Astronomy GR6001 (Radiative Processes) Fall 2019
Due Wed. Nov. 6 Prof. J. Halpern

Homework #8

1. Suppose that mono-energetic electrons of a very large initial γ_0 are injected at a constant rate into a region of uniform magnetic field, and that there is time for them to radiate away all of their energy. Show that in a steady-state situation, the resulting synchrotron spectrum is a power law. What the is value of the spectral index?

2. (a) Show that there is a maximum magnetic field strength for which synchrotron radiation is possible. How does the pitch angle of the electron affect this limit?

   (b) Show that there is a maximum frequency for which synchrotron radiation is possible, which is independent of B. Express it in units of eV.

3. The spectrum of the Crab Nebula can be approximated as a broken power law with segments intersecting at 10^{14} Hz (see attached Figure). Assume that the Crab Nebula is at a distance of 2,000 pc and has a total volume of 3×10^{56} cm^3. A magnetic field strength in the nebula of 5×10^{-4} G is measured from Faraday rotation. For simplicity, assume that conditions in the nebula are spatially uniform.

   (a) Estimate the relativistic γ of electrons emitting at frequencies of 100 MHz (radio), 10^{14} Hz (infrared), and 10^{22} Hz (γ-ray), and compare their gyroradii to the radius of the nebula. Also calculate their synchrotron lifetimes.

   (b) Estimate the total number of relativistic electrons in the nebula.

4. Consider a compact extragalactic radio source at a distance of 300 Mpc, with angular diameter 2 milliarcseconds, observed flux density 1×10^{-25} erg cm^{-2} s^{-1} Hz^{-1} at a synchrotron self-absorption frequency of 10^8 Hz, and spectral index $s = 0.75$ in the optically thin regime. Assume that the source is spherical and has homogeneous properties. Calculate the following:

   (a) Brightness temperature at 10^8 Hz.

   (b) Magnetic field strength and total magnetic energy.

   (c) The relativistic γ of the electrons emitting at 10^8 Hz.

   (d) Total energy in relativistic electrons. Is equipartition satisfied?