1. Suppose that mono-energetic electrons of a very large initial $\gamma_0$ are injected at a constant rate into a region of uniform magnetic field, and that there is time for them to radiate away all of their energy. Show that in a steady-state situation, the resulting synchrotron spectrum is a power law. What is the value of the spectral index?

2. (a) Show that there is a maximum magnetic field strength for which synchrotron radiation is possible. How does the pitch angle of the electron affect this limit?

(b) Show that there is a maximum frequency for which synchrotron radiation is possible, which is independent of $B$. Express it in units of eV.

3. The spectrum of the Crab Nebula can be approximated as a broken power law with segments intersecting at $10^{14}$ Hz (see attached Figure). Assume that the Crab Nebula is at a distance of 2,000 pc and has a total volume of $3 \times 10^{56}$ cm$^3$. A magnetic field strength in the nebula of $5 \times 10^{-4}$ G is measured from Faraday rotation. For simplicity, assume that conditions in the nebula are spatially uniform.

(a) Estimate the relativistic $\gamma$ of electrons emitting at frequencies of 100 MHz (radio), $10^{14}$ Hz (infrared), and $10^{22}$ Hz ($\gamma$-ray), and compare their gyroradii to the radius of the nebula. Also calculate their synchrotron lifetimes.

(b) Estimate the total number of relativistic electrons in the nebula.

4. Consider a compact extragalactic radio source at a distance of 300 Mpc, with angular diameter 2 milliarcseconds, observed flux density $1 \times 10^{-25}$ erg cm$^{-2}$ s$^{-1}$ Hz$^{-1}$ at a synchrotron self-absorption frequency of $10^8$ Hz, and spectral index $s = 0.75$ in the optically thin regime. Assume that the source is spherical and has homogeneous properties. Calculate the following:

(a) Brightness temperature at $10^8$ Hz.

(b) Magnetic field strength and total magnetic energy.

(c) The relativistic $\gamma$ of the electrons emitting at $10^8$ Hz.

(d) Total energy in relativistic electrons. Is equipartition satisfied?