Astronomy GR6001 (Radiative Processes)
Due Wed. Sep. 25

Fall 2019
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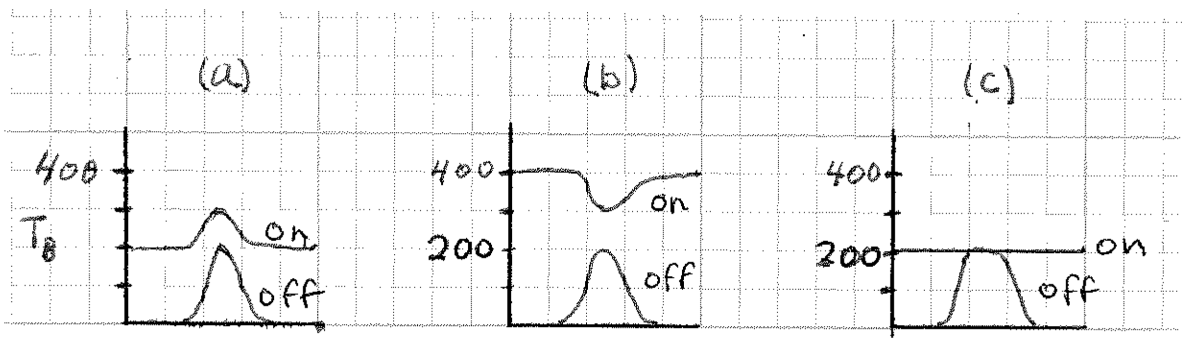
Homework #3

1. The 21 cm hyperfine transition is observed from a cloud of uniform spin temperature, on and off the position of a background continuum source of brightness temperature T_0. Three examples of such on and off spectra are shown. Derive the spin temperature and optical depth at line center in each case.



2. Consider a neutral hydrogen cloud that is optically thin to the 21 cm hyperfine transition, meaning $\tau_\nu \ll 1$ over the entire line profile.

   (a) Show that the brightness temperature T_B of the cloud viewed in emission is related to the spin temperature T_s as

   $$T_B = T_s \tau_\nu.$$

   (b) Convert the arbitrary line profile function $\phi(\nu)$ into $\phi(v_r)$, a function of radial velocity v_r. Note, by definition:

   $$\phi(\nu)\, d\nu = \phi(v_r)\, dv_r$$

   (c) Show that the neutral hydrogen column density in the optically thin case can be expressed as

   $$N_{\rm HI} = 1.82 \times 10^{18} \int T_B\, dv_r \ \ {\rm cm}^{-2},$$

   where v_r is in units of km s^{-1}.

3. Let's see what the spin temperature of the 21 cm line of hydrogen will be if the levels are populated by atomic collisions as well as by emission and absorption of the cosmic microwave background (CMB). The spin temperature T_S is defined from the level populations by

$$\frac{n_1}{n_0} = \frac{g_1}{g_0} e^{-h\nu/kT_S}.$$

In equilibrium, the level populations are calculated using detailed balance:

$$n_0 \left(B_{01} B_\nu + n\, k_{01}\right) = n_1 \left(A_{10} + B_{10} B_\nu + n\, k_{10}\right).$$

Here $n = n_0 + n_1$ is the number density of hydrogen, B_ν is the specific intensity of the CMB, and k_{01} and k_{10} are the collisional excitation and deexcitation rate coefficients. The two collisional rate coefficients are not independent, but are related to each other by the principle of detailed balance in collisional equilibrium,

$$n_0\, k_{01} = n_1\, k_{10}.$$

Therefore

$$\frac{k_{01}}{k_{10}} = \frac{g_1}{g_0} e^{-h\nu/kT_K}$$

where T_K is the kinetic temperature of the atoms.

(a) Show that the spin temperature is

$$T_S = \frac{T_{CMB} + y T_K}{1 + y},$$

where

$$y \equiv \frac{h\nu}{kT_K} \frac{n}{n_{cr}},$$

$T_{CMB} = 2.73$ K, and $n_{cr} = A_{10}/k_{10} \approx 3 \times 10^{-5}$ cm^{-3} is the critical density for deexcitation. You may use the fact that $h\nu \ll kT_S$, and assume that $n \gg n_{cr}$.

(b) Estimate the spin temperature for the two following sets of gas properties:

$n = 20$ cm^{-3}, $T_K = 100$ K

$n = 0.25$ cm^{-3}, $T_K = 8000$ K