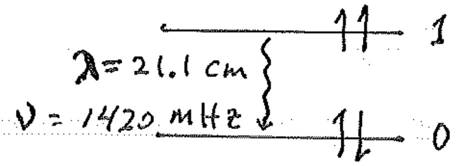
HI 21cm Emission and Absorption - Spin Temperature



The hyperfine splitting of the ground state of hydrogen into levels 0 and 1 total spin.
The level populations are related by the spin temperature, T_s, and the transition frequency ν.

$$\frac{n_1}{n_0} = \frac{g_1}{g_0} e^{-h\nu/kT_s} \qquad \text{where } g_0 = 1 \,,\ g_1 = 3$$

Since $h\nu/k = 0.0682$ is always much smaller than the spin temperature, $n_1/n_0 = 3$ and

$$n_0 = \frac{1}{4} n \,,\quad n_1 = \frac{3}{4} n \,,$$ where n is the total density of HI

The emission and absorption coefficients can be written in terms of the Einstein coefficients

$$j_\nu = \frac{h\nu}{4\pi} n_1 A_{10} \phi(\nu)$$

$$\boxed{A_{10} = 2.88 \times 10^{-15} \, s^{-1}}$$

$$\alpha_\nu = \frac{h\nu}{4\pi} (n_0 B_{01} - n_1 B_{10}) \phi(\nu)$$

where $g_0 B_{01} = g_1 B_{10}$ and $B_{10} = \frac{c^2 A_{10}}{2h\nu^3}$

$$\alpha_\nu = n_0 \frac{g_1}{g_0} \frac{A_{10}}{8\pi} \lambda^2 \left[1 - \frac{n_1}{n_0} \frac{g_0}{g_1} \right] \phi(\nu)$$

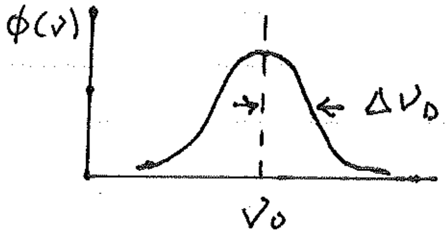
$$\boxed{\alpha_\nu = n_0 \frac{g_1}{g_0} \frac{A_{10}}{8\pi} \lambda^2 \left[1 - e^{-h\nu/kT_s} \right] \phi(\nu)}$$

Since $h\nu \ll kT_s$ the correction for stimulated emission is large, and the absorption coefficient is proportional to T_s^{-1}.

$$\alpha_\nu \approx \frac{3n}{32\pi} A_{10} \frac{hc\lambda}{kT_s} \phi(\nu)$$

On Galactic scales, the optical depth in the 21 cm line can be large. Let's assume a Doppler profile for $\phi(\nu)$ to characterize its width. It's a Gaussian function

$$\phi(\nu) = \frac{1}{\Delta\nu_D \sqrt{\pi}} e^{-\left(\frac{\nu-\nu_0}{\Delta\nu_D}\right)^2}$$



$\Delta\nu_D$ is the "Doppler width", which can be related to a velocity width, b, using the (non-relativistic) Doppler shift.

$$\frac{\Delta\nu_D}{\nu_0} = \frac{b}{c}$$

(If due to thermal motion at temperature T, $b = \sqrt{2kT/m}$)

$$\phi(\nu) = \frac{\lambda_0}{b\sqrt{\pi}} e^{-\left[\frac{c(\nu-\nu_0)}{b\nu_0}\right]^2}$$

Now we can evaluate α_{ν_0}, the absorption coefficient at the center of the line profile $(\nu = \nu_0)$.

$$\alpha_{\nu_0} = \frac{3}{32\pi} \frac{hc}{kT_s} \frac{n}{\sqrt{\pi}} \frac{A_{10}}{b} \lambda_0^2$$

Define column density $N = \int n \, dx$ $[\mathrm{cm}^{-2}]$ and optical depth $\tau_0 = \int \alpha_{\nu_0} dx$.

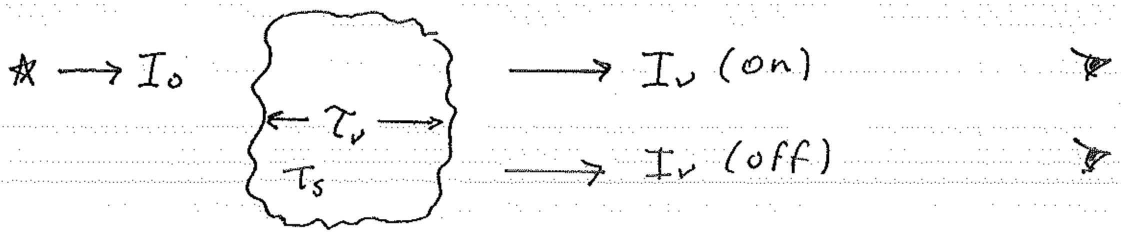
Optical depth can be written in terms of the physical variables as

$$\tau_0 = 3.1 \left(\frac{N}{10^{21}\,\mathrm{cm}^{-2}}\right)\left(\frac{b}{\mathrm{km\,s^{-1}}}\right)^{-1}\left(\frac{T_s}{100\ \mathrm{K}}\right)^{-1}$$

Note that b may or may not be thermal in origin, but if it is, its equivalent temperature is not the spin temperature, in general. This result shows that to measure the mass in HI, we must be able to measure the spin temperature as well as the optical depth across the line profile.

Measurement of Spin Temperature and Optical Depth

If we can measure the brightness of a cloud of HI both in emission and in absorption to a background radio source, then both τ and T_s can be determined. Say the background radio source has intensity I_0 in the continuum around the 1420 MHz HI line, and we observe both "on" and "off" the radio source:



Also, assume that the cloud has uniform T_s and τ_ν on the scale of the separation of the two observing beams. Then we have two relations for T_s and τ_ν from the two measured intensities:

$$I_\nu(\mathrm{on}) = I_0 e^{-\tau_\nu} + S_\nu(1-e^{-\tau_\nu}) \qquad (1)$$

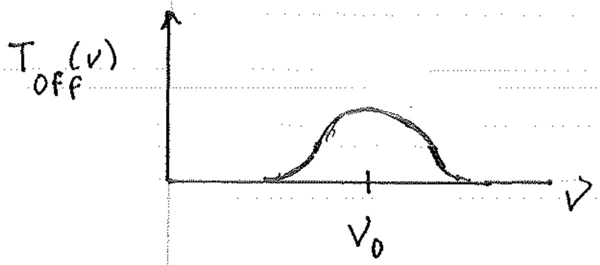
$$I_\nu(\mathrm{off}) = S_\nu(1-e^{-\tau_\nu}) \qquad (2)$$

where $S_\nu \equiv B_\nu(T_s)$, the source function of thermal emission/absorption at the temperature T_s. Since we are in the Rayleigh-Jeans regime where $h\nu \ll kT$, I_ν and S_ν are linearly proportional to T, and we may replace each with their equivalent brightness temperature. Equations (1) and (2) can then be written:

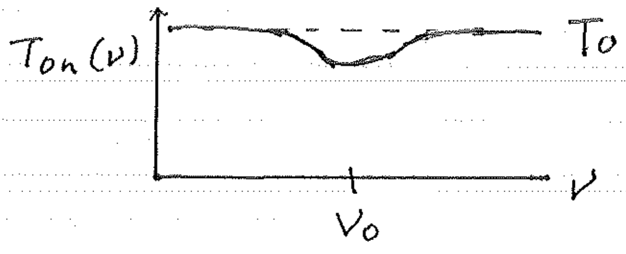
$$T_{on}(\nu) = T_0 e^{-\tau_\nu} + T_s(1-e^{-\tau_\nu}) \qquad (3)$$

$$T_{off}(\nu) = T_s(1-e^{-\tau_\nu}) \qquad (4)$$

The figures illustrate the quantities measured from the spectra:



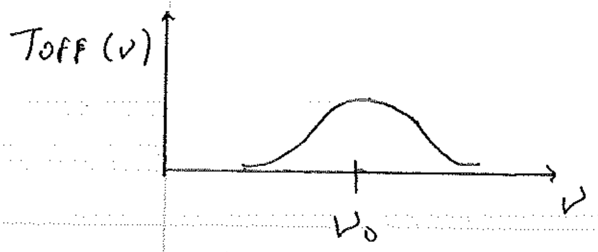
emission



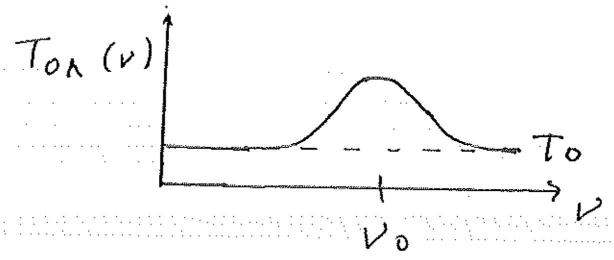
emission + absorption

Note that $T_{on}(\nu)$ must be greater than $T_{off}(\nu)$ at all frequencies. In the particular case that I have drawn in the Figures, $T_0 > T_s$.

It is also possible that $T_0 < T_s$, in which case the spectra might look like the following:



emission



emission + absorption

Solving Equations (3) and (4) for T_s and τ_ν gives:

$$\boxed{\begin{aligned} \tau_\nu &= \ln\left(\frac{T_0}{T_{on}(\nu) - T_{off}(\nu)}\right) . \\ T_S &= \frac{T_0\, T_{off}(\nu)}{T_0 - [T_{on}(\nu) - T_{off}(\nu)]} \end{aligned}}$$

- Note that if $\tau_\nu \ll 1$, then

$$1 + \tau_\nu \approx \frac{T_0}{T_{on} - T_{off}}$$

and $T_{on} - T_{off} \approx T_0$. Only an upper limit on τ_ν can be determined in this case, and a lower limit on T_S.

- If $\tau_\nu \gg 1$, then both T_{on} and T_{off} are equal to T_S, i.e., the cloud is opaque to the background radio source.

A&A 371, 698–707 (2001)
DOI: 10.1051/0004-6361:20010395

Astronomy & Astrophysics

The spin temperature of warm interstellar H I

H. Liszt[1]

National Radio Astronomy Observatory, 520 Edgemont Road, Charlottesville, VA 22903-2475, USA

Received 1 February 2001 / Accepted 15 March 2001

Abstract. Collisional excitation of the $\lambda21$ cm hyperfine transition is not strong enough to thermalize it in warm neutral ("intercloud") interstellar gas, which we show by simultaneously solving the equations of ionization and collisional equilibrium under typical conditions. Coupling of the $\lambda21$ cm excitation temperature and local gas motions may be established by the Ly-α radiation field, but only if strong Galactic Ly-α radiation permeates the gas in question. The Ly-α radiation tends to impart to the gas its own characteristic temperature, which is determined by the range of gas motions that occur on the spatial scale of the Ly-α scattering. In general, the calculation of H I spin temperatures is a more difficult and interesting problem than might have been expected, as is any interpretation of H I spin temperature measurements.

Key words. interstellar medium: general

1. Introduction

It is an article of faith among H I observers that the spin (excitation) temperature $T_{\rm sp}$ of the ground-state $\lambda21$ cm hyperfine transition is equal to the kinetic temperature of the ambient gas. This belief of course arises from the weakness of the transition, the consequent long lifetime against spontaneous emission ($A_{21} = 2.85\ 10^{-15}$ s^{-1} for H I), and the easily-demonstrated dominance of particle collisions with other H-atoms in cool gas. Much of our basic understanding of the phases of the ISM derives directly from measurements of the H I spin temperature, for instance by comparing nearby or overlapping absorption and emission profiles (Dickey et al. 1978; Payne et al. 1982).

But spin temperature measurements often seem to imply unphysical kinetic temperatures: not wildly so, say negative or infinite, but definitely in a range – 1000 to 5000 K – where the interstellar gas is unstable in multiphase models (Wolfire et al. 1995; McKee & Ostriker 1977) and therefore should be so short-lived as to be unobservable. One might be tempted to disparage either theory or the data but Davies & Cummings (1975) took a somewhat less doctrinaire view and showed that, for some assumed combinations of conditions chosen to be representative of gas which might produce pulsar dispersion measures, the density of collision partners in warm or intercloud gas is simply too small to thermalize the line. Although little attention seems to have been paid to this warning, it suggests that we remain open to the possibility that blind faith in the equality of the spin and kinetic temperatures might be misplaced.

Calculation of the H I spin temperature turns out to be a remarkably complex problem, involving intimate knowledge of the ionization and phase structure of the ISM, as well as its topology. Here we update and expand upon the discussion of Davies & Cummings (1975) by actually calculating the ionization and collisional excitation in interstellar H I regions, in the process demonstrating the inability of particle collisions to thermalize the $\lambda21$ cm transition. But we also consider an important mechanism which Davies & Cummings (1975) ignored, whereby the Ly-α radiation field threading and partly produced by the gas tends to impart to the $\lambda21$ cm transition its own effective temperature. Ly-α photons acquire this temperature while undergoing large numbers (10^7 or more) of repeated scatterings on many spatial scales ranging upward from the line-center mean free path, 0.11 $(T_{\rm D}/100\,{\rm K})^{0.5}/n_{\rm H}$ AU ($T_{\rm D}$ is the Doppler temperature), and it is therefore representative of the motions (thermal, turbulent, etc.) which are established in the gas on those scales.

That excitation by light dominates hyperfine excitation locally around individual stars is well known in the context of scattering of Solar Ly-α radiation: Braskén & Kyrölä (1998), for instance, point out that Solar Ly-α radiation dominates in local intercloud H I within 1000 AU of the Sun, and that the sphere of influence of an O-star would necessarily be much larger. In the interstellar context, we will refer to excitation of the hyperfine line by scattered Ly-α radiation as the Wouthuysen-Field mechanism or WF effect following the discussions in Wouthuysen (1952) and Field (1958, 1959) and we find that it opens up interesting possibilities for the spin temperature. At low density in the intercloud medium, warm neutral H I may (or may not) be Dopplerized by Galactic Ly-α photons but

---

Send offprint requests to: H. S. Liszt, e-mail: `hliszt@nrao.edu`

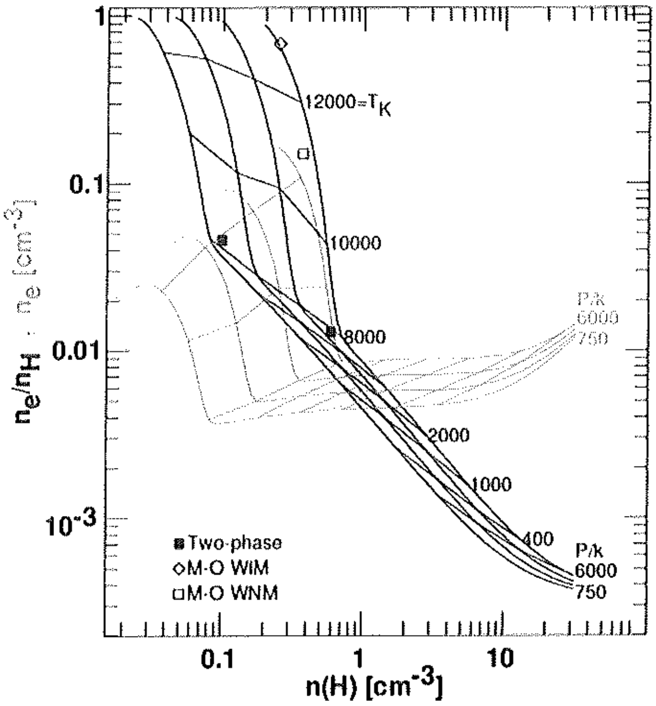


Fig. 1. Ionization equilibrium calculations in atomic gas. The full curves are plots of the ionization fraction n_e/n_H where n_H is the number density of H-nuclei in all forms. Results are shown for four values of the total gas pressure P/k = 750, 1500, 3000, 6000 cm^{-3} K and the kinetic temperature is traced across each set of curves. The shaded curves are plots of the electron *density*. Open symbols represent the ionization fraction in warm neutral and warm ionized gas in the three-phase model of McKee & Ostriker (1977). The filled rectangles show the ionization fraction of warm neutral gas in the two-phase calculations of Wolfire et al. (1995) for their standard case $N w = 10^{19}$ cm^{-2}

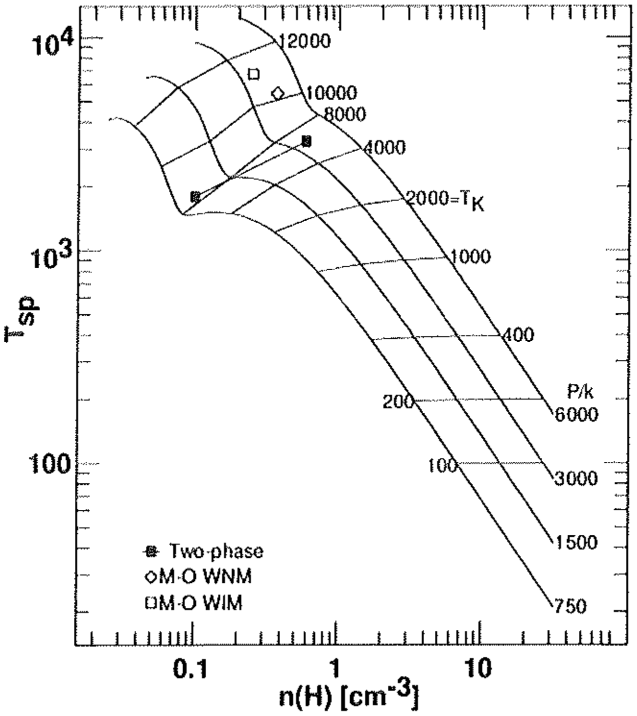


Fig. 2. The spin temperature of the H I λ21 cm transition resulting from the same conditions whose ionization equilibrium is shown in Fig. 1. Symbols represent results for warm gas in the two and three phase calculations of Wolfire et al. (1995) and McKee & Ostriker (1977), see Fig. 1

Table 2. Two-phase spin and kinetic temperatures

Model	P/k (cm^3 K)	T_K(K)	T_{sp}(K)
$N w = 10^{18}$ cm$^{\ 2}$	1600 − 9990	9200 − 5600	3525 − 4620
$N w = 10^{19}$ cm^{-2}	990 − 3600	8700 − 5500	1800 − 3260
$N w = 10^{20}$ cm^{-2}	610 − 1500	8200 − 4900	1035 − 2020
Three-Phase	3700	8000	5400

equilibrium from Wolfire et al. (1995). Table 2 gives the range of pressure and kinetic temperature over which two phases can coexist for various values of $N w$ and it shows that two-phase equilibrium is possible over a wider range in T_K than is sometimes considered when interpreting spin temperature measurements. Comparison of the table entries for the standard case $N w = 1\ 10^{19}$ cm^{-2} with the locations of the filled dark symbols in the figure shows that our results mimic those of Wolfire et al. (1995), as intended. Both sets of calculations produce the same ionization at any given density and temperature.

In Fig. 1, two unfilled symbols represent the results of McKee & Ostriker (1977) for the warm neutral and warm ionized gas (both at 8000 K). The pressure in the McKee-Ostriker model is $P/k = 3700$ K cm^{-3} but the ionization fraction in three-phase warm neutral and ionized gas is considerably higher than that in our calculations. McKee & Ostriker (1977) explain in the caption to their Fig. 1 that the warm neutral medium can only be produced in appreciable quantity at the assumed pressure by assuming a higher than average value of the soft X-ray flux. In the context of our modelling, the soft X-ray flux would have to be increased by a factor of about 50 in order to give the same total ionization which occurs in warm, neutral gas in the McKee-Ostriker model. It is unfortunate that the basic parameters of the three-phase model have not been revised in so long.

4.2. Evaluation of T_{sp}

Figure 2 shows that for kinetic temperatures above 2000 K, and especially at 8000 K, the particle excitation rate is too small to thermalize the λ21 cm line in any model of the ISM, including the McKee-Ostriker three-phase picture. The extreme conditions assumed for the three-phase model result in $T_{sp} \approx 5400$ K in warm neutral gas with $T_K = 8000$ K. In the two-phase model, the spin temperature of warm gas would be smaller, 1800–3200 K for the standard conditions where $T_K = 8700$–5500 K (the models at higher T_K have lower pressure and lower T_{sp}). Table 2 shows the range of spin temperature which results from various conditions of two- and three-phase equilibrium. Figure 3 displays the y-factor for collisional

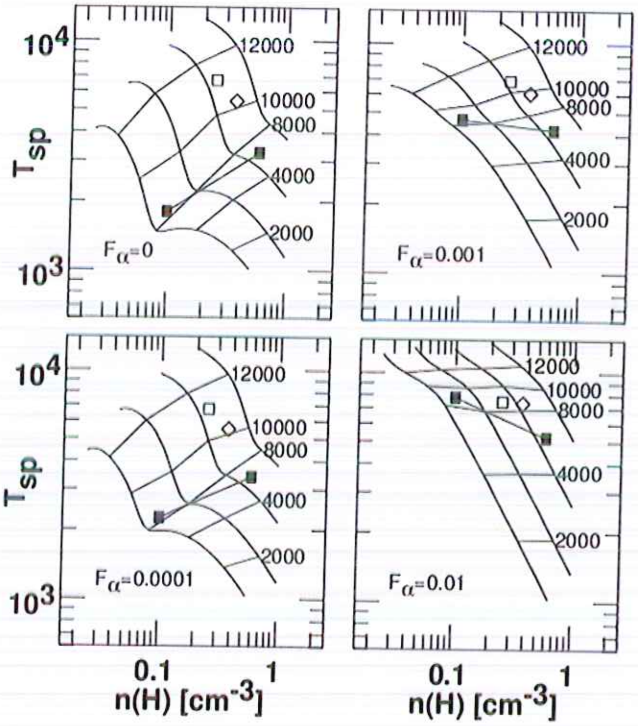


Fig. 5. As in Fig. 2 when a specified fraction F_α of the galactic flux from early-type stars threads the gas and $T_L = T_K$. The panel with $F_\alpha = 0$ repeats behaviour already shown in Fig. 2

the fact that most photons which enter cold H I clouds will be absorbed by dust.

Figure 6 shows our evaluation of the analytic solution (Harrington 1973; Neufeld 1990) to this idealized problem, performed for a gas with $T_K = T_D = 7200$ K and various values of the column density in the H I layer (which determines the loss due to continuous absorption by dust). Although the total column density of H I looking out from the midplane is about 3 10^{20} cm^{-2} (Liszt 1983), that of the intercloud gas is somewhat less, of order half. Figure 6 shows that over most of the volume of this idealized model, the Ly-α radiation field threading the intercloud gas is certainly not less than 1% of Ψ_0. In this case, the spin temperature in the intercloud gas at all but the highest galactic z-heights would be close to equilibrium with the temperature of the ambient galactic Ly-α radiation – T_K, T_D or what have you, depending on how the photon field relaxes in its interactions with the gas.

Cool neutral gas packets dispersed throughout such a scattering medium would clearly be threaded by a substantial fraction of the galactic flux as well, raising the interesting possibility that the spin temperatures measured in strongly absorbing gas are also influenced by Ly-α radiation. A calculation analogous to that used to produce Fig. 5 shows that Ly-α excitation dominates if more than 0.1% of the galactic radiation field threads cool gas. In this case, $T_{sp} = T_D$ rather than $T_{sp} = T_K$ and typical turbulent velocity contributions to T_D will put a 30–120 K floor on measurements of T_{sp} (see Sect. 6.3) *if* the turbulence is established on the scale of the Ly-α scattering.

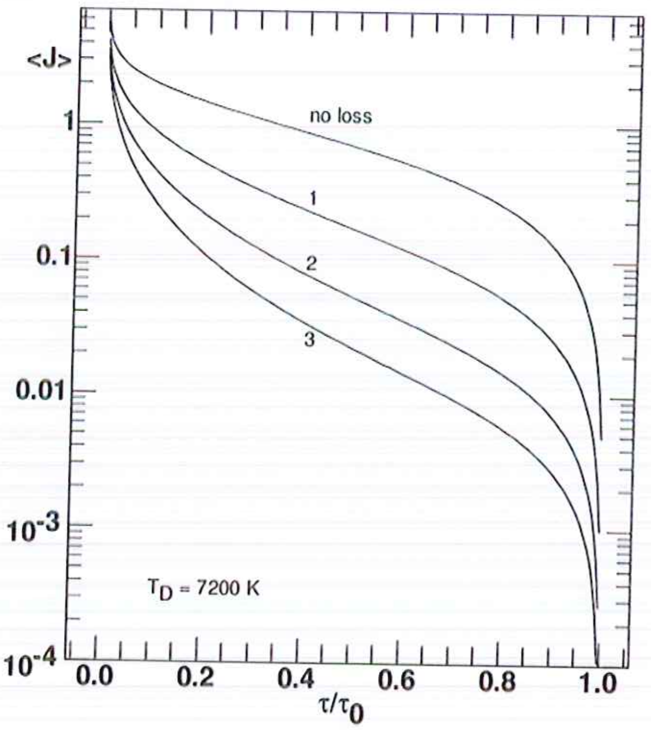


Fig. 6. Variation of the Ly-α radiation field inside a slab of warm H I harboring a unit plane source $\delta(\tau)$ at the midplane, plotted against distance from the slab center τ/τ_0; see Sect. 6.1 of the text. Results for a lossless slab are shown, and for N_H = 1, 2, and 3 10^{20} cm^{-2}

6.2. A multi-phase ISM

Neufeld (1991) pointed out that a multiphase ISM structure may foster escape of Ly-α radiation and that individual neutral gas clouds – warm or cool – may not be threaded by much of the ambient Ly-α flux. If most of the volume of the ISM is in a contiguous hot phase having negligible neutral hydrogen or Ly-α absorption, scattered radiation escapes relatively easily even when neutral gas clouds have a very high surface covering factor. Photons scatter off individual clouds as if they were particles of a gas having a Doppler temperature corresponding to the cloud-cloud velocity dispersion, 6 km s^{-1}, appreciably narrowing the interstellar line profile if the photons are scattered sufficiently often before escape. The crucial issue is the fraction and connectedness of the ISM occupied by very hot gas in which hydrogen is totally ionized, as opposed even to 6000–10000 K H I intercloud gas which will still present a sizable opacity to Ly-α radiation.

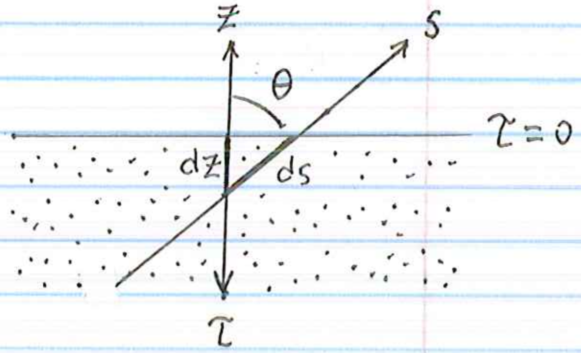
The fraction of incident photons which will be reflected back from the outer boundary of an isolated, discrete cloud is of order $1 - 1/(\tau_0\phi(x))$ where x is the normalized frequency shift of the incident radiation from line center (Neufeld 1990): see Sect. 5.1 here. An analytic solution to the problem of radiation incident on a lossless slab is presented by Neufeld (1990) in his Eqs. (2.19)–(2.22); calculation is considerably simplified when Neufeld's Eq. (2.22)[4]

---

[4] Note that the quantity plotted in Fig. 2 of Neufeld (1990) is $(a\tau_0)^{1/3}J(\tau_0,x)$, not $(a\tau_0)J(\tau_0,x)$.

Radiative Transfer in Plane-Parallel Geometry

The Semi-Infinite Atmosphere



$$\frac{dI_\nu}{ds} = -\alpha_\nu (I_\nu - S_\nu)$$

$$ds = \frac{dz}{\cos\theta} \qquad \text{let } \mu = \cos\theta$$

$I_\nu(z, \mu)$ is a function of depth <u>and</u> angle

$$\mu \frac{\partial I_\nu}{\partial z} = -\alpha_\nu (I_\nu - S_\nu)$$

Let $d\tau_\nu = -\alpha_\nu \, dz$ (negative because τ_ν increases inward)

$$\boxed{\mu \frac{\partial I_\nu}{\partial \tau_\nu} = I_\nu - S_\nu}$$

with $I_\nu(\tau_\nu, \mu)$

Equation of Radiative Transfer

What is the emergent intensity as a function of angle?

$$\frac{\partial I_\nu}{\partial \tau_\nu} e^{-\tau_\nu/\mu} - \frac{I_\nu}{\mu} e^{-\tau_\nu/\mu} = -\frac{S_\nu}{\mu} e^{-\tau_\nu/\mu}$$

$$\frac{\partial}{\partial \tau_\nu} \left[I_\nu e^{-\tau_\nu/\mu} \right] = -\frac{S_\nu}{\mu} e^{-\tau_\nu/\mu}$$

$$-\int_0^\infty d \left[I_\nu e^{-\tau_\nu/\mu} \right] = \int_0^\infty \frac{S_\nu}{\mu} e^{-\tau_\nu/\mu} \, d\tau_\nu$$

$$\boxed{I_\nu(0, \mu) = \int_0^\infty S_\nu(\tau_\nu) \, e^{-\tau_\nu/\mu} \, \frac{d\tau_\nu}{\mu}}$$

emergent intensity

The source function is a function of depth, in general.

Just as we took moments of the specific intensity, we can take moments of the Equation of Radiative Transfer by multiplying by $\mu = \cos\theta$ and integrating over solid angle. The goal is to find approximate solutions of the equation by using isotropic or nearly isotropic properties of the higher moments in simple situations to specify S_ν.

Zeroth moment of Radiative Transfer Equation

$$J_\nu = \frac{1}{4\pi}\int I_\nu \, d\Omega = \frac{1}{2}\int_{-1}^{1} I_\nu \, d\mu \qquad (\text{since } d\Omega = -d\mu \, d\varphi)$$

Then $$\frac{1}{2}\int_{-1}^{1} \mu \frac{\partial I_\nu}{\partial \tau_\nu} d\mu = \frac{1}{2}\int_{-1}^{1} I_\nu \, d\mu - \frac{1}{2}\int_{-1}^{1} S_\nu \, d\mu$$

$$\boxed{\frac{1}{4\pi}\frac{dF_\nu}{d\tau_\nu} = J_\nu - S_\nu}$$

The zeroth-moment equation contains flux F_ν which is the first moment. Taking moments of the transfer equation does not, by itself, aid in its solution since you introduce a new variable. Same problem with the...

First moment of Radiative Transfer Equation

$$\frac{1}{2}\int_{-1}^{1} \mu^2 \frac{\partial I_\nu}{\partial \tau_\nu} d\mu = \frac{1}{2}\int_{-1}^{1} \mu I_\nu \, d\mu - \cancel{\frac{1}{2}\int_{-1}^{1} \mu S_\nu \, d\mu}$$

$$\boxed{c\frac{dP_\nu}{d\tau_\nu} = F_\nu}$$ introduces radiation pressure

Let's assume that flux F_ν is constant with depth, as may be the case in a stellar atmosphere. Then the zeroth moment-equation gives $S_\nu \approx J_\nu$.

Second, let's assume the Eddington approximation, $\boxed{P_\nu = \frac{u_\nu}{3}}$

$$P_\nu \approx \frac{u_\nu}{3} = \frac{4\pi}{3c} J_\nu \quad , \quad \text{which becomes} \quad P_\nu = \frac{4\pi}{3c} S_\nu$$

Integrating the first-moment equation gives

$$c P_\nu = F_\nu (\tau_\nu + \tau_0)$$ τ_0 is the constant of integration

Therefore $S_\nu = \frac{3}{4\pi} F_\nu (\tau_\nu + \tau_0)$

Now we can solve for the emergent intensity

$$I_\nu(0,\mu) = \frac{3}{4\pi} F_\nu \int_0^\infty (\tau_\nu + \tau_0)\, e^{-\tau_\nu/\mu} \frac{d\tau_\nu}{\mu}$$

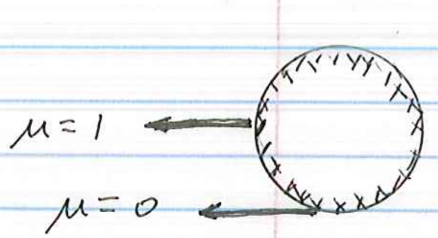
$$I_\nu(0,\mu) = \frac{3}{4\pi} F_\nu (\mu + \tau_0)$$

The constant τ_0 can be determined by requiring the flux at the surface to be the first moment of the emergent specific intensity

$$F_\nu = 2\pi \int_0^1 \mu\, I_\nu(0,\mu)\, d\mu$$ (outward θ only)

$$F_\nu = 2\pi \int_0^1 \mu \frac{3}{4\pi} F_\nu (\mu + \tau_0)\, d\mu = \frac{1}{2} F_\nu + \frac{3}{4} F_\nu \tau_0$$

$$\Rightarrow \tau_0 = 2/3$$



$$I_\nu(0,\mu) = \frac{3}{4\pi} F_\nu (\mu + 2/3)$$
$$I_\nu(0,1) = \frac{5}{4\pi} F_\nu$$

$$\boxed{I_\nu(0,\mu) = I_\nu(0,1)\, \frac{3}{5} \left(\mu + \frac{2}{3}\right)}$$

Limb Darkening

The limb darkening result says that the intensity (brightness) of the center of the disk of the star ($\mu=1$) is 5/2 the intensity at the limb ($\mu=0$).

Making a further approximation, of "grey opacity" (τ_ν independent of frequency), we can find the temperature as a function of optical depth. To do this, integrate the first-moment equation over ν.

$$\int_0^\infty c P_\nu \, d\nu = \int_0^\infty F_\nu (\tau_\nu + 2/3) \, d\nu$$

$$c P = \sigma T_{eff}^4 (\tau + 2/3) \qquad F = \sigma T_{eff}^4$$

Effective temperature T_{eff} is defined by total flux.

Since $P \equiv \frac{4}{3} \frac{\sigma}{c} T^4$ in the Eddington approximation

$$\boxed{T^4 = \frac{3}{4} T_{eff}^4 (\tau + 2/3)}$$

In the approximation of grey opacity, the kinetic temperature at an optical depth of 2/3 is equal to the effective temperature, the latter defined in terms of the total radiation energy flux.

At the "surface" of the star, where $\tau = 0$, the kinetic temperature is $(1/2)^{1/4}$ of the effective temperature

$$T(\tau=0) = 0.841 \, T_{eff}$$

In summary, the Eddington approximation applies where the source function (and the specific intensity) can be approximated as linear functions of optical depth. In this case $P_\nu = u_\nu/3$ even though intensity is not isotropic.