Annual Reviews of Astronomy and Astrophysics (2002)

Adopting a reasonable cosmology and accounting for the increase in the universal matter density with redshift, the mass limit for a given SZE survey flux sensitivity is not expected to change more than a factor of ~2–3 for any clusters with $z > 0.05$.

SZE surveys therefore offer an ideal tool for determining the cluster-density evolution. Analyses of even a modest survey covering ~10 square degrees will provide interesting constraints on the matter density of the universe. The precision with which cosmological constraints can be extracted from much larger surveys, however, will be limited by systematics due to our insufficient understanding of the structure of clusters, their gas properties, and their evolution.

Insights into the structure of clusters will be provided by high-resolution SZE observations, especially when combined with other measurements of the clusters. Fortunately, many of the cluster properties derived directly from observational data can be determined in several different ways. For example, the gas-mass fraction can be determined by various combinations of SZE, X-ray, and lensing observations. The electron temperature, a direct measure of a cluster's mass, can be measured directly through X-ray spectroscopy or determined through the analysis of various combinations of X-ray, SZE, and lensing observations. Several of the desired properties of clusters are therefore over-constrained by observation, providing critical insights to our understanding of clusters, and critical tests of current models for the formation and evolution of galaxy clusters. With improved sensitivity, better angular resolution, and sources out to $z \sim 2$, the next generation of SZE observations will provide a good view of galaxy cluster structure and evolution. This will allow, in principle, the dependence of the cluster yields from large SZE surveys on the underlying cosmology to be separated from the dependence of the yields on cluster structure and evolution.

We outline the properties of the SZE in the next section and provide an overview of the current state of the observations in "Status of Observations." This is followed in "Sky Surveys with the Sunyaev-Zel'dovich Effect" by predictions for the expected yields of upcoming SZE surveys. In "Cosmology from Sunyaev-Zel'dovich Effect Survey Samples" we provide an overview of the cosmological tests that will be possible with catalogs of SZE-selected clusters. This is followed by a discussion of backgrounds, foregrounds, contaminants, and theoretical uncertainties that could adversely affect cosmological studies with the SZE and a discussion of observations that could reduce or eliminate these concerns. Throughout the paper, $h$ is used to parametrize the Hubble constant by $H_0 = 100\,h$ km s$^{-1}$ Mpc$^{-1}$.

## THE SUNYAEV-ZEL'DOVICH EFFECT

### Thermal Sunyaev-Zel'dovich Effect

The SZE is a small spectral distortion of the CMB spectrum caused by the scattering of the CMB photons off a distribution of high-energy electrons. We focus only on the SZE caused by the hot thermal distribution of electrons provided by the ICM of galaxy clusters. CMB photons passing through the center of a massive cluster have only a ≈ 1% probability of interacting with an energetic ICM electron. The resulting inverse Compton scattering preferentially boosts the energy of the CMB photon by roughly $k_B T_e / m_e c^2$, causing a small ($\lesssim$1 mK) distortion in the CMB spectrum. Figure 1 shows the SZE spectral distortion for a fictional cluster that is over 1000 times more massive than a typical cluster to illustrate the small effect. The SZE appears as a decrease in the intensity of the CMB at frequencies below $\lesssim$218 GHz and as an increase at higher frequencies.

The derivation of the SZE can be found in the original papers of Sunyaev & Zel'dovich (Sunyaev & Zel'dovich 1970, 1972), in several reviews (Sunyaev & Zel'dovich 1980a, Rephaeli 1995, Birkinshaw 1999), and in a number of more recent contributions that include relativistic corrections (see below for references). This review discusses the basic features of the SZE that make it a useful cosmological tool.



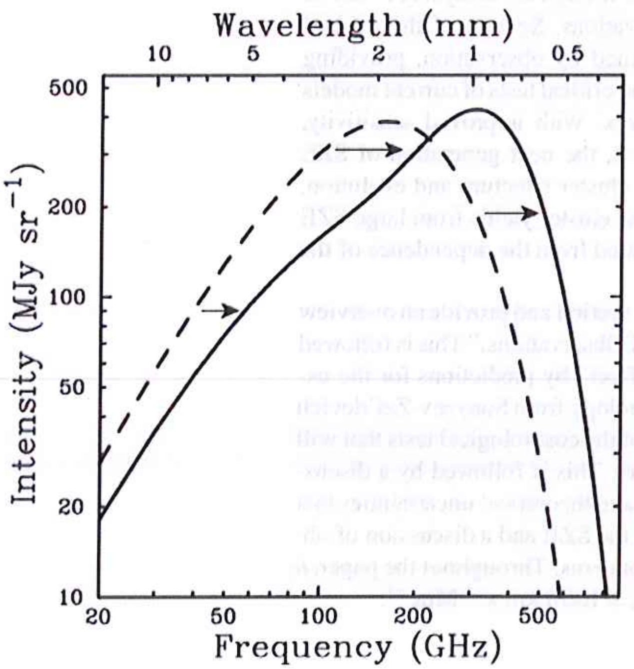


**Figure 1** The cosmic microwave background (CMB) spectrum, undistorted (*dashed line*) and distorted by the Sunyaev-Zel'dovich effect (SZE) (*solid line*). Following Sunyaev & Zel'dovich (1980a) to illustrate the effect, the SZE distortion shown is for a fictional cluster 1000 times more massive than a typical massive galaxy cluster. The SZE causes a decrease in the CMB intensity at frequencies $\lesssim$218 GHz and an increase at higher frequencies.

The SZE spectral distortion of the CMB expressed as a temperature change $\Delta T_{SZE}$ at dimensionless frequency $x \equiv \frac{h\nu}{k_B T_{CMB}}$ is given by

$$\frac{\Delta T_{SZE}}{T_{CMB}} = f(x)\, y = f(x) \int n_e \frac{k_B T_e}{m_e c^2} \sigma_T \, d\ell, \tag{1}$$

where $y$ is the Compton $y$-parameter, which for an isothermal cluster equals the optical depth, $\tau_e$, times the fractional energy gain per scattering, $\sigma_T$ is the Thomson cross-section, $n_e$ is the electron number density, $T_e$ is the electron temperature, $k_B$ is the Boltzmann constant, $m_e c^2$ is the electron rest mass energy, and the integration is along the line of sight. The frequency dependence of the SZE is

$$f(x) = \left( x \frac{e^x + 1}{e^x - 1} - 4 \right) (1 + \delta_{SZE}(x, T_e)), \tag{2}$$

where $\delta_{SZE}(x, T_e)$ is the relativistic correction to the frequency dependence. Note that $f(x) \to -2$ in the nonrelativistic and Rayleigh-Jeans (RJ) limits.

It is worth noting that $\Delta T_{SZE}/T_{CMB}$ is independent of redshift, as shown in Equation 1. This unique feature of the SZE makes it a potentially powerful tool for investigating the high-redshift universe.

Expressed in units of specific intensity, common in millimeter SZE observations, the thermal SZE is

$$\Delta I_{SZE} = g(x) I_0 y, \tag{3}$$

where $I_0 = 2\,(k_B T_{CMB})^3/(hc)^2$ and the frequency dependence is given by

$$g(x) = \frac{x^4 e^x}{(e^x - 1)^2} \left( x \frac{e^x + 1}{e^x - 1} - 4 \right) (1 + \delta_{SZE}(x, T_e)). \tag{4}$$

$\Delta T_{SZE}$ and $\Delta I_{SZE}$ are simply related by the derivative of the blackbody with respect to temperature, $|dB_\nu/dT|$.

The spectral distortion of the CMB spectrum by the thermal SZE is shown in Figure 2 (*solid line*) for a realistic massive cluster ($y = 10^{-4}$) in units of intensity (*left panel*) and RJ brightness temperature (*right panel*). The RJ brightness is shown because the sensitivity of a radio telescope is calibrated in these units. It is defined simply by $I_\nu = (2k_B\nu^2/c^2)T_{RJ}$, where $I_\nu$ is the intensity at frequency $\nu$, $k_B$ is Boltzmann's constant, and $c$ is the speed of light. The CMB blackbody spectrum, $B_\nu(T_{CMB})$, multiplied by 0.0005 (*dotted line*) is also shown for comparison. Note that the spectral signature of the thermal effect is distinguished readily from a simple temperature fluctuation of the CMB. The kinetic SZE distortion is shown by the dashed curve (see "Kinetic Sunyaev-Zel'dovich Effect," below). In the nonrelativistic regime it is indistinguishable from a CMB temperature fluctuation.

The gas temperatures measured in massive galaxy clusters are around $k_B T_e \sim$ 10 keV (Mushotzky & Scharf 1997, Allen & Fabian 1998) and are as high as $\sim$17 keV in the galaxy cluster 1E 0657-56 (Tucker et al. 1998). The mass is expected to scale with temperature roughly as $T_e \propto M^{2/3}$. At these temperatures

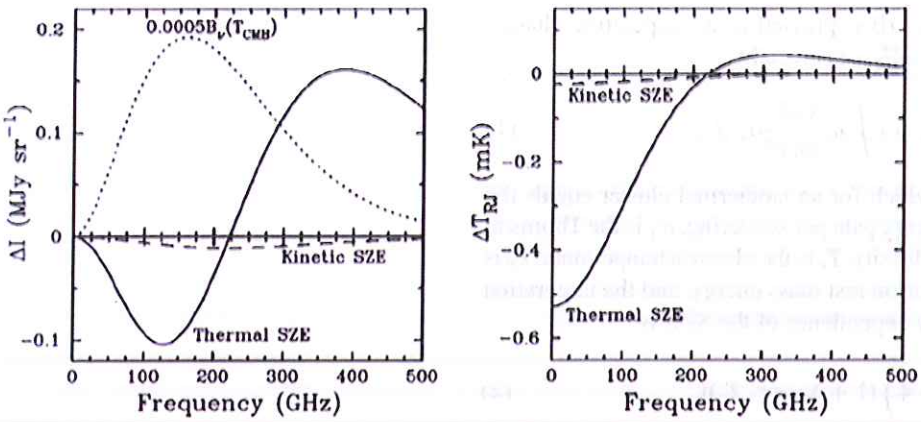


**Figure 2** Spectral distortion of the cosmic microwave background (CMB) radiation due to the Sunyaev-Zel'dovich effect (SZE). The left panel shows the intensity and the right panel shows the Rayleigh Jeans brightness temperature. The thick solid line is the thermal SZE and the dashed line is the kinetic SZE. For reference the 2.7 K thermal spectrum for the CMB intensity scaled by 0.0005 is shown by the dotted line in the left panel. The cluster properties used to calculate the spectra are an electron temperature of 10 keV, a Compton $y$ parameter of $10^{-4}$, and a peculiar velocity of 500 km $s^{-1}$.

electron velocities are becoming relativistic, and small corrections are required for accurate interpretation of the SZE. There has been considerable theoretical work that includes relativistic corrections to the SZE (Wright 1979; Fabbri 1981; Rephaeli 1995; Rephaeli & Yankovitch 1997; Stebbins 1997; Itoh et al. 1998; Challinor & Lasenby 1998, 1999; Sazonov & Sunyaev 1998a,b; Nozawa et al. 1998b; Molnar & Birkinshaw 1999; Dolgov et al. 2001). All of these derivations agree for $k_B T_e \lesssim 15$ keV, appropriate for galaxy clusters. For a massive cluster with $k_B T_e \sim 10$ keV ($k_B T_e / m_e c^2 \sim 0.02$), the relativistic corrections to the SZE are on the order of a few percent in the RJ portion of the spectrum but can be substantial near the null of the thermal effect. Convenient analytical approximations to fifth order in $k_B T_e / m_e c^2$ are presented in Itoh et al. (1998).

Particularly relevant for finding clusters with an SZE survey is the integrated SZE signal. Because the SZE signal is the integrated pressure, integrating over the solid angle of the cluster provides a sum of all of the electrons in the cluster weighted by temperature. This provides a relatively clean measure of the total thermal energy of the cluster. Integrating the SZE over the solid angle of the cluster, $d\Omega = dA/D_A^2$, gives

$$\int \Delta T_{SZE}\, d\Omega \propto \frac{N_e \langle T_e \rangle}{D_A^2} \propto \frac{M \langle T_e \rangle}{D_A^2}, \tag{5}$$

where $N_e$ is the total number of electrons in the clusters, $\langle T_e \rangle$ is the mean electron temperature, $D_A$ is the angular diameter distance, and $M$ is the mass of the cluster (either gas or total mass as $M_{gas} = M_{total} f_g$, where $f_g$ is the gas-mass fraction). The integrated SZE flux is simply the temperature-weighted mass of the cluster divided by $D_A^2$. The angular diameter distance $D_A(z)$ is fairly flat at high redshift. Also, a cluster of a given mass will be denser and therefore hotter at high redshift because the universal matter density increases as $(1+z)^3$. Therefore, one expects an SZE survey to detect all clusters above some mass threshold with little dependence on redshift (see "Mass Limits of Observability," below).

The most important features of the thermal SZE are that (*a*) it is a small spectral distortion of the CMB of order $\sim 1$ mK, which is proportional to the cluster pressure integrated along the line of sight [Equation 1]; (*b*) it is independent of redshift; (*c*) it has a unique spectral signature with a decrease in the CMB intensity at frequencies $\lesssim 218$ GHz and an increase at higher frequencies; and (*d*) the integrated SZE flux is proportional to the temperature-weighted mass (total thermal energy) of the cluster, implying that SZE surveys will have a mass threshold nearly independent of redshift.

## Kinetic Sunyaev-Zel'dovich Effect

If the cluster is moving with respect to the CMB rest frame, there will be an additional spectral distortion due to the Doppler effect of the cluster bulk velocity on the scattered CMB photons. If a component of the cluster velocity, $v_{pec}$, is projected along the line of sight to the cluster, then the Doppler effect will lead to an observed distortion of the CMB spectrum referred to as the kinetic SZE. In the nonrelativistic limit the spectral signature of the kinetic SZE is a pure thermal distortion of magnitude

$$\frac{\Delta T_{SZE}}{T_{CMB}} = -\tau_e \left(\frac{v_{pec}}{c}\right), \tag{6}$$

where $v_{pec}$ is along the line of sight; i.e., the emergent spectrum is still described completely by a Planck spectrum, but at a slightly different temperature, lower (higher) for positive (negative) peculiar velocities (Sunyaev & Zel'dovich 1972, Phillips 1995, Birkinshaw 1999) (see Figure 2).

Relativistic perturbations to the kinetic SZE are due to the Lorentz boost to the electrons provided by the bulk velocity (Nozawa et al. 1998a, Sazonov & Sunyaev 1998a). The leading term is of order $(k_B T_e/m_e c^2)(v_{pec}/c)$ and for a 10 keV cluster moving at 1000 km s$^{-1}$ the effect is about an 8% correction to the nonrelativistic term. The $(k_B T_e/m_e c^2)^2(v_{pec}/c)$ term is only about 1% of the nonrelativistic kinetic SZE, and the $(v_{pec}/c)^2$ term is only 0.2%.

## Polarization of the Sunyaev-Zel'dovich Effect

The scattering of the CMB photons by the hot intracluster medium (ICM) electrons can result in polarization at levels proportional to powers of $(v_{pec}/c)$ and $\tau_e$. The largest polarization is expected from the anisotropic optical depth to a given location in the cluster. For example, toward the outskirts of a cluster one expects to see a concentric (radial) pattern of the linear polarization at frequencies at which the thermal SZE is positive (negative). Sazonov & Sunyaev (1999) presented plots of the polarization pattern. Nonspherical morphology for the electron distributions will lead to considerably complicated polarization patterns. The peak polarization of this signal will be of order $\tau_e$ times the SZE signal, i.e., of order $0.025(k_B T_e/m_e c^2)\tau_e^2$ times the CMB intensity. For a massive cluster with $\tau_e = 0.01$, the effect would be at the 0.1 $\mu$K level toward the edge of the cluster. In principle, this effect could be used to measure the optical depth of the cluster and therefore separate $T_e$ and $\tau_e$ from a measurement of the thermal SZE (see Equation 1).

It can be shown that polarization of the SZE comes entirely from the quadrupole component of the local radiation field experienced by the scattering electron. In the case above, the quadrupole component at the outskirts of the cluster is caused by the anisotropy in the radiation field in the direction of the cluster center due to the SZE. Sunyaev and Zel'dovich discussed polarization due to the motion of the cluster with respect to the CMB and transverse to our line of sight (Sunyaev & Zel'dovich 1980b; see also Sazonov & Sunyaev 1999). In this case, the quadrupole comes from the Doppler shift. They found the largest terms to be of order $0.1\tau_e(v_{pec}/c)^2$ and $0.025\tau_e^2(v_{pec}/c)$ of the CMB intensity. The latter term, second order in $\tau_e$, can be thought of as imperfect cancellation of the dipole term due to the anisotropic optical depth. Using $\tau_e = 0.01$ and a bulk motion of 500 km s$^{-1}$ results in polarization levels of order 10 $nK$, far beyond the sensitivity of current instrumentation.

The CMB as seen by the cluster electrons will have a quadrupole component and therefore the electron scattering will lead to linear polarization. This mechanism could possibly be used to trace the evolution of the CMB quadrupole if polarization measurements could be obtained for a large number of clusters binned in direction and redshift (Kamionkowski & Loeb 1997; Sazonov & Sunyaev 1999). Sazonov and Sunyaev calculated the expected polarization level and found that the maximum CMB quadrupole–induced polarization is 50 $(\tau_e/0.01)$ nK, somewhat higher than the expected velocity-induced terms discussed above. The effect is again too small to expect detection in the near future. However, by averaging over many clusters, detecting this polarization might be possible with future satellite missions.

## STATUS OF OBSERVATIONS

In the 20 years following the first papers by Sunyaev and Zel'dovich (Sunyaev & Zel'dovich 1970, 1972) there were few firm detections of the SZE despite a considerable amount of effort (Birkinshaw 1991). Over the past several years, however, observations of the effect have progressed from low S/N detections and upper limits to high confidence detections and detailed images. In this section we briefly review the current state of SZE observations.

The dramatic increase in the quality of the observations is due to improvements both in low-noise detection systems and in observing techniques, usually using