Homework #4

1. Consider radiation from a semi-infinite atmosphere of uniform temperature that is emitting, absorbing, and scattering. Assume for simplicity that \( \epsilon_\nu \equiv \alpha_\nu / (\alpha_\nu + \sigma_\nu) \) and \( B_\nu \) are independent of optical depth \( \tau_\nu \). Using the two-stream approximation and radiative diffusion equation:

(a) Show that the mean intensity \( J_\nu \) at any optical depth is less than \( B_\nu \).

(b) Express the outgoing intensity \( I_\nu \) at the surface in terms of \( B_\nu \) and \( \epsilon_\nu \).

(c) Derive the flux \( F_\nu \) from the surface and compare it to \( \pi B_\nu \), the flux of a blackbody.

2. A linear polarizer is a device that transmits 100% of the intensity of light polarized along one axis, and none of the light polarized perpendicular to that axis. A right-circular polarizer and left-circular polarizer are defined in a similar way. Using the general form for the \( E \)-field of a plane wave propagating in the \( z \)-direction (for simplicity evaluate it at \( z = 0, t = 0 \)),

\[
\vec{E}(z,t) = (\varepsilon_1 e^{i\phi_1} \hat{x} + \varepsilon_2 e^{i\phi_2} \hat{y}) e^{i(kz-\omega t)},
\]

prove that these expressions for the Stokes parameters:

\[
I = \varepsilon_1^2 + \varepsilon_2^2 \\
Q = \varepsilon_1^2 - \varepsilon_2^2 \\
U = 2\varepsilon_1\varepsilon_2 \cos (\phi_1 - \phi_2) \\
V = 2\varepsilon_1\varepsilon_2 \sin (\phi_1 - \phi_2)
\]

correspond to the following definitions:

(I) Total intensity

(Q) Difference between intensity transmitted by a linear polarizer oriented along the \( x \)-axis and one oriented along the \( y \)-axis

(U) Difference between intensity transmitted by a linear polarizer oriented at 45° to the \( x \)-axis and one oriented at 135° to the \( x \)-axis

(V) Difference between intensity transmitted by a right-circular polarizer and a left-circular polarizer

3. Using the expressions for the differential Thomson scattering cross-section \( d\sigma_T / d\Omega \) of unpolarized light, and of 100% linearly polarized light, show that the total Thomson cross section \( \sigma_T \) is the same in either case.