d_{i2}|^2$$

The value of  $gf$  within a multiplet is proportional to the statistical weight of the upper level of the transition. For hydrogen Lyman  $\alpha$ , for example

Total  $gf = 0.8324$  (multiplet)

$J = \frac{1}{2} \rightarrow J = \frac{1}{2}$      $gf = 0.2775$

$J = \frac{1}{2} \rightarrow J = \frac{3}{2}$      $gf = 0.5549$



The statistical weight of the level is  $2J+1$

The statistical weight of the multiplet is  $(2S+1)(2L+1)$

Electric Dipole Selection rules for hydrogen

1. Single-electron transitions
2.  $\Delta L = \pm 1$
3.  $\Delta m = 0, \pm 1$
4.  $\Delta S = 0$
5.  $\Delta j = 0, \pm 1$  (but  $j=0 \nrightarrow j=0$ )
6.  $\Delta m_j = 0, \pm 1$  (but  $m_j=0 \nrightarrow m_j=0$  if  $\Delta j=0$ )

Electric Dipole Selection rules for multi-electron atoms

$\Delta S = 0$   
 $\Delta L = 0, \pm 1$   
 $\Delta J = 0, \pm 1$  (but  $J=0 \nrightarrow J=0$ )

photon must carry away one unit of angular momentum

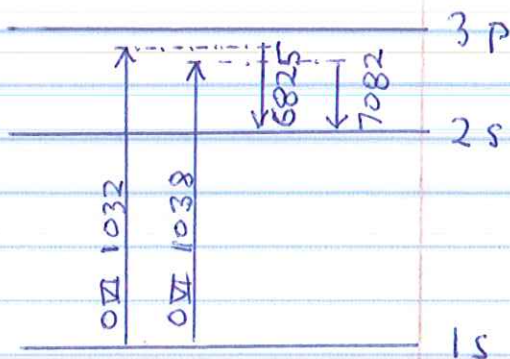


## Raman Scattering (in symbiotic stars)

Fine-structure lines of  $OVI$  1032, 1038 Å

are scattered by neutral hydrogen in the ground state to an intermediate state

that decays to  $n=2$ , emitting the Raman bands at 6825, 7082 Å

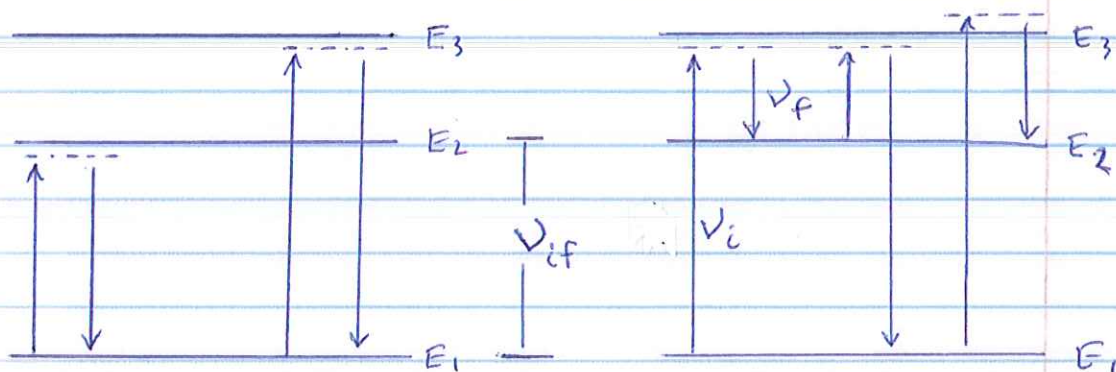


If  $\nu_i$  is the frequency of the incident photon and  $\nu_f$  is the frequency of the scattered photon, then

$$\nu_f = \nu_i - \nu_{if} \quad \text{or} \quad E_i + h\nu_i = E_f + h\nu_f$$

where  $\nu_{if}$  corresponds to the energy difference between the initial and final states of the hydrogen atom.

Raman scattering is closely related to Rayleigh scattering, and both differ from resonance line scattering in that the intermediate state is not a true bound state.



Rayleigh  
(elastic)

Raman  
(inelastic)

Rayleigh scattering may be considered a special case of Raman scattering when  $\nu_i = \nu_f$

Line-broadening Incident lines of intrinsic width  $\Delta\nu_i$  will result in scattered lines of width  $\Delta\nu_f = \Delta\nu_i$  by energy conservation. In wavelength units,  $\lambda_f = c/\nu_f$  and

$$\Delta\lambda_f = \frac{c}{\nu_f^2} \Delta\nu_f = \frac{\lambda_f^2}{c} \Delta\nu_i = \frac{\lambda_f^2}{c} \frac{c}{\lambda_i^2} \Delta\lambda_i$$

$$\text{so } \frac{\Delta\lambda_f}{\Delta\lambda_i} = \left(\frac{\lambda_f}{\lambda_i}\right)^2$$

$$\text{or } \frac{\Delta\nu_f}{\Delta\nu_i} = \frac{\lambda_f}{\lambda_i} \quad \left(\Delta\nu = \frac{\Delta\lambda}{\lambda}\right)$$

Apparent velocity broadening is increased by the ratio

$$\frac{6825}{1032} = 6.6 \quad \text{or} \quad \frac{7082}{1038} = 6.8$$

Cross Sections - now with quantum effects included

$$\text{Rayleigh scattering } \sigma_{\text{Rayleigh}} = \sigma_T \left[ \sum_n \frac{f_{1n}}{(\lambda/\lambda_{1n})^2 - 1} \right]^2$$

$f_{1n}$  = oscillator strengths of Lyman lines  $1 \rightarrow n$ , assuming atom in ground state

Classically  $\lambda \gg \lambda_{1n}$  so  $\sigma_{\text{Ray}} \propto \lambda^{-4}$

Raman Scattering

$$\sigma_{\text{Raman}} = \frac{\sigma_T}{16} \nu_i \nu_f^3 \left[ \sum_n \sqrt{\frac{(gf)_{fn} (gf)_{in}}{\nu_{fn} \nu_{in}}} \frac{(\nu_{in} + \nu_{fn})}{(\nu_{in} - \nu_i)(\nu_{fn} + \nu_i)} \right]^2$$

where  $\nu_i$  is the frequency of the incident photon,  $\nu_f$  the frequency of the scattered photon,  $\nu_{in}$  and  $\nu_{fn}$  are the frequencies of the transitions between initial or final



state of the atom and the level  $n$ . In each term  $n \neq i$  and  $n \neq f$ .

Rayleigh scattering is the case in which  $i=f$ . Then  $\nu_i = \nu_f$ ,  $\nu_{in} = \nu_{fn}$ . The Raman cross section then reduces to

$$\begin{aligned} \sigma &= \frac{\sigma_T}{16} \nu_i^4 \left[ \sum_n \frac{(gf)_{in}}{\nu_{in}} \frac{2\nu_{in}}{(\nu_{in}^2 - \nu_i^2)} \right]^2 \\ &= \frac{\sigma_T}{4} \left[ \sum_n (gf)_{in} \frac{\nu_i^2}{(\nu_{in}^2 - \nu_i^2)} \right]^2 \\ &= \frac{\sigma_T}{4} \left[ \sum_n \frac{(gf)_{in}}{(\lambda_i/\lambda_{in})^2 - 1} \right]^2 \end{aligned}$$

In the ground state of hydrogen ( $^2S_{1/2}$ )  $g=2$  and  $i=1$

$$\sigma = \sigma_{\text{Rayleigh}} = \sigma_T \left[ \sum_n \frac{f_{1n}}{(\lambda/\lambda_{1n})^2 - 1} \right]^2$$

Oscillator strengths for the Lyman series are ( $1s \rightarrow np$ ):

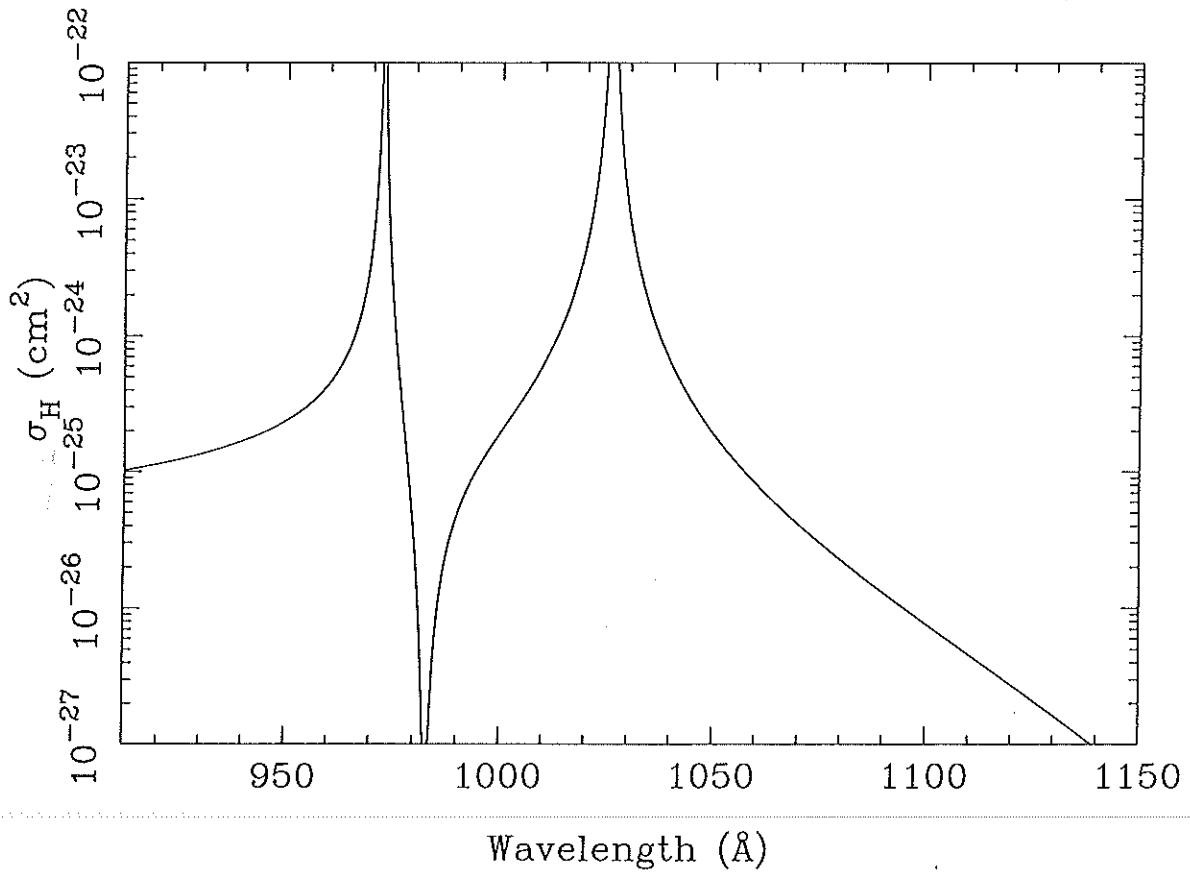
$$f_{1n} = \frac{2^8}{3n^3} \frac{(1 - \frac{1}{n})^{2n-4}}{(1 + \frac{1}{n})^{2n+4}} \quad n=2, 3, 4, \dots$$

In the case of Rayleigh scattering of OIII 1032, 1038, approximate the cross section as the sum of  $n=2-4$

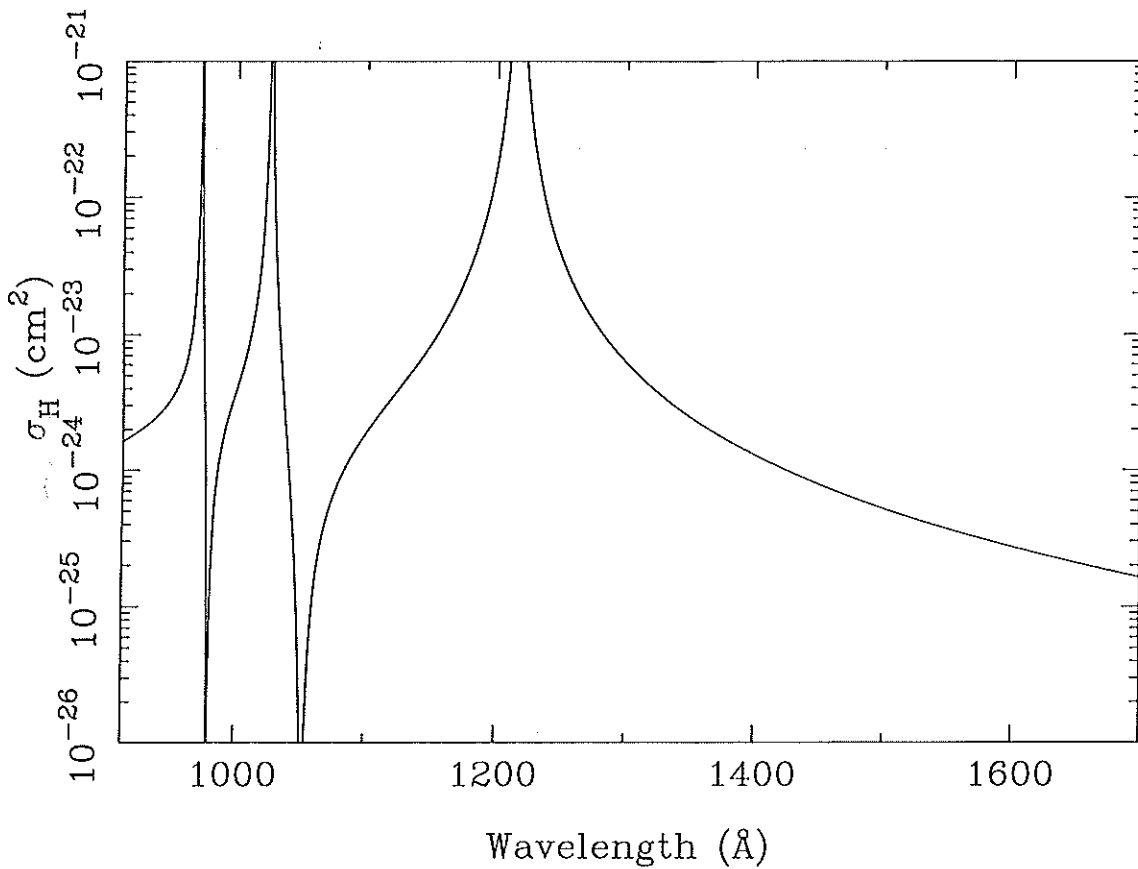
$f_{12} = 0.4162$	$\lambda_{12} = 1215.67 \text{ \AA}$
$f_{13} = 0.0791$	$\lambda_{13} = 1025.72 \text{ \AA}$
$f_{14} = 0.0290$	$\lambda_{14} = 972.54 \text{ \AA}$

$\lambda_i (\text{\AA})$	$\sigma (\text{Rayleigh})$
1031.91	$1.85 \times 10^{-23} \text{ cm}^2$
1037.61	$2.84 \times 10^{-24} \text{ cm}^2$

Raman scattering on Hydrogen (approx  $n \leq 4$ )



Rayleigh scattering on Hydrogen (approx  $n \leq 4$ )



Raman scattering cross sections can be approximated by summing over  $n=3,4$  (not  $n=1$  or  $2$ , since those are the initial and final states of the hydrogen atom, only  $s \rightarrow p$  oscillator strengths contribute. The initial and final states are both  $s$  states, while the intermediate state ( $n$ ) must be a  $p$  state in order to satisfy  $\Delta l = 1$ .  $g=2$  in all cases

$f_{13} (s \rightarrow p) = 0.0791$	$\lambda_{13} = 1025.72 \text{ \AA}$	$L\gamma\beta$
$f_{14} (s \rightarrow p) = 0.0290$	$\lambda_{14} = 972.54 \text{ \AA}$	$L\gamma\delta$
$f_{23} (s \rightarrow p) = 0.4351$	$\lambda_{23} = 6562.74 \text{ \AA}$	$H\alpha$
$f_{24} (s \rightarrow p) = 0.1028$	$\lambda_{24} = 4861.29 \text{ \AA}$	$H\beta$

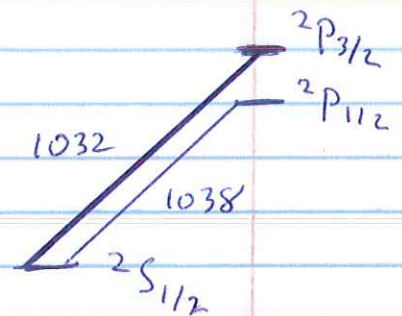
$\lambda_i (\text{\AA})$	$\lambda_f (\text{\AA})$	$\sigma (\text{Raman})$
1031.91	6826.72	$3.65 \times 10^{-24}$
1037.61	7084.22	$9.31 \times 10^{-25}$

### Line Intensity Ratios

If the OVI lines are optically thin they will have an intensity ratio

$$\frac{I(1032)}{I(1038)} = 2$$

according to the relative statistical weights of their upper levels, which are  $2J+1$ .



If the Raman scattered lines are optically thin, their intensity ratios will be proportional to the ratio of the incident OVI lines, times the cross section for scattering

$$\frac{I(6826)}{I(7084)} = \frac{I(1032) \sigma(1032)}{I(1038) \sigma(1038)} = \frac{2 \times 3.65 \times 10^{-24}}{9.31 \times 10^{-25}} = 7.8$$

Optical depth effects will tend to reduce these ratios.