Thermal Compton Scattering

In the rest frame of the electron, a scattered photon always loses energy. But in an electron gas, the photons may gain energy if the electron temperature is high enough.

$$\lambda_1' - \lambda_0' = \frac{h}{mc}(1-\cos\theta')$$

$$h\nu_1' = \frac{h\nu_0'}{1+\frac{h\nu_0'}{mc^2}(1-\cos\theta')}$$ electron rest frame

$$\frac{\Delta E'}{E_0'} = \frac{h\nu_1' - h\nu_0'}{h\nu_0'} = -\frac{h\nu_1'}{mc^2}(1-\cos\theta') \approx -\frac{h\nu_0'}{mc^2}(1-\cos\theta')$$

Averaging over all angles $0 < \theta' < \pi$

$$\frac{\Delta E'}{E_0'} \approx -\frac{E_0'}{mc^2}$$ fractional energy lost in the electron rest frame

In the lab frame there is an additional increase in energy, of order β^2, as we can show, due to inverse Compton scattering power.

$$P_{IC} = \frac{4}{3}\sigma_T c \gamma^2 \beta^2 U_{ph}$$

For a thermal distribution of non-relativistic electrons, we may approximate $\gamma^2\beta^2 \approx \beta^2$.

Since $\frac{1}{2}m\langle v^2\rangle = \frac{3}{2}kT$, $\langle\beta^2\rangle = \frac{3kT}{mc^2}$

Then, for an electron density n_e, the volume emissivity is

$$n_e P_{IC} = \frac{4kT}{mc^2}\sigma_T c\, n_e U_{ph} \quad \left[\frac{\text{erg}}{\text{cm}^3\,\text{s}}\right]$$

If the photon energy E_0 is $\ll 4kT$, the fraction of energy gained is

$$\frac{\Delta E}{E_0} = \frac{4kT}{mc^2}$$ per scattering

Adding the positive and negative contributions

$$\frac{\Delta E}{E_0} = \frac{4kT}{mc^2} - \frac{E_0}{mc^2}$$

Photons will gain energy if $E_0 < 4kT$. Or if photons and electrons are in equilibrium due to multiple scatterings, then $\langle E \rangle = 4kT$.

Now let's assume that $E_0 \ll 4kT$. The degree to which repeated Compton scatterings changes the energy of the photons is given by the Compton "y" parameter

$$\boxed{y \equiv \frac{\Delta E}{E_0} \max(\tau_s, \tau_s^2)}$$

which is the product of the fractional energy gain per scattering, times the mean number of scatterings. The latter is τ_s^2 when $\tau_s \gg 1$, from the random walk.

So $$y = \frac{4kT}{mc^2} \max(\tau_s, \tau_s^2)$$

After N scatterings, the energy of the photon is

$$E(N) = E_0 \, e^{N\left(\frac{4kT}{mc^2}\right)} \qquad \left[\text{because } \frac{dE}{dN} = E\,\frac{4kT}{mc^2}\right]$$

So $E(N) = E_0 e^y$

When $E(N) = 4kT$, the photon stops gaining energy, and Comptonization is said to be saturated. This occurs at a critical value of y corresponding to a critical value of τ_s, assumed to be $\gg 1$.

$$\frac{4kT}{E_0} \simeq e^{y_{crit}} \qquad \text{or} \qquad \tau_{crit} = \sqrt{\frac{mc^2}{4kT} \ln\left(\frac{4kT}{E_0}\right)}$$

Repeated scattering from relativistic electrons of small optical depth (an approximate treatment)

A photon's energy is multiplied by $\frac{4}{3}\gamma^2$, on average upon each scattering from an electron of energy γ. After k scatterings, the frequency of the photon will be

$$\nu \simeq \nu_0 \left(\frac{4}{3}\gamma^2\right)^k$$

and whatever the width $d\nu_0$ of the seed spectrum is, the scattered spectrum will be increased in width by the same factor

$$d\nu \approx d\nu_0 \left(\frac{4}{3}\gamma^2\right)^k$$

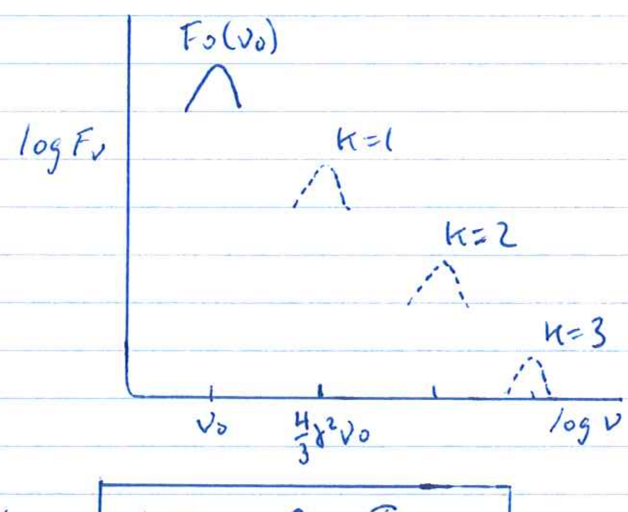
The probability that a photon is scattered **k** times is τ_s^k <u>if τ_s is small</u>. Therefore, the scattered spectrum will have a specific flux $F(\nu)$ that can be approximated as

$$F(\nu) = F_0(\nu_0)\, \tau_s^{\,k}$$

$$\ln\left[\frac{F(\nu)}{F_0(\nu_0)}\right] = k \ln \tau_s$$

$$\ln\left(\frac{\nu}{\nu_0}\right) = k \ln\left(\frac{4}{3}\gamma^2\right)$$

$$\ln\left(\frac{F(\nu)}{F_0(\nu_0)}\right) = \frac{\ln \tau_s}{\ln\left(\frac{4}{3}\gamma^2\right)} \ln\left(\frac{\nu}{\nu_0}\right)$$



$$F(\nu) = F_0(\nu_0) \left(\frac{\nu}{\nu_0}\right)^{-\alpha}, \quad \text{where} \quad \boxed{\alpha \equiv \frac{-\ln \tau_s}{\ln(4\gamma^2/3)}}$$

If the seed spectrum is distributed over a wide range of frequency ν_0, then the scattered spectrum at frequency ν can be generalized by integrating over $F_0(\nu_0)$ for $\nu_0 \leq \nu$. Only seed photons with $\nu_0 < \nu$ contribute to $F(\nu)$.

$$\nu F(\nu) = \int_0^{\nu} F_0(\nu_0) \left(\frac{\nu}{\nu_0}\right)^{-\alpha} d\nu_0$$

or $$F(\nu) = \nu^{-(\alpha+1)} \int_0^{\nu} F_0(\nu_0)\, \nu_0^{\alpha}\, d\nu_0$$

An example is the <u>Comptonized Blackbody</u> (see figures below)

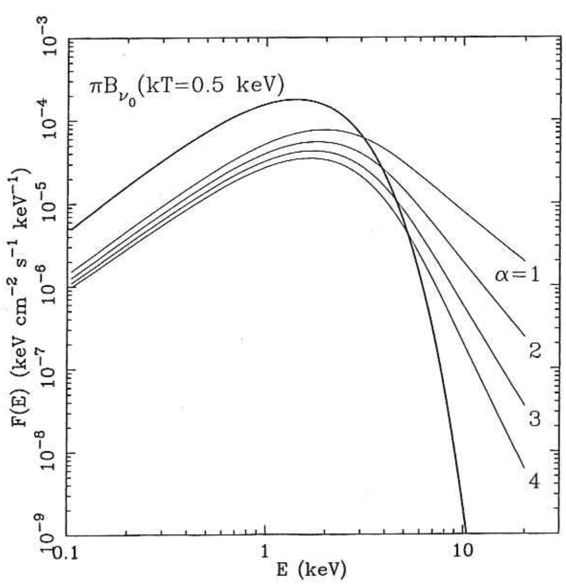
$$F(\nu) = \nu^{-(\alpha+1)}\, \pi \int_0^{\nu} B_{\nu_0}(T)\, \nu_0^{\alpha}\, d\nu_0$$

It has a power-law "tail" of index $\alpha+1$. In this approximation it is required that $\tau_s \ll 1$ and $h\nu \ll \gamma mc^2$. From the definition of α,

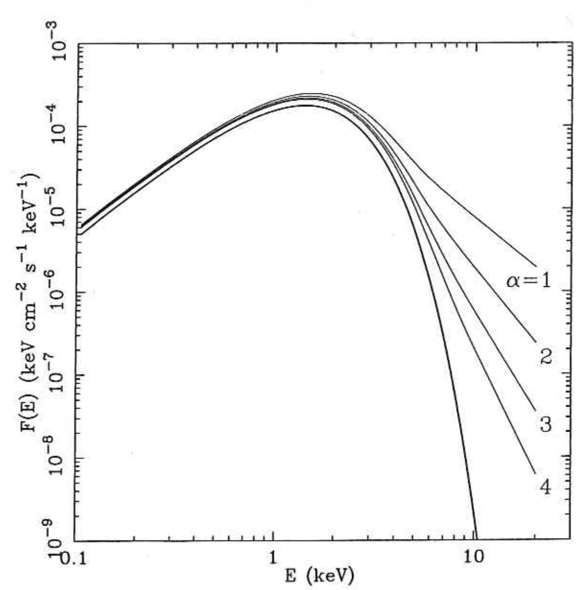
$$\alpha \equiv \frac{-\ln \tau_s}{\ln(4\gamma^2/3)}, \quad \text{it follows that} \quad \left(\frac{4\gamma^2}{3}\right)^{\alpha} \tau_s = 1$$

which can be rewritten $$\left(\frac{4\gamma^2}{3}\right)\tau_s = \left(\frac{4\gamma^2}{3}\right)^{1-\alpha}$$

This expression is less than 1 as long as $\alpha > 1$. The left side is exactly the compton y parameter for the case $\tau_s < 1$. So large α corresponds to small y



Blackbody and Compton scattered spectra for different α.



Sum of blackbody and Compton scattered spectrum.