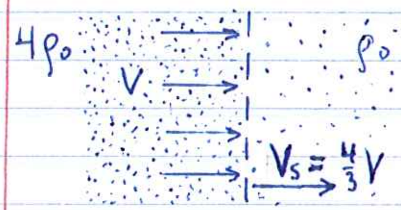
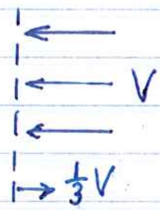
Diffusive Shock Acceleration (DSA) (1st order Fermi acceleration)

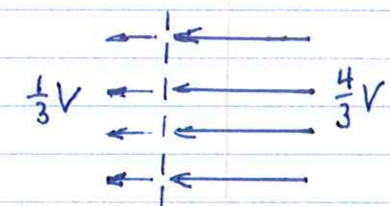
A power-law distribution of non-thermal particle energies can result from multiple reflections across a shock front, which acts as a pair of moving mirrors approaching each other. In an adiabatic strong shock, the density compression is a factor of 4. In the following diagrams the dashed line is the shock.



frame of reference of unshocked medium

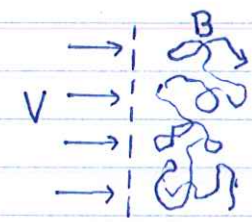


frame of reference of shocked medium

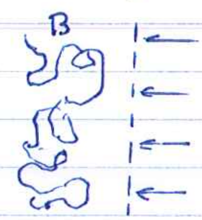


frame of reference of shock

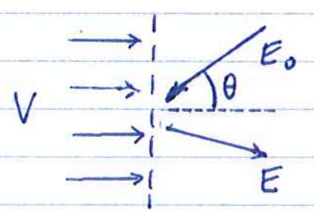
Conservation of mass requires that $\rho V = \text{const.}$ in the frame of the shock. If the medium also contains a magnetic field, it can trap and reflect relativistic particles that cross the shock. Crossing the shock twice is the equivalent of reflection from an approaching mirror.



frame of reference of unshocked medium



frame of reference of shocked medium



A relativistic particle reflected from a mirror moving with $\beta = V/c$, as shown in the homework, will get energy boost

$$E = \gamma^2 E_0 (1 + 2\beta\cos\theta + \beta^2)$$

In the limit $\beta \ll 1$, $\gamma^2 \approx 1 + \beta^2$, and $E = E_0(1 + 2\beta\cos\theta + 2\beta^2)$, or

$$E \approx E_0 (1 + 2\beta\cos\theta) \quad \text{to first order in } \beta.$$

For an isotropic distribution of angles, the probability that a particle will cross the shock at an angle θ is:

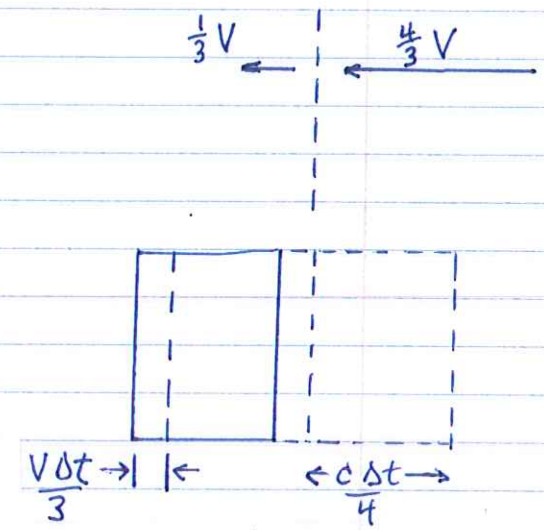
$$P(\theta) = 2\cos\theta\sin\theta \qquad \text{where} \quad \int_0^{\pi/2} P(\theta)\,d\theta = 1$$

The energy upon reflection from the shock is, on average

$$E = E_0 \int_0^{\pi/2} (1+2\beta\cos\theta)\, 2\cos\theta\sin\theta\, d\theta = 2\int_0^1 (1+2\beta\mu)\mu\, d\mu$$

$$E = E_0\left(1+\tfrac{4}{3}\beta\right)$$

For an isotropic distribution of angles, the rate at which particles cross the shock in either direction is $\frac{1}{4}nc$, where n is their density. (This is a flux, similar to $F_\nu = \pi I_\nu$ for an isotropic intensity, where $u_\nu = \frac{4\pi}{c} I_\nu$, so $F_\nu = \frac{1}{4} c u_\nu$). In addition, assume that particles will escape when they are advected away from the shock, at velocity $\frac{V}{3}$ and with flux $nV/3$. In the diagram, during the time that $\frac{nc\Delta t}{4}$ particles enter the box, $\frac{nV\Delta t}{3}$ are advected away and escape. The fraction of particles lost during one round trip cycle is, therefore, $(V/3)/(c/4) = \frac{4}{3}\beta$. The probability that a particle does not escape is $1-\frac{4}{3}\beta$.



If we start with N_0 total particles, the number remaining after k round-trip cycles is

$$N = N_0\left(1-\tfrac{4}{3}\beta\right)^k$$

and their average energy will be

$$E = E_0\left(1+\tfrac{4}{3}\beta\right)^k$$

Combing N and E as follows ...

$$\frac{\ln (N/N_0)}{\ln (E/E_0)} = \frac{\ln (1-\frac{4}{3}\beta)}{\ln (1+\frac{4}{3}\beta)}$$

$$\frac{N}{N_0} = \left(\frac{E}{E_0}\right)^{\frac{\ln(1-\frac{4}{3}\beta)}{\ln(1+\frac{4}{3}\beta)}}$$

N is the total number of particles whose energy is greater than or equal to E.

$$N = N(\geq E) = \int_E^\infty N(E)\, dE$$

Therefore $N(E)\, dE = \text{const.}\, E^{\left[\frac{\ln(1-\frac{4}{3}\beta)}{\ln(1+\frac{4}{3}\beta)} - 1\right]}$

Since $\ln(1-\frac{4}{3}\beta) \approx -\frac{4}{3}\beta$
and $\ln(1+\frac{4}{3}\beta) \simeq \frac{4}{3}\beta$,

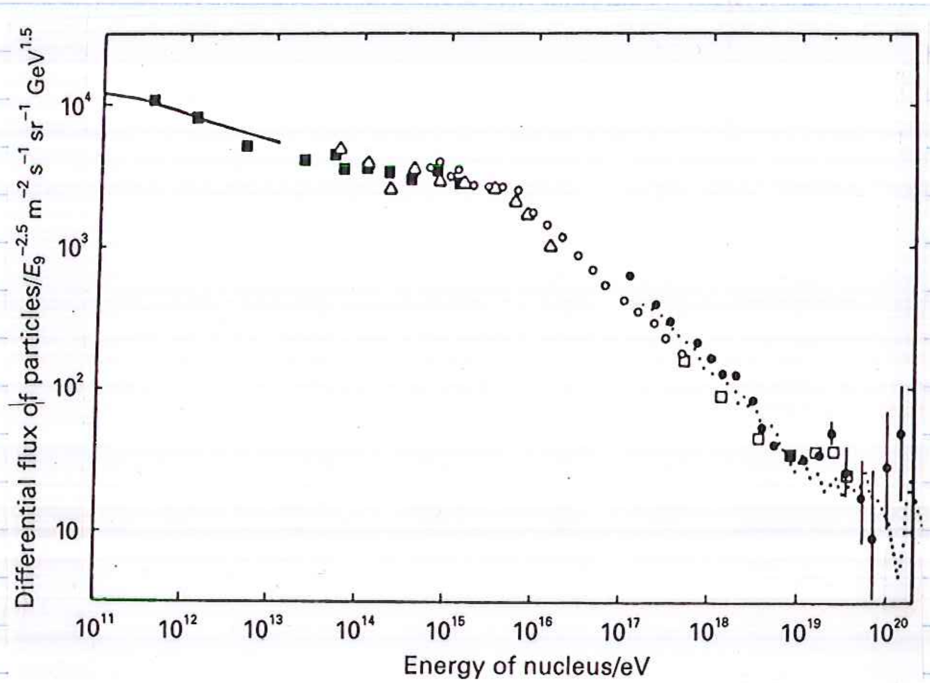
$N(E)\, dE = \text{const.}\, E^{-2}\, dE$, thus $p = 2$

This result depended on the compression ratio being 4, thus the advection velocity being $V/3$. You can show that for a compression ratio r, the index of the energy distribution is

$$p = \frac{r+2}{r-1}$$

Spectrum of Cosmic Rays multiplied by $E^{2.5}$

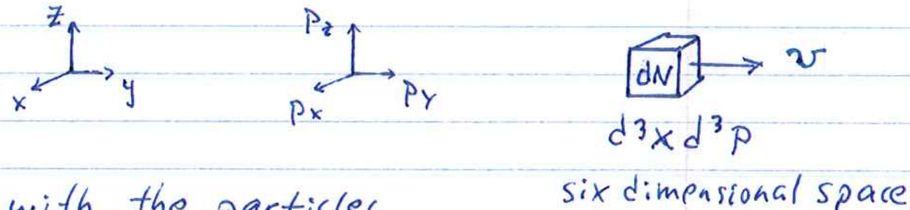
$F(E)\, E^{2.5}$ has units $\left[\frac{\text{GeV}^{1.5}}{\text{m}^2\ \text{s ster}}\right]$



Special Results

1. I_ν/ν^3 is a Lorentz invariant. To prove this, we first need to show that phase space density f is a Lorentz invariant. Phase space density is the number of particles dN per unit volume $d^3x\, d^3p$ in phase space.

$$f = \frac{dN}{d^3x\, d^3p}$$



In the frame moving with the particles

$$d^3x' = dx'\, dy'\, dz'$$
$$d^3p' = dp'_{x'}\, dp'_{y'}\, dp'_{z'}$$

In the lab frame, due to Lorentz contraction in x,

$$d^3x = \frac{1}{\gamma}\, dx'\, dy'\, dz'$$

While for the momentum

$$dp_x = \gamma\,(dp'_{x'} + \beta\, \cancel{dE'}^{\,0}/c)$$
$$dp_y = dp'_{y'}$$
$$dp_z = dp'_{z'}$$

To first order in p, the particles in the box have the same energy in the comoving frame, so $dE' = 0$

Then $d^3x\, d^3p = d^3x'\, d^3p'$. Since the number of particles dN is invariant, phase space density is invariant:

$$f = \frac{dN}{d^3x\, d^3p} = \frac{dN}{d^3x'\, d^3p'}$$

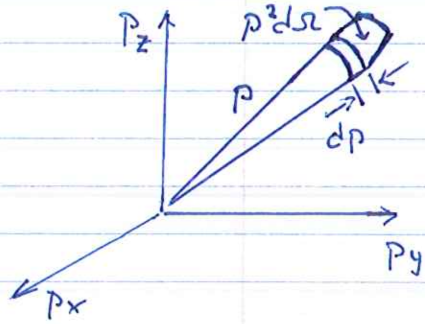
In the case of photons, the phase space density can be related to specific intensity I_ν. Remember that

$$\frac{I_\nu}{c} = u_\nu(\Omega) \quad \left[\frac{\text{erg}}{\text{cm}^3\ \text{Hz ster}}\right]$$

"Specific energy density per steradian"

$u_\nu(\Omega)\, d\nu\, d\Omega$ has units $\left[\frac{\text{ers}}{\text{cm}^3}\right]$, as does $\frac{I_\nu}{c} d\nu\, d\Omega$

In terms of phase space density f we would write this as



$f\, h\nu\, p^2\, dp\, d\Omega \quad \left[\frac{\text{ers}}{\text{cm}^3}\right]$

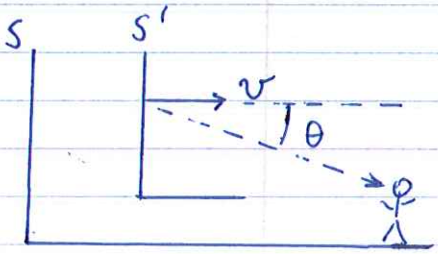
Therefore $\quad f\, h\nu\, p^2\, dp = \frac{I_\nu}{c} d\nu$

Since $p = \frac{h\nu}{c}$, $\quad \frac{f\, h^4 \nu^3 d\nu}{c^3} = \frac{I_\nu}{c} d\nu$

and $\frac{I_\nu}{\nu^3} \propto f$, so $\frac{I_\nu}{\nu^3}$ is a Lorentz invariant

$$\boxed{\frac{I'_{\nu'}}{(\nu')^3} = \frac{I_\nu}{\nu^3}}$$

- Example: The specific intensity of a moving source can be found using the Doppler shift.



Since $\nu' = \nu\gamma(1-\beta\cos\theta)$

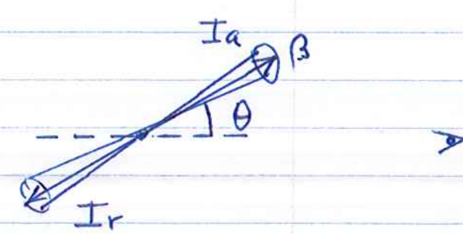
$$I_\nu = \frac{I'_{\nu'}}{\gamma^3(1-\beta\cos\theta)^3}$$

For a source with a power-law spectrum of the form $I'_{\nu'} \propto (\nu')^\alpha$ observed at frequency ν,

$I'_{\nu'} = I'_\nu \left(\frac{\nu'}{\nu}\right)^\alpha \qquad$ then $\quad \frac{I'_{\nu'}}{(\nu')^3} = \frac{I'_\nu}{(\nu')^3}\left(\frac{\nu'}{\nu}\right)^\alpha = \frac{I_\nu}{\nu^3}$

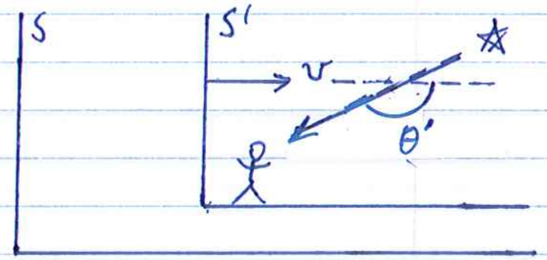
so $\quad I_\nu = I'_\nu \left(\frac{\nu}{\nu'}\right)^{3-\alpha}$

- Example: For oppositely directed jets of equal intensity, it follows that



$$\frac{I_a}{I_r} = \left(\frac{1+\beta\cos\theta}{1-\beta\cos\theta}\right)^{3-\alpha}$$

- Example: Dipole anisotropy of the cosmic microwave background due to the motion of the solar system with respect to the CMB frame. The CMB has the Planck spectrum:



$$I_\nu = \frac{2h\nu^3}{c^2} \frac{1}{e^{h\nu/kT}-1}$$

$$I'_{\nu'} = I_\nu \left(\frac{\nu'}{\nu}\right)^3$$

$$I'_{\nu'} = \frac{2h(\nu')^3}{c^2} \frac{1}{e^{h\nu/kT}-1}$$

The observer is in the primed frame, so the Doppler shift becomes

$$\nu = \nu' \gamma (1+\beta\cos\theta')$$

$$I'_{\nu'} = \frac{2h(\nu')^3}{c^2} \frac{1}{e^{h\nu'\gamma(1+\beta\cos\theta')/kT}-1}$$

The observed spectrum is still a blackbody, with temperature

$$T' = \frac{T}{\gamma(1+\beta\cos\theta')}$$

In the limit $\beta \ll 1$, $\gamma = 1$, and $T' \approx T(1-\beta\cos\theta')$

The dipole anisotropy of the CMB has been used to measure the velocity of the solar system

$$\frac{\Delta T}{T} = \frac{T'_{max} - T}{T} = \beta = 1.23\times10^{-3}, \text{ or } v = 369 \pm 1 \text{ km s}^{-1}.$$

In addition, the aptly named Planck mission has measured the relativistic aberration of the CMB anisotropies (small scale fluctuations) to make an independent determination of $v = 384 \pm 78 \pm 115$ km s^{-1}, where the first error is statistical and the second is systematic. Relativistic aberration compresses and stretches the fluctuations, which can be measured as a shift of the spatial power spectrum over the sky...

Planck 2013 results. XXVII. Doppler boosting of the CMB: Eppur si muove★

ABSTRACT

Our velocity relative to the rest frame of the cosmic microwave background (CMB) generates a dipole temperature anisotropy on the sky which has been well measured for more than 30 years, and has an accepted amplitude of $v/c = 1.23 \times 10^{-3}$, or $v = 369\,\mathrm{km\,s^{-1}}$. In addition to this signal generated by Doppler boosting of the CMB monopole, our motion also modulates and aberrates the CMB temperature fluctuations (as well as every other source of radiation at cosmological distances). This is an order 10^{-3} effect applied to fluctuations which are already one part in roughly 10^5, so it is quite small. Nevertheless, it becomes detectable with the all-sky coverage, high angular resolution, and low noise levels of the *Planck* satellite. Here we report a first measurement of this velocity signature using the aberration and modulation effects on the CMB temperature anisotropies, finding a component in the known dipole direction, $(l, b) = (264^\circ, 48^\circ)$, of $384\,\mathrm{km\,s^{-1}} \pm 78\,\mathrm{km\,s^{-1}}$ (stat.) $\pm$ $115\,\mathrm{km\,s^{-1}}$ (syst.). This is a significant confirmation of the expected velocity.

Key words. cosmology: observations – cosmic background radiation – reference systems – relativistic processes

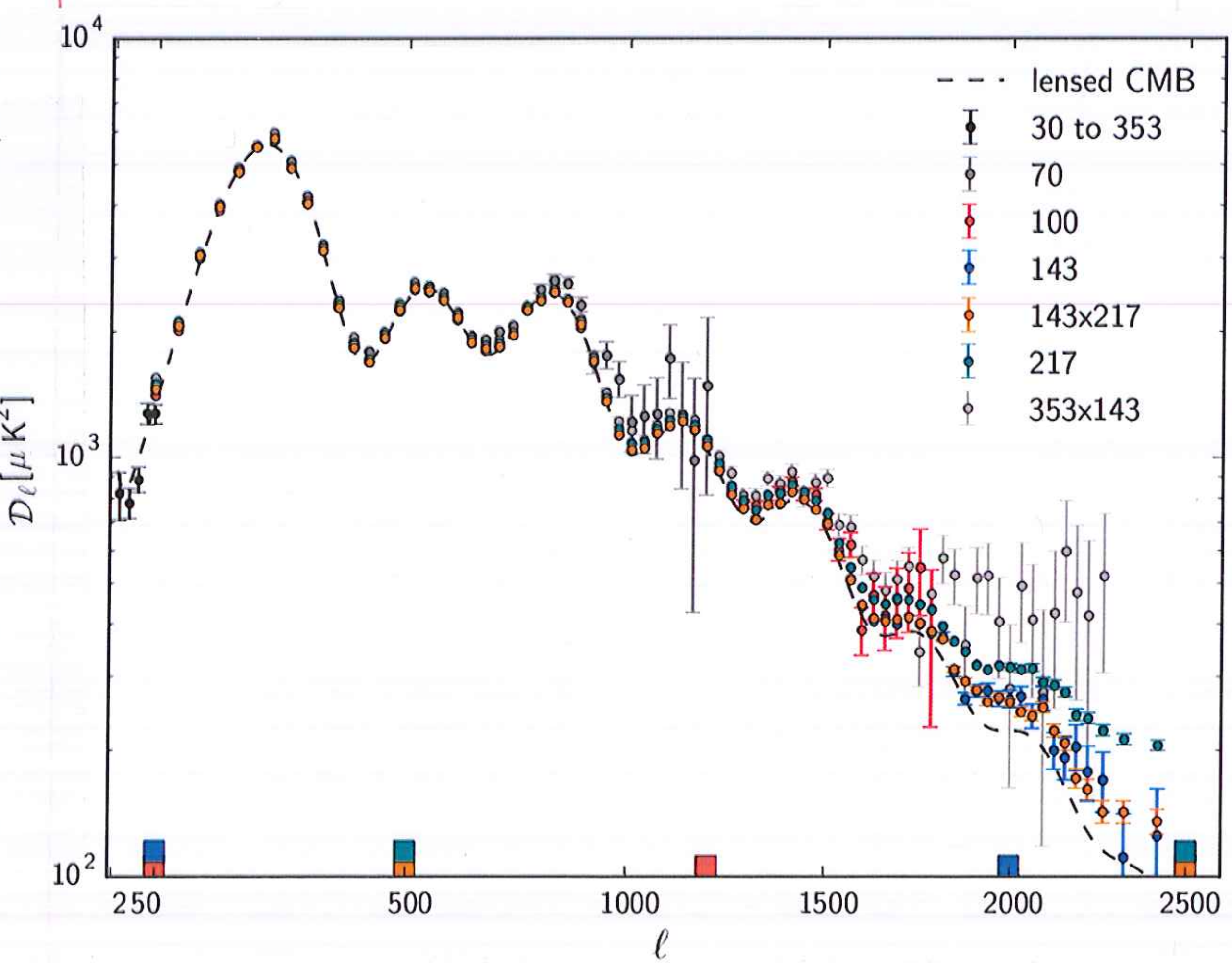
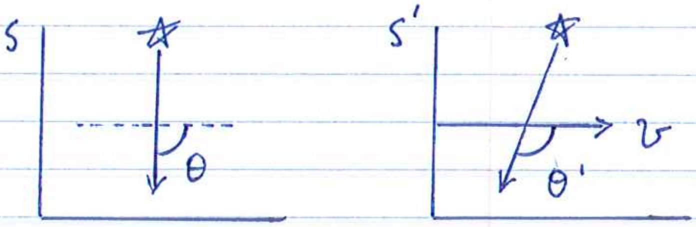


Fig. 11. *Planck* power spectra and data selection. The coloured tick marks indicate the ℓ-range of the four cross-spectra included in CamSpec (and computed with the same mask, see Table 4). Although not used, the 70 GHz and 143 × 353 GHz spectra demonstrate the consistency of the data. The dashed line indicates the best-fit *Planck* spectrum.

- Example: Relativistic aberration of the CMB fluctuations

$$\cos\theta' = \frac{\cos\theta - \beta}{1-\beta\cos\theta}$$



when $\theta = \frac{\pi}{2}$, $\cos\theta' = -\beta$

For $\beta = 1.23\times10^{-3}$, $\theta' = 90^\circ.07$

The maximum aberration is $0^\circ.07$ or $4'.2$.
(Note, the aberration due to the Earth's orbital 30 km s^{-1} velocity around the Sun is $20''.6$).
This corresponds to about 1 part in 1000 of the angular distance (0.07/90), which shifts the $l=1000$ multipole by about 1.

- Example: Predicted detection of the same kinematic dipole distortion in galaxy survey number counts

If galaxy number counts are isotropic on large scales

$N(\theta,\phi) = N/4\pi$ [number of galaxies per steradian]

In the moving frame

$$N'(\theta',\phi')\,d\Omega' = N(\theta,\phi)\,d\Omega$$

On page 99 of the notes we transformed solid angle using aberration. Let $\mu = \cos\theta$, $d\Omega = d\mu\, d\phi$

$$\mu = \frac{\mu'+\beta}{1+\beta\mu'} \qquad \nu = \nu'\gamma(1+\beta\mu')$$

$$\frac{d\mu}{d\mu'} = \frac{1+\beta\mu' - \beta(\mu'+\beta)}{(1+\beta\mu')^2} = \frac{1}{\gamma^2(1+\beta\mu')^2} = \frac{d\Omega}{d\Omega'}$$

$$N'(\theta',\phi') = \frac{N}{4\pi}\frac{1}{\gamma^2(1+\beta\mu')^2} \approx \frac{N}{4\pi}(1-2\beta\mu')$$

The limiting magnitude of detection is m'_{lim}, where

$$m'_{lim} - m_{lim} = -2.5 \log (f'_{lim} / f_{lim})$$

The flux of a galaxy is $f' = I' d\Omega'$. If the spectrum of a galaxy is a power law, $f = \text{const.}\ \nu^{\alpha}$, then

$$\frac{f'_{\nu'}}{f_{\nu'}} = \frac{I'_{\nu'}\, d\Omega'}{I_{\nu'}\, d\Omega} = \left(\frac{\nu'}{\nu}\right)^{3-\alpha} \left(\frac{\nu'}{\nu}\right)^{-2} = \left(\frac{\nu'}{\nu}\right)^{1-\alpha}$$

and $m'_{lim} = m_{lim} - 2.5(1-\alpha) \log (\nu'/\nu)$

If the number of galaxies brighter than m_{lim} is a function

$$N(< m_{lim}) = \text{const.}\ 10^{x\, m_{lim}}$$

where $x \approx 0.11$, then

$$\frac{N'(< m'_{lim})}{N(< m_{lim})} = 10^{x(m'_{lim} - m_{lim})} = \left(\frac{\nu'}{\nu}\right)^{-2.5x(1-\alpha)}$$

$$= (1 - \beta \cos \theta')^{2.5x(1-\alpha)}$$

$$\approx 1 - 2.5x(1-\alpha)\beta \cos \theta'$$

Combining the number counts due to aberration on the previous page with the change of numbers due to the change of limiting magnitude, we have

$$N'(< m'_{lim}) = N(< m_{lim})(1 - 2\beta\mu')\left[1 - 2.5x(1-\alpha)\beta\mu'\right]$$

Keeping terms to first order in β,

$$N'(< m'_{lim}) = N(< m_{lim})\left[1 - 2\beta\mu'(1 + 1.25x(1-\alpha))\right]$$

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KINEMATIC DIPOLE DETECTION WITH GALAXY SURVEYS: FORECASTS AND REQUIREMENTS

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ABSTRACT

Upcoming or future deep galaxy samples with wide sky coverage can provide independent measurement of the kinematic dipole—our motion relative to the rest frame defined by the large-scale structure. Such a measurement would present an important test of the standard cosmological model, as the standard model predicts the galaxy measurement should precisely agree with the existing precise measurements made using the cosmic microwave background. However, the required statistical precision to measure the kinematic dipole typically makes the measurement susceptible to bias from the presence of the local-structure-induced dipole contamination. In order to minimize the latter, a sufficiently deep survey is required. We forecast both the statistical error and the systematic bias in the kinematic dipole measurements. We find that a survey covering ~75% of the sky in both hemispheres and having ~30 million galaxies can detect the kinematic dipole at 5σ, while its median redshift should be at least $z_{\rm med} \sim 0.75$ for negligible bias from the local structure.

Key words: galaxies: statistics – large-scale structure of universe

1. INTRODUCTION

Measurements of the motion of our solar system through the cosmic microwave background (CMB) rest frame represent one of the early successes of precision cosmology. This so-called kinematic dipole corresponds to a velocity of (369 ± 0.9) km s^{-1} in the direction $(l, b) = (263^\circ\!.99 \pm 0^\circ\!.14, 48^\circ\!.26 \pm 0^\circ\!.03)$ (Hinshaw et al. 2009). The kinematic dipole has even been detected (though not *as* precisely measured) by observing the relativistic aberration in the CMB anisotropy that it causes, which is detected via the coupling of high CMB multipoles in Planck (Aghanim et al. 2014).

Independently, the past few decades have seen significant progress in measuring the dipole in the distribution of extragalactic sources. The contribution of our motion through the large-scale structure (LSS) rest frame—the kinematic dipole—also leads to relativistic aberration, this time of galaxies or other observed LSS sources. We define the dipole amplitude via the amount of its "bunching up" of galaxies in the direction of the dipole

$$\frac{\delta N(\hat{n})}{\bar{N}} = A\,\hat{d}\cdot\hat{n} + \epsilon(\hat{n}), \qquad (1)$$

where N is the galaxy number in an arbitrary direction $\hat{n}$, $\hat{d}$ is the dipole direction, and ϵ is random noise. The dipole amplitude A is approximately (but not exactly) equal to our velocity through the LSS rest frame in units of the speed of light; the precise relation is given in the following section.

However, the dominant contribution to the LSS dipole is typically not our motion through the LSS rest frame, but rather the fluctuations in structure due to the finite depth of the survey. The dipole component of the latter—the so-called "local-structure dipole" in the nomenclature of Gibelyou & Huterer (2012)—has amplitude $A \sim 0.1$ for shallow surveys extending to $z_{\max} \sim 0.1$, but is significantly smaller for deeper surveys. The local-structure dipole is the dominant signal at multipole $\ell = 1$ in all extant LSS surveys. It has been measured and reported either explicitly (Baleisis et al. 1998; Blake & Wall 2002; Hirata 2009; Gibelyou & Huterer 2012; Rubart & Schwarz 2013; Appleby & Shafieloo 2014; Fernández-Cobos et al. 2014; Yoon et al. 2014; Alonso et al. 2015), or as part of the angular power spectrum measurements. No LSS survey completed to date therefore had a chance to separate the small kinematic signal from the larger local-structure dipole contamination due to insufficient depth and sky coverage. This will change drastically with the new generation of wide, deep surveys.

Standard theory based on the adiabatic initial perturbations predicts that the kinematic dipole measured by the LSS should agree with the one measured by the CMB. Detection of an anomalously large (or small) dipole or the disagreement of its direction from that of the CMB dipole could indicate new physics: for example, the presence of superhorizon fluctuations in the presence of isocurvature fluctuations (Turner 1991; Erickcek et al. 2008; Zibin & Scott 2008; Itoh et al. 2010). Clearly, a kinematic dipole detection and measurement represent an important and fundamental consistency test of the standard cosmological model.

2. METHODOLOGY

2.1. Theoretical Signal

The expected LSS kinematic dipole signal amplitude is given by (Burles & Rappaport 2006; Itoh et al. 2010)

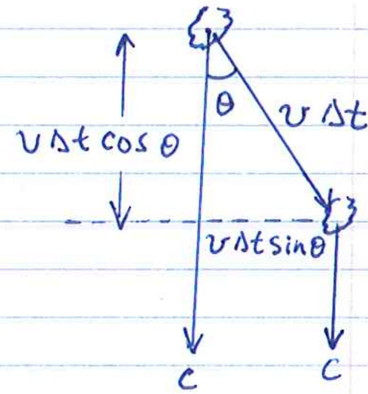
$$A = 2\tilde{\beta} = 2[1 + 1.25x(1 - p)]\beta \qquad (2)$$

where $\beta = v/c = 0.00123$ (assuming the CMB dipole). The contribution 2β comes from relativistic aberration, while the correction $[1 + 1.25x(1 - p)]$ corresponds to the Doppler effect; here x is the faint-end slope of the source counts, $x \equiv d\log_{10}[n(m < m_{\rm lim})]/dm_{\rm lim}$, and p is the logarithmic slope of the intrinsic flux density power-law, $S_{\rm rest}(\nu) \propto \nu^p$.

Clearly, the parameters x and p depend on the population of sources selected by the survey, and on any population drifts as a function of magnitude. We now estimate these parameters—note also that we only need the quantity A to set our fiducial

2. Superluminal Motion ($v_{app} > c$)

$$\Delta t_{app} = \Delta t - \frac{v \Delta t \cos\theta}{c}$$



This is the time between the arrival of signals that were emitted at points 1 and 2, a time Δt apart.

The apparent transverse velocity on the sky is

$$v_{app\perp} = \frac{v \Delta t \sin\theta}{\Delta t_{app}} = \frac{v \sin\theta}{1 - \frac{v}{c}\cos\theta}$$

$$\boxed{\beta_{app\perp} = \frac{\beta \sin\theta}{1 - \beta\cos\theta}}$$

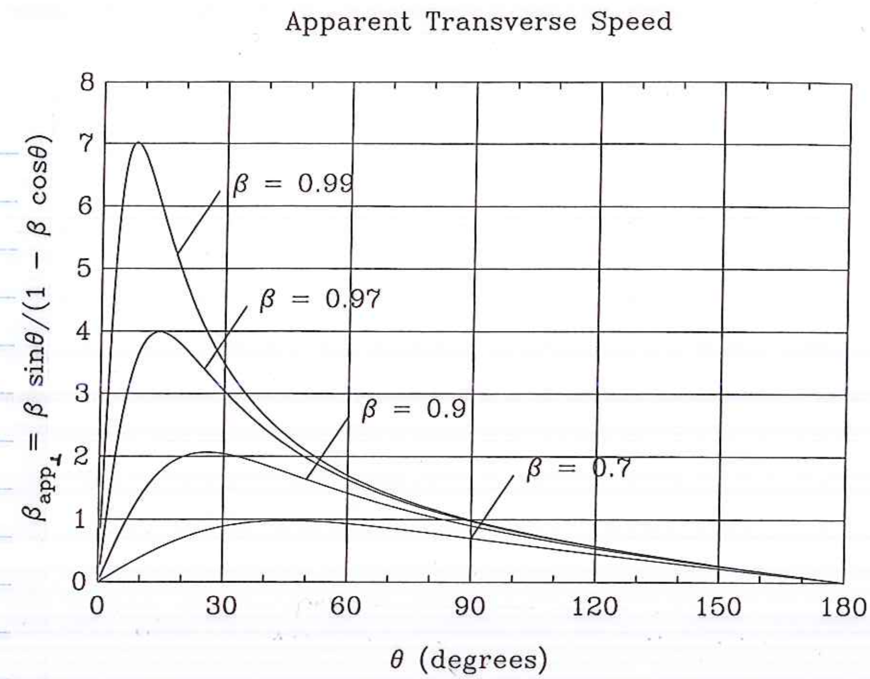
$\beta_{app\perp}$ is maximized at a specific angle θ_c

$$\frac{d\beta_{app\perp}}{d\theta} = \frac{\beta\cos\theta(1-\beta\cos\theta) - \beta^2\sin^2\theta}{(1-\beta\cos\theta)^2} = 0$$

when $\cos\theta_c = \beta$

Then $\beta_{app\perp}(\max) = \dfrac{\beta\sqrt{1-\beta^2}}{1-\beta^2} = \beta\gamma$

But apparent superluminal motion is not a relativistic effect because it does not depend on time dilation, Lorentz contraction, or any other effect of Lorentz transformations. It's just an illusion.



Note that longitudinal motion can also be superluminal. Parallel to the line of sight,

$$v_{app\parallel} = \frac{v \Delta t \cos\theta}{\Delta t_{app}} = \frac{v \cos\theta}{1 - \beta\cos\theta}$$

$$\beta_{app\parallel} = \frac{\beta\cos\theta}{1-\beta\cos\theta}$$

$\beta_{app\parallel}$ is maximized for $\theta = 0$. Then

$$\beta_{app}(max) = \frac{\beta}{1-\beta}$$

In the limit $\beta \to 1$ $\quad \beta = \left(1 - \frac{1}{\gamma^2}\right)^{1/2} \to 1 - \frac{1}{2\gamma^2}$

So $\beta_{app_\parallel}(max) \to 2\gamma^2$ as $\beta \to 1$

This effect is very important in viewing extremely relativistic jets or explosions, such as gamma-ray bursts.