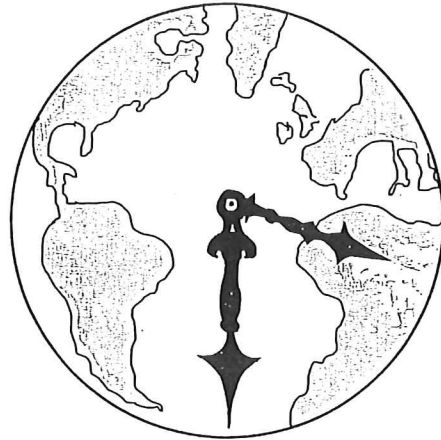


X. ATOMS, STARS, AND STELLAR EVOLUTION

The Age of The World



As the 19th century progressed, the early-century puzzles (caloric versus kinetic theory, continuum versus atoms, particles versus waves, mechanical versus heat energy, electricity and magnetism) seemed to resolve. The conservation of energy was recognized as a powerful new principle – as consequential as the law of gravity. That law of gravity was itself spectacularly vindicated to high precision, by the discovery of a new planet (Neptune, in 1846), precisely where Newtonian gravity predicted it to be. Electricity and magnetism, which had been regarded as utterly strange in years past, were found by Faraday and Maxwell to obey regular mathematical laws. The problem of stellar parallax – a huge problem since the days of Copernicus – was finally solved, with Bessel's 1838 discovery of parallax in the star 61 Cygni. A feeling was spreading among physicists that all the important laws of physics had been discovered, and the 20th century would be mainly a mopping-up operation. It was the medieval “Great Chain of Being” all over again. Oh sure, there were issues yet to be understood – like life, and human consciousness. But Darwin's work suggested that these too might well be governed by laws of strict determinism. Those laws could be discovered in the next century.

A famous advocate of this view, and perhaps the most famous physicist of his era, was Lord Kelvin (= William Thomson, before receiving his lordship). Kelvin tried to calculate how old the Earth was. It was well known, from studies in deep mines, that the Earth gets hotter and hotter as you go deeper down. He reasoned that this is probably heat left over from the formation of the Earth – the release of potential energy as all that matter (6×10^{24} kg worth) slowly fell onto the proto-Earth. The idea is that *the Earth hasn't completely cooled yet*. His calculations showed that to match the temperature data, the age of the Earth should be in the range 20-50 million years.

He also calculated the age of the Sun. Again, the energy source is gravity, which yields a total energy $E = GM^2/R$. Assuming present-day values for M and R , this implies a total energy available (mks units) of

$$E = GM^2/R = (6.7 \times 10^{-11}) (2 \times 10^{30})^2 / (7 \times 10^8) = 4 \times 10^{41} \text{ J}.$$

Since the Sun's luminosity is now 4×10^{26} J/s, it has been around for approximately $E/L = (4 \times 10^{41} \text{ J}) / (4 \times 10^{26} \text{ J/s}) = 10^{15} \text{ s}$. This is about 3×10^7 years, or 30 million years.

So 30 million years is about the right number, regardless of whether you use the Sun or the Earth as your laboratory. The energy source is gravitation. Problem solved.

Of course, there were howls from other quarters of society, especially from religious purists who wished to rely on the sequence of generations (who begat who) in Genesis. Archbishop Ussher did his scholarly (?) best to do the addition, and arrived at a precise date for the origin of the world: Oct. 23, 4004 B.C. You can read about it, and about its disputation in a famous 1925 criminal trial*, here:

<http://law2.umkc.edu/faculty/projects/ftrials/scopes/ussher.html>

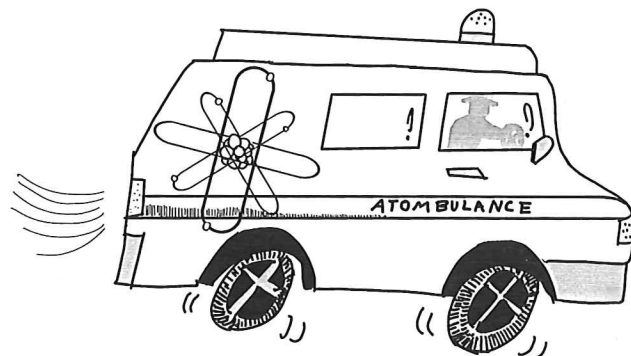
*And also in the fictional drama *Inherit The Wind*, a 1955 play and 1960 movie. A great story about the collisions of science, religion, and law.

There were also critiques of Kelvin's estimate from geologists and biologists. Geologists were starting to learn about processes of mountain-building, and continental drift. They knew that vast stretches of time – a few hundred million years or more – were needed to reproduce the features of earth and sea that are around us. There are fossils of marine life at 15,000 feet in the Alps; how did such things get there? Apparently the Alps were once at sea-level! And some of these fossils are of sea-creatures which no longer exist; so the evolution-minded biologists were also uncomfortable with a “young” Earth (30 million years or so).

None of the critiques bothered Kelvin. In his view, the “inferior sciences” could carp and complain away, but it didn't change the basic reality: the world is constructed according to the principles of *physics*.

But as the 19th century approached its end, the world is about to change.

Atoms, and more specifically nuclei, to the rescue



The dawn of the nuclear age is sometimes said to be August 6, 1945, when the first nuclear (“atomic”) bomb destroyed Hiroshima. But the real birthday should probably be November 8, 1895, when Wilhelm Roentgen made an accidental discovery in his laboratory. Roentgen was experimenting with cathode-ray tubes – a very common piece of equipment in 1895 physics labs. In this device, a very high voltage causes a stream of electrons to flow from the cathode to the anode of an evacuated tube. Across the room were some phosphorescent crystals, being readied for an unrelated experiment. Whenever Roentgen turned on the cathode-ray tube (CRT) in the darkened room, the crystals glowed. Then he wrapped the CRT in black paper, and again turned it on. Again the crystals glowed. Probably he uttered some close cousin of the words which famously accompany a new discovery – not “Eureka!” or “Aha!”, like in the comics... but something more like “that’s funny” (das ist lustig!). He replaced the crystals with a phosphorescent screen; same result. He put his hand in front of the screen, and now he could see a silhouette of his hand – or more correctly, the *bones* of his hand.

He had discovered X-rays.

No one knew what these rays were (hence the name), but they caused an immediate sensation. All physics labs had CRTs, so his results were quickly verified and extended. The unknown rays travelled in a straight line and very fast (later found to be the speed of light). They had no electric charge. They were not stopped by air, or glass, or paper, or even skin... but they were stopped by metal and bones. Their usefulness in medical diagnosis was obvious, and within a few years they were used by battlefield doctors to find bullets in wounded soldiers.



The continuing mystery of their nature – they show the *insides* of people! – invited speculations about the “spirit world”. Some newspapers reported discoveries of “Z-rays” and “N-rays”, and suggested that these too were rays from the spirit world. These reports were all bogus, but probably sold a lot of papers.

A few months later, the French physicist Henri Becquerel was studying the properties of fluorescent minerals, which emit their own light after being exposed to sunlight. He used photographic plates to study that fluorescence. There was no existing theory to understand the fluorescence. But he had heard about X-rays, and wondered if sunlight might cause the mineral (he used a salt of uranium) to emit X-rays as well as visible light. His initial results seemed to confirm this. After a few hours of exposure to sunlight, the little lump of uranium sulfate was placed on top of a photographic plate which remained tightly wrapped in black paper. After

developing the plate, he saw a blurry image of the little uranium lump, showing that something had been emitted by the uranium sulfate and penetrated the black paper. X-rays, right? That must have been exciting.

Then it became cloudy in Paris, and he could not repeat the experiment for a few days. He stored some new and unexposed photographic plates in a drawer, together with the uranium sulfate. The next day, he found that some of the plates were ruined, seemingly exposed to light, despite being tightly wrapped in dark paper.

Maybe sunlight had nothing to do with the result; maybe the uranium sulfate *itself* produced rays which penetrated black paper and exposed the photographic plates. X-rays, presumably.

His subsequent experiments showed that every compound containing uranium had this property. He concluded that uranium itself, with no assistance from sunlight, cathode-rays, or alliance with any other element in a compound, spontaneously emits X-rays. But just to be safe, he called them “uranic rays”.



By good fortune, a young Polish woman, Marie Sklodowska, had just arrived in Paris. She was superbly trained in the separation of heavy elements, and was looking for a PhD topic (women could not get PhDs in Poland). Becquerel's work was really tempting to Marie. Over the next 10 years, she, her husband Pierre Curie, and Becquerel established the new science of **radioactivity**. They found that:

- (1) many heavy elements are intrinsically “radioactive”;
- (2) the emission of these rays always came **from the atoms themselves**, regardless of what molecules they formed, or what compounds contained them;
- (3) they were not X-rays at all, but carried a charge; the most common ones (“alpha rays”) had a charge of +2 and a mass of +4 atomic mass units.

This last point was a sensation. An element is *defined* by its positive charge. If uranium, with an atomic number of 92, emits a particle of charge +2, then it's no longer uranium, but thorium. Apparently it ejects a helium nucleus: charge +2, mass +4. And it does this *spontaneously*, not with the intervention of a nearby physicist. The uranium spontaneously transmutes into a different element, and in the process ejects something new, a helium nucleus. In the notation that became standard, the uranium decay can be written as:



The accompanying electrons just go along for the ride. This is an instability of the *nucleus*.

But this distinction between an atom and its nucleus is an anachronism; no one would have understood this distinction until a young New Zealander, Ernest Rutherford, began his famous “scattering” experiments in 1909 at the Cavendish Laboratory in England. Rutherford knew that you can't actually *see* atoms. Their sizes are 10,000 times smaller than the wavelength of light, so light just sweeps over them, like ocean waves over a small piece of driftwood. You can't detect the presence of the driftwood from its effect on the waves.

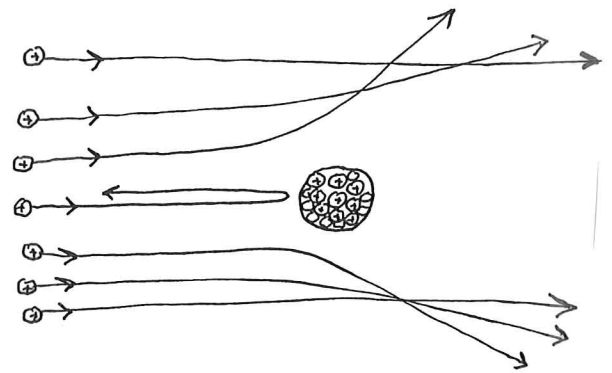
So he decided to study atoms by firing other atoms at them.

This doesn't sound like an obviously winning strategy either, but it's the best he could think of. He knew that the helium atoms emitted by radium (which is many times more radioactive than uranium) come flying off with very high speed. And he knew that there was one metal (gold) that could be hammered down to extreme thinness – so thin that if you shot helium atoms through it, most would encounter no gold atoms at all, but some small fraction would encounter precisely **one**. So most of the helium atoms would just stream straight through, but a few oddballs would be heading straight for a gold atom, scatter off of it, and he could study the scattering pattern to learn the size of the gold atom. (If you watch a blind person tapping ahead with a cane to appraise the nature of the object before him, you get the general idea.)

Then Rutherford built the apparatus and did the experiment. The helium atoms (also called alpha-rays, because by this time two other types of radioactivity – beta-rays and gamma-rays – had been discovered*) mostly just streamed through, but a few scattered through angles of 1 or 2 or 3 degrees. From this he could measure the size of the gold atom. Nice experiment.

*At the time of discovery, the nature of new types of particles is hardly ever known. But they always become the subject of new experiments, and eventually four kinds of “elementary” particles were identified. Gamma-rays are ordinary photons – just like light/radio/infrared/ultraviolet/X-rays – travelling at the speed of light. Their symbol is $0\gamma^0$, indicating zero charge and zero rest mass. Beta-rays are ordinary electrons – symbolized by $_{-1}e^0$ to indicate a charge of -1 and a rest mass of (approximately) zero – which can sometimes fly out of nuclei with great speed. Alpha-rays are ordinary helium nuclei, and are therefore not really “elementary” - since they consist of two protons and two neutrons. But they're so commonly produced in radioactivity, and so important in the early history of the subject, that the name sticks.

Figure x. Charged particles (alpha-rays) passing near a gold nucleus. Most are deflected through small angles – just a few degrees. But a few (less than one in a thousand) are reflected straight back, to Rutherford's astonishment.



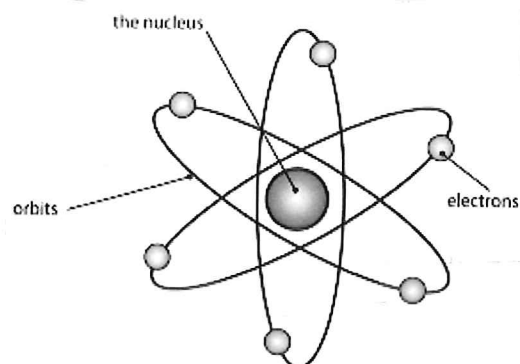
Then his assistant, Hans Geiger, asked Rutherford “don't you think that young Marsden, who we are training in methods of radioactivity research, should have an experiment of his own?” Geiger thought that it would be a useful exercise for Marsden – an undergraduate student – to modify the equipment so as to measure any alpha-particles scattered through very large angles (more than 45 degrees). Of course he didn't think there would be any, since he couldn't see how the highly energetic alpha particles would be affected much by the ultra-thin gold foil. It seemed like bullets fired into hot butter. But then Marsden did the experiment... and found that some of the alpha-particles were scattered at very large angles, even up to 180 degrees. Rutherford later wrote:

“It was quite the most incredible event that ever happened to me in my life. It was almost as incredible as if you fired a 15-inch shell into a sheet of tissue paper, and it came back and hit you”.

Even more extreme than bullets into butter.

He soon realized how both results could be understood: by supposing that both atoms (helium and gold) had all their positive charge concentrated in a very tiny region he called the “nucleus”. That way, some tiny fraction of the incident alpha-particles would, by chance, be heading straight for a nucleus. If the nucleus were really small, the alphas would get very close to the target nucleus, and the electrostatic force of repulsion would be enormous – so big that the alpha would be hurled back right where it came from! By measuring the *rate* at which this happens, he could measure the size of the nucleus relative to the size of an atom.

The result was amazing: the radius of atoms is about 10^{-10} m, and the radius of the nucleus is just 10^{-15} m. Pretty tiny. This led to the famous “Rutherford” or “solar system” model of the atom, in which all of the positive charge resides in the tiny nucleus, and the negatively charged particles (“electrons”) orbit the nucleus like planets around the Sun. Since the electrical force follows an inverse-square law, just like gravity, the analogy is pretty good. It's still the model* we teach today – called the “semi-classical” model, since it treats both nucleus and electrons as



simple particles, while in quantum theory they are fuzzier and more complex.

*Unfortunately you can't really draw it to scale. If the nucleus is 1 mm, then the electrons orbit at a radius of 100 m. Truly, the atom is *mostly empty space*.

Now that we understand what atoms really look like, up close and personal, let's return to the issue of understanding the origin of solar energy. (Remember, as astronomers, that's what we're trying to solve; before all these discoveries in atomic and nuclear physics, we were stuck with totally inadequate solutions: chemical and gravitational energy.)

In the first decade of the 20th century, experimental results came pouring in. Marie and Pierre Curie showed that *all* of the elements heavier than lead – not just the celebrity elements radium and uranium – decayed naturally. Most decayed through emission of alpha particles (which were now understood to be helium nuclei). These particles came ripping out with great velocity, and therefore kinetic energy. *That* could certainly heat a star. In terms of Einstein's famous formula for mass-energy equivalence, the radioactive decay of a mass M would produce an energy of about $0.001Mc^2$ – compared to $0.00000001Mc^2$ for gravitational energy, Kelvin's favorite theory.* These numbers were well known to Rutherford.

*And compared to Mc^2 , the energy you get out from matter-antimatter annihilation. That can't be important in the Sun's energy generation, since it would require that large quantities of the Sun be actually made of antimatter. Mc^2 is just a nice benchmark in comparing efficiencies of energy generation.

Furthermore, his experiments revealed the decay times for all these heavy elements; armed with these decay times, you can learn the age of a rock by measuring the ratio of parent nuclei to daughter nuclei. In a famous talk delivered to the Royal Society in 1904, he discussed the problem of solar energy, presenting Lord Kelvin's young-Earth theory, which assigned an age of ~30 million years. Then, pulling a small rock out of his pocket, he said “I have here in my hand a rock which I **know** is 800 million years old.” And then he explained how that age could be known by measuring the amount of parent and daughter nuclei in a rock, when the decay timescale is known. (In the example several pages back, it would be the U/Th ratio.)

That was conclusive proof that the Earth was very old, as the geologists had thought. But to believe that the *Sun* has been shining with its present luminosity for that long, powered by radioactive decays, you'd have to believe that the Sun is made mostly of radioactive elements. That seemed doubtful, since these elements are very rare on Earth. Ever optimistic, Rutherford was unfazed; he bragged that

“what is possible in the Cavendish Laboratory might not be too difficult in the Sun”.

As more data from nuclear physics labs accumulated, a new candidate theory emerged: **nuclear fusion**. Just as heavy nuclei release energy when they decay, light nuclei release energy when they fuse into heavier nuclei. Some physicists – especially Jean Perrin and Arthur Eddington – realized that hydrogen “burning” (four hydrogens burning to form one helium) was especially powerful. If a hydrogen mass M burns to form He, then $0.007Mc^2$ of energy is released. As an energy source, that's $\sim 7x$ more efficient than radioactivity. If the Sun's mass is entirely hydrogen, and if that entire mass (2×10^{30} kg) burns to form helium, then the total energy produced is

$$E = 0.007 Mc^2 = (0.007) (2 \times 10^{30} \text{ kg}) (3 \times 10^8 \text{ m/s})^2 = 1.36 \times 10^{45} \text{ J}.$$

If the Sun radiates at 1 solar luminosity throughout its life, then it will last

$$t = E/L = 1.36 \times 10^{45} \text{ J} / (4 \times 10^{26} \text{ J/s}) = 3 \times 10^{18} \text{ s},$$

which is 100 billion years. Wow. This is plenty long enough to satisfy any constraints which come from geology or biology!

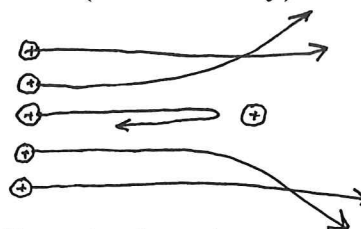
This doesn't mean that the Sun has been around for 100 billion years. It just means that the fuel supply could conceivably last for that long.

OK, it's energetically possible. But IS IT TRUE? Are stars actually powered by the fusion of hydrogen into helium?

This was very problematic. A serious physics problem appeared to make it impossible. Two free protons wandering around in a stellar interior repel each other with an electrostatic force proportional to $1/r^2$. For very small r , this force becomes enormous, and you might think that you could never get two protons to actually *touch*. Since protons are extremely tiny, the collision has to be *directly* head-on (unlikely)... PLUS they have to be moving really fast (also unlikely).

People* tried to calculate these matters from theory. The direct head-on part was not a big problem; the Sun is so big, that if even one in a trillion close encounters was directly head-on, that would be

*Or more correctly, a bunch of theoretical physicists. But they're people too.

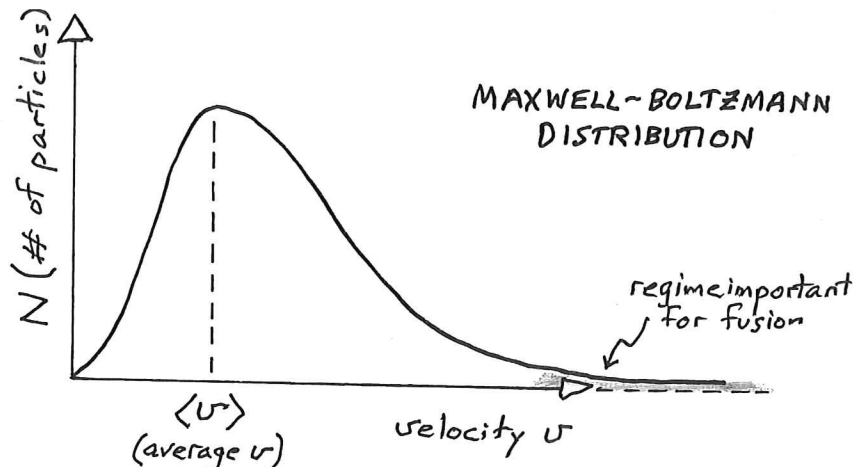


*the electrostatic
repulsion of protons*

enough. But the required speed was a huge problem. We know what the temperature at the Sun's center is: about 13 million K. That's the temperature needed to produce enough pressure to balance the force of gravity pressing in. So we can deduce the mean velocity of protons there from the basic equation of the kinetic theory of gases:

$$\left(\frac{1}{2}\right) m v^2 = \left(\frac{3}{2}\right) kT.$$

This yields $v = 5 \times 10^5$ m/s, or roughly $v = 0.002$ c. That seems pretty fast: 0.2% of the speed of light. But it's FAR from enough speed to overcome the electrostatic repulsion (the "Coulomb barrier") between protons. You would need around 30x more speed (around 0.05c). So then people calculated how many protons in the Sun's core are moving with speeds 30x the average speed. This can be readily calculated. In any gas containing a very large number of particles colliding and re-shuffling energy, the distribution of velocities will follow a so-called "Maxwell-Boltzmann distribution". It's sort of like a bell curve, but slightly different – and is shown below. At any temperature, there's a most likely speed, and some number of particles moving at 30x the average speed – way out on the extreme right side of the curve:



When you evaluate this, you find something very discouraging: the number of particles expected to be moving at 30x the average speed, over the entire Sun, is *less than one*.* But if fusion powers the Sun, then you need to burn 6 trillion tons

 *Considering the Sun's huge mass, does that seem counter-intuitive? Imagine there were such a particle. On its next encounter with a nearby particle, a picosecond later, it would surrender roughly half its energy (sometimes called "the equipartition principle"; particles share their wealth, unlike humans). In another picosecond, it surrenders about half of what's remaining. Pretty soon it's just an average particle. If you're mathematically inclined, punch in e^{-v^2} (roughly the Maxwell-Boltzmann distribution at high v) for $v = 30$, and watch your calculator protest.

 of hydrogen every second to explain the Sun's luminosity. So as far as

mathematics and classical physics goes, that's the end of the story. Ain't a single proton moving fast enough in the Sun for nuclear fusion.

Many physicists felt that this was a death blow to the fusion theory for the Sun's energy. Others felt that since the competing theories were disqualified on more basic grounds (the abysmal failure to produce enough energy), the fusion theory just had to be right. Sir Arthur Eddington, in his famous 1925 book *The Internal Constitution of the Stars*, eloquently and sardonically came to its defense:

We do not argue with those critics who maintain that the interiors of stars are not hot enough for this purpose. We simply tell them to go and find a hotter place."

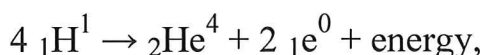
A touch of highbrow profanity from a Cambridge don.



Something else happened in 1925: the various paradoxes of classical physics vis-a-vis the new experimental results of atomic physics were explained in an entirely new theory: the theory of **quantum mechanics**. Quantum theory gave explanations for the experimental results, and also “reduced” to the familiar results of classical physics for objects larger than, say, a few million atoms. But the explanations for events in the microworld were undeniably strange. You don't get something for nothing... and the big loser in quantum theory is everyday human understanding. Intuition – long considered the cornerstone of a physicist's education – had to surrender to the mathematical formality of quantum mechanics. It became harder to be a good physicist in 1925. And even today, although the correctness of quantum mechanics is not challenged, the precise interpretation of it is still much debated.

In quantum mechanics, a single particle doesn't have a precise energy, but only a mean energy. It's a particle, but also a wave – as famously discovered by de Broglie. This is at the heart of quantum theory. Waves are spread out in space, not precisely located like the “particle” of classical physics. In particular, suppose a proton's kinetic energy is only sufficient to approach within, say, 2 proton radii of a target proton. That seems to be not enough; to trigger the nuclear force, the particles have to touch. But if both protons are somewhat spread out in space, then they have *some* probability, however low, of getting close enough (“touching”) to trigger the nuclear force which will bind them. This is called *quantum tunneling*. Particles can sometimes tunnel through energy barriers, even if the barrier exceeds the particle's kinetic energy. It has no counterpart in classical physics. (If it did, I could occasionally run through a concrete wall, or high-jump 50 feet. If you don't succeed, try, try again.)

So once in a great while, a proton will tunnel through and fuse with a target proton. This will trigger a series of nuclear reactions, which amounts to



where **energy** is the difference in mass between the input (4 protons) and the output (a helium nucleus and 2 positrons). This difference amounts to $0.007Mc^2$, where M is the original mass of the 4 protons.

In 1929, Robert Atkinson and Fritz Houtermans made this calculation rigorously. They set $T = 13$ million K, accounted for both quantum tunneling and the Maxwell-Boltzmann distribution, and got the rest of the physics right. Their result was that H fusion should proceed at the rate of about $0.3\text{ }M_{\odot}$ per 10 billion years, which is about the observed rate required to yield 1 solar luminosity (“6 trillion tons of hydrogen per second”). To put it another way, the average solar proton fuses with its neighbor about every 30 billion years*. **Stars are very, very slow fusion reactors . . . and only can manage it by virtue of their great mass.**

*But this doesn't mean that the Sun *lasts* 30 billion years. Nuclear reactions occur only in the core, and typically more than half of the star's mass remains forever in the “envelope”, where it's too cool for nuclear reactions.

They did, however, have to assume the Sun to be made mostly of hydrogen. We now know this to be correct. But at the time it was doubted by most astronomers – because hydrogen is not that common on Earth, and more specifically because the “spectral lines” of hydrogen in the Sun's spectrum are quite weak. The really strong spectral lines in the Sun are due to calcium, sodium, magnesium, and iron – respectable elements all, but of no relevance to the hydrogen-fusion theory. *

*Most astronomers prior to the 1950s did not have a good physics education, and could not have thoroughly appreciated, or even understood, the nuclear-physics arguments discussed above. The term “astrophysics” – now basically a synonym for astronomy – was rarely used.

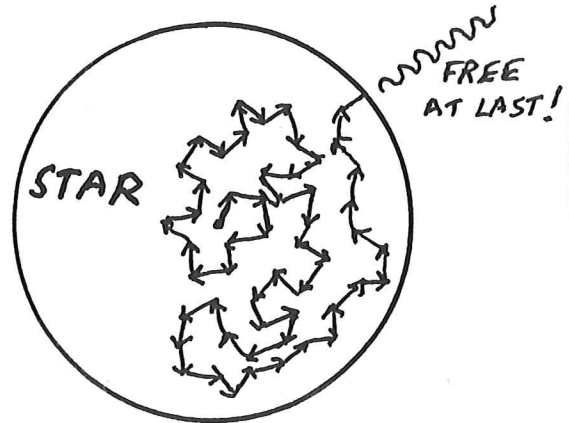
Someone needed to jump up and prove that hydrogen was not only common, but the dominant element in the Sun.

And finally, in this long story winding from Babylonia through Greece and Italy to 20th-century Germany, the scene shifts to the good ol' U.S. of A.



Arthur Eddington started the ball rolling in the early 1920s. One of his students was the young Cecilia Payne, who could not obtain a degree in England (Cambridge University did not grant degrees to women until 1948). She emigrated to the United States, began graduate study at Harvard, and in 1925 wrote what Otto Struve later called “undoubtedly the most brilliant PhD thesis ever written in astronomy”. Payne figured out how to learn the relative abundances of elements from the strength of the “lines” in astronomical spectra. The strength of the lines matters, but so does the temperature of the stars. It turns out that hydrogen lines are only strong in stars with surface temperatures near 10,000 K – much hotter than the Sun's surface (6000 K). So the weakness of hydrogen in the Sun and similar stars was mainly due to temperature, not abundance. After many more years of research, we learned that the outer parts of stars – not the cores, where actual nucleosynthesis occurs – are amazingly uniform in their composition: 72% H, 26% He, 2% everything else (often called “metals” in astronomical jargon). About 99% of stars are like this; they're called **Population I stars**, because they were studied first. The other 1% of stars are extremely deficient in metals, so their breakdown is around 73% H, 27% He, 0.1% everything else. They're called **Population II stars**.

With all the hurly-burly going on in the core, why are the surface abundances so uniform? The answer lies in the nature of heat transfer. All stars are supported by the heat generated in the core. But that heat slowly leaks to the surface by the outward diffusion of **photons** emitted by the hot gas in the core. Photons, not the actual atoms which are making the heat by nucleosynthesis in the core! The photons emitted in the core travel only a few centimeters before being absorbed, but are then re-emitted, absorbed, re-emitted again... and gradually diffuse out from core to surface, after a few zillion more absorptions and re-emissions. This is a decidedly zigzag path out to the star's surface, so it usually takes millions of years for the surface of the star to change even slightly, in response to events in the core.*



*Think of it this way. Your house is probably heated by pipes containing very hot water. By some combination of radiation/convection/conduction, they heat your house, which then radiates that heat to the outside. Over the years, a tremendous amount of heat is lost to the outside air. But the water and the pipes don't leave – just the heat.

On the astronomical side, Payne's discovery was a key breakthrough. But the formulation (discovery? invention? take your pick) of quantum mechanics in 1925

made it possible to actually *calculate* the theoretical nuclear-reaction rates, and therefore energy flows, in the star. That's what Atkinson and Houtermans did for the Sun, and got the right answer. It became possible to calculate “model stars” – first laboriously by hand, then easily when computers became widely available in the 1950s-60s. These models could be matched to existing data – the known masses, luminosities, radii, and surface temperatures of the stars.

And the fit was really good. The great observed variety in stars could be expressed in simple formulas: on the main sequence, stars followed these simple prescriptions (expressed in solar units):

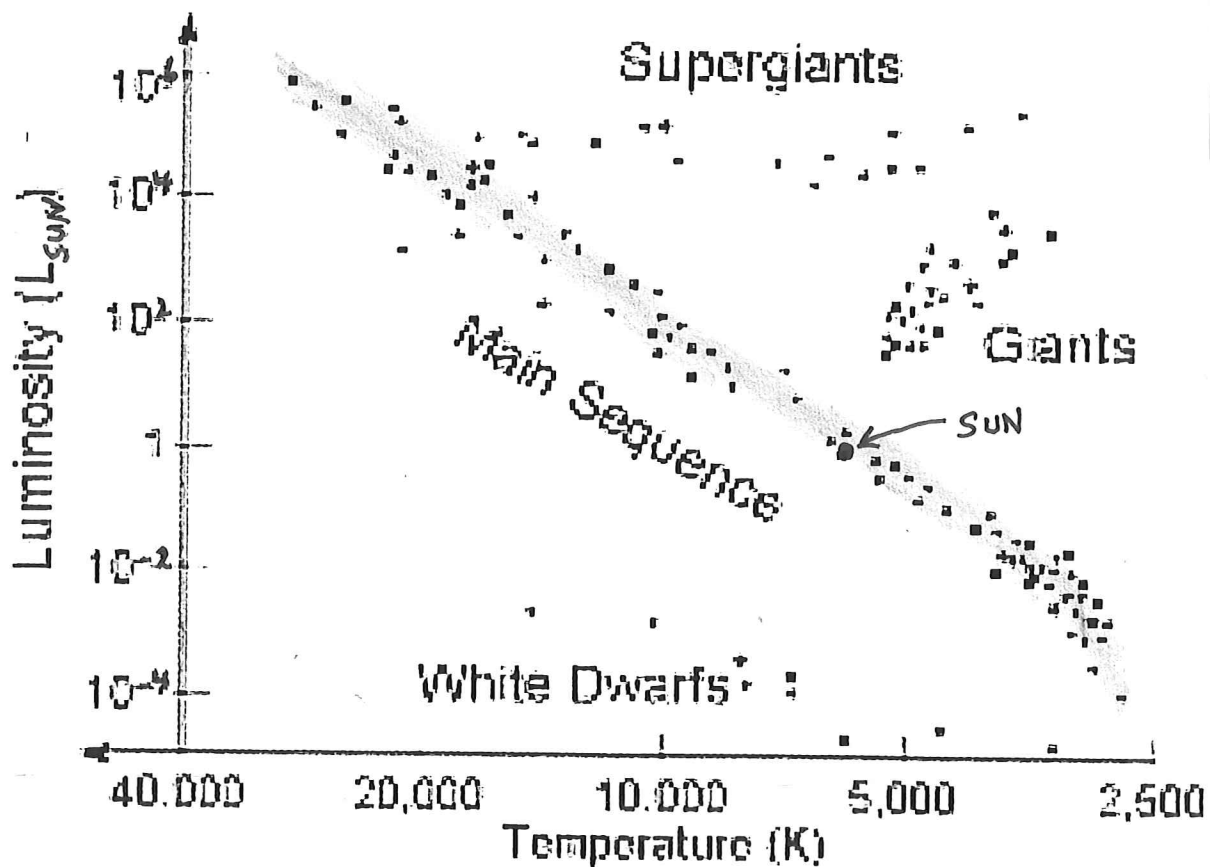
$$\text{Radius: } R/R_{\odot} = M/M_{\odot}$$

$$\text{Luminosity: } L/L_{\odot} = (M/M_{\odot})^4$$

$$\text{Temperature: } T/T_{\odot} = (M/M_{\odot})^{0.5}$$

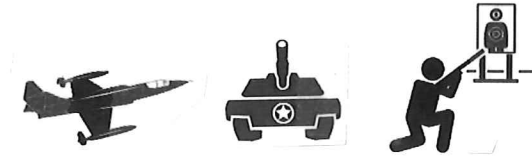
Mass determines pretty much everything. Very nice! By around 1940, the structure and properties of main-sequence stars were considered a solved problem.

Much of stellar evolution can be depicted in the Hertzsprung-Russell (or “color-magnitude”) diagram. This is usually represented as a graph of luminosity versus temperature (or color, which signifies temperature). Here's a schematic version of the H-R diagram:



Generations of astronomers (the people who actually look through telescopes, not the nuclear physicists) have labored to measure all these things. Fairly detailed H-R diagrams, similar to the one shown above, were available by ~1940. But a detailed understanding of stars *not* on the main sequence still remained elusive.

THE 1940s



Then came a decade mostly lost to astronomy. The world had plenty of other problems to attend to. You know the story.

THE 1950s - 70s

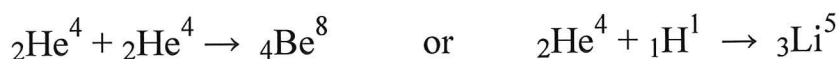


Many external factors juiced up astronomical progress in the 1950s-60s. Peace (more or less). The development of computers. Economic prosperity in the “great powers”. The Space Race between the USA and USSR, fueled primarily by fear that the other side would achieve military advantage by developing super-weapons, or by using space to launch attacks. Each country's financial investment in physics and astronomy grew manyfold. Nuclear energy generation in stars is not all that different from nuclear-energy generation in weapons or reactors, so many astronomers shared in the much bigger pie of *military* research funds. Astronomers interested in the evolution of stars made regular pilgrimages* to Los Alamos National Laboratory – mainly a government lab for weapons research, but also home to a group of nuclear physicists with special interests in stars. Armed with the latest computers and latest lab data on nuclear-reaction rates, these people became the high priests of stellar evolution.

*I made one myself, as a young postdoc. I must have done some science that week, but only remember one thing. There was a 5-minute trek down a long corridor to reach the men's room. There was another men's room a few feet from my office, but it was in the “classified” part of the floor, and had an armed guard posted outside. Whatever went on in there remained a mystery to me. But I got some exercise amid my long-forgotten calculations.

After another 10-20 years and thousands of research papers, the physical understanding of post-main-sequence evolution of stars became a mostly solved problem, too. It's a vastly harder problem, because there are some phases of very rapid evolution... and because the nuclei involved are quite heavy – much more complex than hydrogen! That's why we needed the high priests plus Uncle Sam's labs, computers, and dough.

The biggest problem was: how to get beyond helium? Can stars make anything beyond helium? Once you have manufactured He from H, the only heavier possibilities are beryllium-8 or lithium-5:



If stars could make these elements, they could just keep on going, eventually making all the abundant elements in the cosmos. But beryllium-8 and lithium-5 are violently unstable; they both fall apart in 10^{-23} seconds. To illustrate just how short this is, it's the time it takes a proton, travelling at the speed of light, to traverse its own diameter! In other words, the hypothetical Be^8 and Li^5 just fall apart *immediately*. So the origin of elements heavier than helium was mysterious, and many astronomers thought that everything heavier than helium must have been made in the Big Bang*.

*Assuming they believed in the Big Bang. In the mid-20th century, some did, some didn't.

Enter Fred Hoyle, probably the most creative astronomer of the 20th century. Hoyle was very skeptical about the Big Bang, and therefore needed a way to make heavy elements in stars. He knew a lot of nuclear physics. He knew that these lifetimes of freshly assembled nuclei depend on the kinetic energy of the particles which produce them, and calculated from theory that for a temperature of about 100 million K, the lifetime of the newly created Be^8 should be not 10^{-23} seconds, but $\sim 10^{-13}$ seconds. He then tried to calculate whether that was long enough to sneak another He^4 nucleus in there to smash into the incipient but unstable Be^8 nucleus, and thereby make a stable nucleus (C^{12}). In other words, to produce this reaction:



He found that for densities and temperatures (about a hundred million K) characteristic of the helium cores of red giants, 10^{-13} seconds *is* enough for a third helium to smash its way in there and save the day. It is now famous as the “triple-alpha process”, because helium nuclei are called alpha-particles in the nuclear biz. Because of this quirky *resonance* in the formation of the intermediate product beryllium-8, stars can produce carbon... and from then on, there are no serious problems in producing all heavier nuclei, up to iron. And a damn good thing for us carbon-based life forms; it's pretty hard – as in **impossible** – to imagine any interesting life forms arising strictly from the chemical properties of hydrogen and helium!

That solves the *nuclear* dilemma of how to get beyond helium. The result of all these fancy calculations by the high priests and their acolytes amount to the following. Start with a 1 solar-mass star like the Sun. Hydrogen-burning in the core lasts about 10 billion years. All this while, the forces balance, the pressures balance, the energy produced in the core equals the energy radiated away at the surface. The star is stable. But the helium “ash” just piles up in the core, and it's inert – unable to burn since helium burning requires a temperature of a hundred million K. Lacking an internal energy source, the core starts to contract. The gravitational energy thereby released, along with the nuclear energy still released by H-burning in the shell *around* the core, produces a lot of heating in and around the core. The extra heat now starts to swell the star; the outer surface expands, and the star is on the way to becoming a **red giant**. Now the triple-alpha burning starts, raising the luminosity and swelling the star further. Now it's a full-fledged red giant. Mercury is vaporized, and probably also Venus.

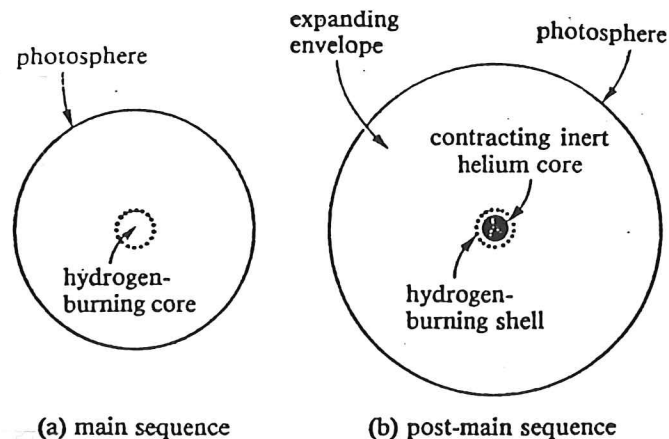


Figure The structure of a star (a) on the main sequence and (b) as it begins to leave the main sequence because of core-hydrogen exhaustion.

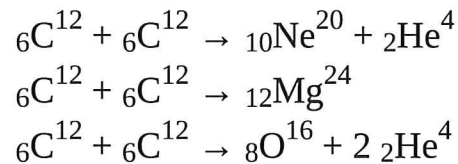
The red giant's outer parts are very weakly bound by the star's gravity. They just sort of float away, forming a little wispy shell of gas around the star. This is called a **planetary nebula**, because it looks round and (very) vaguely like a planet. The inner parts of the star, which we used to call the star's *core*, are all that's left. While the red giant was expanding and sloughing off its envelope, the core has actually been contracting under its own gravity... and by the time the planetary nebula has formed, the core has shrunk to about the size of the Earth. But it's very hot – initially about 200,000 K, and gradually cooling to ~10,000 K. We call such objects **white dwarfs**, and identify them as the bluish faint stars at the lower left of the H-R diagram. Below is a modern photo of the most famous white dwarf in the sky (“Sirius B”, the companion to Sirius A, the brightest star in the sky; a lot of fame wrapped up in that binary-star system!). Obviously Sirius B is much fainter than Sirius A, even though the two stars are of similar temperature and at the identical distance. So Sirius B must be vastly smaller – about 200 times smaller in radius.

https://upload.wikimedia.org/wikipedia/commons/1/18/Sirius_A_and_B_Hubble_photo.editted.PNG

Roughly one solar-mass in a sphere of about one Earth-radius. That's a lot of density! A penny of white-dwarf material would weigh as much as the Great Pyramid of Khufu. And forget about getting out of bed in the morning: you would weigh 100 billion tons.

White dwarfs are plenty interesting (I make a living from them)... but nothing dramatic lies ahead for a solitary white dwarf. They just radiate their energy away, gradually cooling, until they eventually crystallize. The whole star is then one gigantic crystal – usually carbon (and hence a gigantic diamond).

For more massive stars, late evolution will be more exciting. Higher temperatures are reached, and the carbon will burn to yield heavier nuclei, via reactions like these:



If there is any envelope (the low-density outer part of the star) remaining, these burning episodes may send the star ascending up the red giant branch a second time, producing a still more luminous star (a “red supergiant”). This could cause a second or third planetary-nebula phase, as new fuels are ignited in the core.

Notice that all the products of the nuclear reactions described so far are multiples of helium (${}_2\text{He}^4$). Why? Because the hydrogen in the core is long gone; it was entirely burned during the star's main-sequence life. There was plenty of unburnt hydrogen in the envelope, but that envelope is expelled during the planetary-nebula phase. So as further nucleosynthesis occurs, the multiples of ${}_2\text{He}^4$ are generally favored (${}_8\text{O}^{16}$, ${}_{10}\text{Ne}^{20}$, ${}_{12}\text{Mg}^{24}$, ${}_{14}\text{Si}^{28}$, etc.). Toss in a few protons and neutrons here and there, and you can make all the elements up to iron in this manner.

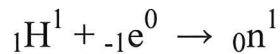
Computer models show that low-mass stars like the Sun evolve to carbon, eject a planetary nebula, and leave a white dwarf consisting of nearly pure carbon. Stars of slightly higher mass have higher-mass cores, which evolve beyond carbon to oxygen, neon, and magnesium (or some combination). The limit comes for stars which start with about $6 M_{\odot}$. Such a star ascends the red-giant branch several times, puffs off a few planetary-nebula shells, and winds up with a tiny core consisting mostly of iron. This too is a white dwarf, but with a mass near $1.4 M_{\odot}$. Such a star is courting disaster, as we'll see shortly.

What exactly is a white dwarf? A few were discovered by astronomers in the 19th century, but their physics was basically mysterious until 1926, when Ralph Fowler applied the newly developed quantum theory to an old star which has exhausted its nuclear fuel sources. The core continues to lose energy by radiating its heat outward, and so must begin to contract. When its density gets very high, it actually becomes governed by the laws of quantum mechanics, not classical physics. Why? Chemistry and physics students are familiar with the principle that atoms only have room for a certain number of electrons: two in the inner shell, eight in the next, etc. The inner shell has the least energy, the second shell slightly more, the third shell slightly more, etc. This numerical restriction is called the “Pauli exclusion principle”, and in atoms acts to determine the *sizes* of atoms. Now the core of a very old star has a super-high density, and you just can't cram more low-energy electrons into it – the low-energy states are all filled. Pauli rises from the grave to exclude any more low-energy electrons. So as the core grows in mass, the new electrons can only go in *high*-energy states... and thus constitute a gas of particles with very high energies, resisting further compression. This balance of forces (gravity versus the outward pressure of the gas of electrons) is exactly analogous to the balance in normal stars, where ordinary gas pressure pushes outward against gravity.

Astronomers commonly use the term *degenerate dwarf*, because the star has radiated away nearly all its “optional” energy (heat) and degenerated to its bare minimum, where the outward pressure is based strictly on **density**. Essentially nothing more will happen to these stars; they'll just radiate their feeble remaining thermal energy away, at an ever decreasing rate. They're just the fading remnant of a former star – the most common type of stellar corpse.

But degenerate dwarfs are not stable above 1.4 M_{\odot} . Stars with a birth mass greater than $\sim 6 M_{\odot}$ will, after sloughing off their outer layers, grow a 1.4 M_{\odot} degenerate core, and *keep going*. Big mistake. These cores are too massive to support themselves against their own gravity. If you're in the business of shipping iron ore, you know how much iron ore you can load before the ship will sink. Same here. Buoyancy in water has a limit, and so does the “buoyancy” of degenerate electrons, courtesy of quantum theory (the Pauli exclusion principle). Now the core begins to collapse. This only makes the imbalance worse, because gravity is now stronger. The collapse continues, and is doomed to accelerate unless some new and fantastically strong force stops it.

Such a force exists. According to computer simulations, this accelerating collapse takes only a few seconds. When the core reaches ~ 10 km in radius, it hits a brick wall. Why? Well, at this radius the protons are so close to each other as to be virtually touching... and theory shows that under these circumstances, the protons and electrons merge to form neutrons:

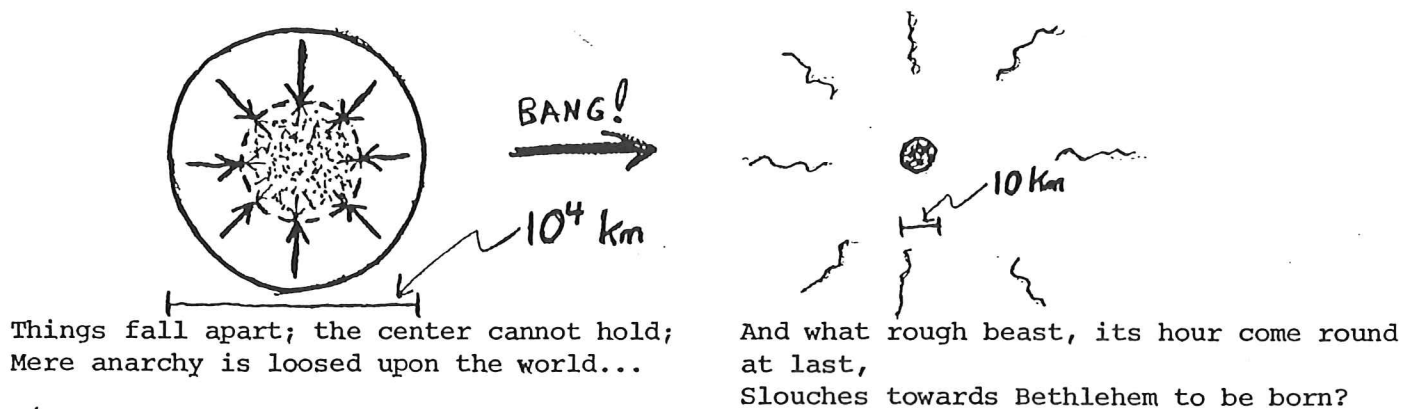


Think about this (wake up). All 10^{57} protons merge with 10^{57} electrons to make a star (just the core!) which consists purely of neutrons. A *neutron star*!

When it does this, the collapse halts suddenly, because at a radius of 10 km, the neutrons reach closest packing – they're just jammed up right next to each other, like jelly beans in a jar.. The gravity is immense, because the density ρ of a $1.4 M_{\odot}$ star with a radius of only 10 km is

$$\rho = M / [(4/3)\pi R^3] = (1.4 \times 2 \times 10^{30} \text{ kg}) / [(4/3)\pi (10^4 \text{ m})^3] \approx 10^{18} \text{ kg/m}^3.$$

This is equal to the density of the atomic nucleus, since that's essentially what a neutron star is: a gigantic nucleus with a charge of zero and an atomic mass of 10^{57} .



As Yeats might well have noted but apparently forgot to, a huge amount of gravitational energy is released in this collapse. From the formula $E_{\text{grav}} = GM^2/R$, you can easily calculate how much: roughly 10^{47} J. This is about a hundred times more energy than the star radiated during its entire previous lifetime. And this energy is released in just a few seconds!

But... **HOW DO WE KNOW ALL THIS?**

Great question. Remember the phrase a while back: “according to computer simulations”. It's time to inject some humility – always a good idea – into this discussion.

Some of the theory was worked out in 1934 by the Caltech astronomers Fritz Zwicky and Walter Baade (two gentlemen* whose fingerprints are all over the

*Well, maybe Baade was. By all accounts of his colleagues, Zwicky was the most disagreeable and cantankerous guy they had ever known.

history of 20th century astronomy). But theoretical predictions have little impact until the moment when observation and experiment have weighed in.

That moment would wait a long time. In 1967 Jocelyn Bell, an English graduate student, was studying the “twinkling” pattern of distant radio sources. These were quasars, and the origin of their twinkling was pretty well known: when radio waves pass through distant interstellar gas clouds, these clouds produce irregular changes in brightness – exactly as the Earth's atmosphere makes stars twinkle. She was trying to learn what could be learned from the pattern of the twinkling. Then she noticed that one of them varied in a strictly periodic manner. It flashed on and off every 1.3373 seconds – exactly!

That was ridiculous. Imagine if you saw a star wink on and off, precisely every 1.3373 seconds. You might think it was a flashing light on a balloon. But unlike a balloon, it was moving across the sky, rising and setting every 24 hours. Then she found three more of these objects – all flashing with a strict period in the range 1-2 seconds. In each case the period appeared to be ultra-precise – at least as precise as the clocks Bell had available to measure the timing.

Bell took these strange findings to discuss with her faculty advisor, Tony Hewish. Every graduate student knows how these meetings go. It can be pretty annoying, and it's easy to imagine. Did you calibrate the antennas on a known source? Did you avoid the radio frequencies used by military and civilian transmissions? Was there a solar flare going on at the time? And probably a dozen more worries.

Bell endured and answered the questions, and they carried out two additional tests. They measured the precise time of rising and setting, and found that all the sources reappeared in the sky with a period of 23 hours 56 minutes – the “sidereal day”. Not 24 hours, the basic period of earthly activities... but 23 hours 56 minutes, the basic period with which *cosmic* sources reappear in the sky. They also measured precise positions in the sky, and compared them to the best-quality photographs of the night sky. These photographs showed many millions of stars, but there was no match for the precise position. These fantastically precise emitters of radio waves (“pulsating stars”, hence **pulsars**) come from blank spots on the sky.

Blank spots on the sky. That eliminated just about every known type of

astronomical object, except one: a distant white dwarf which was pulsating or rotating very fast. A white dwarf is intrinsically quite faint, because it is so small (roughly 1 Earth radius). So maybe it could be faint enough – if very distant – to be invisible in the night-sky photographs. And theorists calculated that a very massive white dwarf (say $1.3 M_{\odot}$, near the cutoff of $1.4 M_{\odot}$) could possibly rotate or vibrate that fast. No one had ever found a white dwarf that massive, or rotating anywhere near that fast... and no one had ever found any radio emission at all from a known white dwarf. So the white dwarf theory was far from promising.

There was another possibility. Maybe the pulses were of artificial origin – interstellar radio transmissions from other civilizations. Maybe the extraterrestrials were broadcasting a series of regular pulses, to attract attention... and would then follow with instructions for world peace, or demands for planetary surrender. The British astronomers facetiously called the four objects LGM-1, 2, 3, and 4 – for “Little Green Men”*.

*Fortunately for us, the alien theory turned out to be false. Otherwise, how would we have known which of the four groups of aliens we should surrender to?

A few weeks later, Bell knocked on Hewish's door, and to her surprise found a high-level meeting of British astronomers under way – discussing the interpretation and significance of her discovery. (She wasn't exactly thrilled at being excluded from the meeting.) They went through all the facts and speculations discussed above, and published the results in February 1968. By this time the alien hypothesis had leaked out to the media. The media went slightly crazy over it, and ufologists went more than slightly crazy.

Within a year, many more pulsars were discovered, including two that were smack dab in the centers of supernova remnants – the shells of gas blown out by ancient supernovae. People remembered a prediction of Zwicky and Baade – that supernovae were powered by the release of gravitational energy released in the sudden collapse of a massive star, and would lead to the creation of a neutron star. Interesting! Furthermore, the fastest of all pulsars ($P = 33$ milliseconds) was found right in the center of the youngest and most famous supernova remnant – the Crab Nebula.

A convincing theory came out of all this. Sudden collapse of an old star's core, down to neutron-star dimensions (about 10 km), would release enough energy to expel the outer parts of the star (the “envelope”) at great velocity, producing the visible shell that is the supernova remnant. The freshly-created neutron star is spinning really fast, because it must conserve angular momentum during the collapse. In principle, the spin period could be as short as a few milliseconds. But as time goes on, a sort-of propeller mechanism drives gas

outward and contributes to the powering of the nebula.

Here's a good historical account of Jocelyn Bell's discovery*:

<https://www.aps.org/publications/apsnews/200602/history.cfm>

*Which, by the way, produced a Nobel Prize – not for Bell but for Tony Hewish, her adviser. This generated a who-deserves-the-credit controversy which endures to this day. The discovery of insulin is an even more egregious case. (See *Glory Enough For All*, perhaps the greatest movie ever made about scientific research.)

Be Scientific with OL' DOC DABBLE.

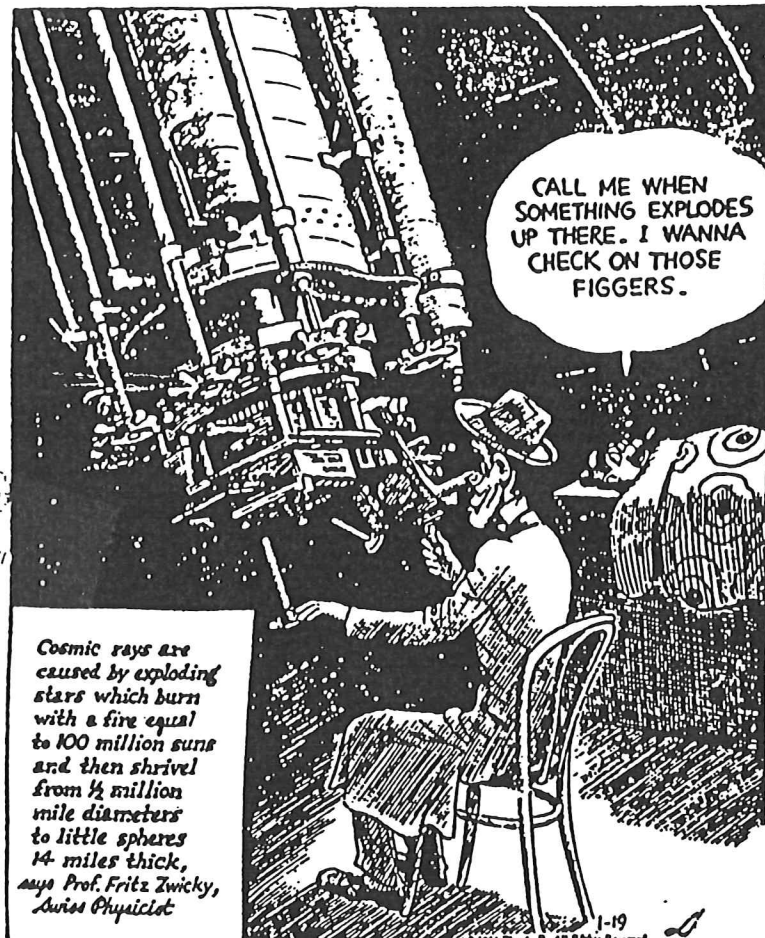


FIGURE : This cartoon appeared in the *Los Angeles Times* for January 19, 1934. In the box at the left is one of the most concise predictions of supernova properties ever published. The prediction of neutron stars in supernova explosions ("little spheres 14 miles thick") is especially interesting. Ol' Doc Dabble presents a model of empiricism, as he anticipates the mode of operation of modern observational astronomers, both by his readiness to observe, and his willingness to let someone else search for the supernovae!

Something else happens in the core collapse.

As we've seen throughout this chapter, nuclear evolution of their cores is the central engine of stars. All stars burn hydrogen; stars near $1 M_{\odot}$ will later burn helium to form carbon, and more massive stars will go through the burning of carbon, oxygen, neon, and magnesium. Each of these processes is **exothermic** – it heats the core, which serves to both power the star's luminosity and support the star against gravity. But the end of the line comes with iron. Why? Because nuclear reactions beyond iron absorb energy (“endothermic”) and thereby act to *cool* the core. This is disastrous. The creation of each nucleus beyond iron lowers the pressure, tipping the balance in favor of gravity. The core contracts, the higher density increases the reaction rate, making more heavy nuclei, which cools the core further... and so on. The process is violently unstable, the core collapses all the way down to neutron-star density, and a neutron star is formed in a few seconds. In the process, most heavy nuclei (heavier than iron) are formed. This is called *r-process nucleosynthesis* (r for rapid; it's certainly rapid!). Some of these nuclei are quickly broken down again to neutrons, as the neutron star is formed; others are violently expelled by all the energy released in the collapse. These are flung out into the interstellar medium, along with all the lighter elements outside the core. This is how stellar evolution seeds the interstellar medium with heavy elements (beyond helium), there to form the raw material to make new stars.

This subject was all the rage in theoretical astrophysics during the 1970s and 80s. Accurate measurements of the relative abundances of each element were being churned out by laboratories, using the Earth's rocks, the Moon's rocks, meteorites... and by astronomers, using the spectra of stars (including the Sun). Great material for theorists to fuss over and try to explain. The research journals themselves started to explode.

Then there's ...

THE EXPLOSION ITSELF

In that famous series of papers in the 1930s, Zwicky and Baade got most everything right – many years before their ideas became well accepted. Until then, it was not even understood that “super-novae” existed. The problem was that supernovae are extremely rare – the last one observed in our galaxy was Kepler's Star, in 1604. But by the middle of the 20th century, a few dozen very luminous stellar explosions were observed, and we knew their approximate distances since most were in external galaxies (distances to nearby galaxies became relatively easy to measure). They were sometimes as bright as the entire rest of the galaxy. This finally gave a sensible way to interpret what we now know as the most famous supernova in history: the supernova that made the Crab Nebula. In July of the year

1054, a star suddenly erupted from invisibility to become the brightest star in the sky, remaining visible in full daylight for 23 days, and in the night sky for another 2 years. It was observed assiduously by Chinese and Korean astronomers, although strangely, this amazing sight seems to have left no written record in other parts of the world (Europe? The Americas?). The Chinese charts and the modern nebula are in the same position on the sky, and modern measurements of the nebula's expansion show that it began expanding about 900 years ago. Q.E.D. Then, to top it all, a rapidly rotating pulsar was found in the middle of the Crab Nebula. And as people started to explore new frontiers (X-rays, gamma rays, synchrotron radiation, etc.), the Crab Nebula and its pulsar never disappointed. The Crab is a gift to astronomy that keeps on giving.

Nowadays, there are many surveys, some robotic, which specifically search for supernovae. With nightly photographs of thousands of galaxies, they're easy to discover, and the brightness can be readily measured. We know now that the *theoretical* figure estimated earlier for the gravitational energy released in the formation of a neutron star ($E = GM^2/R = 10^{47}$ J) is roughly correct for these incredibly luminous events. They're rare, of course. A star can only do it once, and only massive stars need apply. For galaxies like the Milky Way, it's estimated that roughly one is produced per 100 years. Naturally, we're getting pretty irritated that we haven't seen one since 1604!

AND THE DEBRIS: LIFE FROM DEATH

Finally, let's consider the effects of these supernova explosions on the interstellar medium. There's an old problem in astrophysics: how to understand the observed abundances of nuclei in space? How much H, He, C, N, O, Ne, Fe, etc.? Most people used to think that they're just left over from some previous phase of the universe that we don't know anything about. In the 1950s, Fred Hoyle pointed out that because supernovae eject huge quantities of stellar material, which must have undergone nuclear processing in order to meet the star's energy needs, you might expect that supernovae would greatly enrich the interstellar medium with heavy elements ("metals," as they're called).

Of course, other stars expel gas into space, too. A "solar wind" streams out from the Sun, causing the beautiful aurora borealis (and australis). Similar and more powerful winds stream out of many stars. In a planetary-nebula event, a star ejects several tenths of a solar mass – and these events are much more common than supernovae. But stellar-wind and planetary-nebula processes merely eject the outer parts of stars, which consist of matter which has not been enriched by the nuclear processes in the core. That doesn't change the composition of the interstellar medium – any more than evaporation from the oceans changes the composition of the Earth's atmosphere. H_2O to H_2O . There is a quite general principle of stellar evolution: except possibly in an explosion, **material in the core**

and in the envelope never mix. But a supernova is the mother of all explosions, and can eject most or even all* of the star.

*Although some stellar explosions certainly form neutron stars – as proven by the Crab pulsar, for example – we still do not know if all of them do. There's enough energy in the implosion to disrupt the star completely, leaving a supernova remnant in the sky but no pulsar. Detailed study of the remnants may eventually clarify this question, but the answer is still unknown.

That's the early-21st-century understanding of these matters. When we look out at the thousands of nearby stars, we see only their surfaces, which have not been polluted by any nuclear processing in their cores. Hence we see them with just the same distribution of elements they were born with: pretty close to 72% H, 26% He, 2% everything else. Main-sequence stars, giants, even protostars... they all show pretty similar abundances. Their vastly different status and histories (newborn/midlife/elderly?) are written mainly in the stellar cores.

Thanks to the explosions, the Galaxy slowly ramps up its percentage of heavy elements. **The debris shed by ancient stars forms the raw material which will form new stars.** That's the principle.

Note that we owe *our* existence, with our well-known reliance on carbon chemistry, to a couple of accidents of nuclear physics, which permit formation of carbon in stellar interiors, and later violently expel it into space, along with many heavier elements. Walt Whitman summed it up, sixty years before the physicists worked out the fine details:

I believe a leaf of grass is no less than the journey-work of the stars.

The Good Grey Poet

