RELATIVITY: A CRISIS RESOLVED, AND ANOTHER CREATED

In the last two decades of the 19th century, physicists tended to congratulate themselves on the predictive power of "classical physics" - the great edifice which Newton had begun two centuries earlier. And indeed, there was plenty to celebrate:

(1) the laws of mechanics were beautifully unified by the conservation of energy;
(2) the ancient question of “what is matter” appeared to be finally solved (atoms and molecules);
(3) the laws of gas dynamics (the ideal gas lawwere established and found to be just “Newton’s Laws in the microworld”;
(4) the machines of the industrial revolution exemplified all the ways in which one form of energy converts to another;
(5) Newton’s Laws were found to operate precisely far out in the solar system (Neptune) and even in binary star systems thousands of light-years distant;
(6) the temperatures, masses, and distances of stars were finally measured; and
(7) the laws of electricity and magnetism were established – with the experiments of Michael Faraday and the equations of James Clerk Maxwell.

On April 27, 1900, Lord Kelvin – the dean of British science at the time – gave a famous speech in which he bragged of these accomplishments but also cited two major clouds on the physics horizon which were still unexplained. Considering what these clouds turned out to be, it has become famous as the “Two Clouds” speech. Kelvin exuded confidence that further work would resolve these apparent difficulties. What a prophecy that turned out to be! It turned out to be resoundingly true, but not by fine-tuning a few details in theory or experiment. The first cloud was the unexplained shape of the blackbody curves (“ultraviolet catastrophe”), which led to Planck's quantum theory. And the second was the null result of the Michelson-Morley experiment... which led to relativity theory.

We discussed the blackbody curves and the quantum theory of radiation in Chapter 8. Now let’s consider the Michelson-Morley experiment, perhaps the most unexpected result in the history of experimental physics.

Albert Michelson was a young physics professor at the Case Institute of Technology, with a great talent for measuring things to very high precision. He was no revolutionary; he once said that all the fundamentals of physics were known, and the future lay in working out the details "to the sixth decimal place". He chose an odd subject for his experiment: the motion of the Earth. It had been known for more than a century that the Earth moves at 30 km/s – in one direction in January, and in the opposite direction in July. Light travels at 300,000 km/s, relative to the medium which carries it. That medium was called the *luminiferous ether*, or simply the ether.

The ether was much discussed in the 1880s and 1890s. Just about everyone believed in its existence. It had some very strange properties: it had to be incredibly tenuous – since planets orbit in it without any friction, and since you can easily see stars thousands of light-years distant. And it had to be thousands of times more rigid than steel, since the speed of light was so great. Despite such oddities, most physicists believed in it, because all waves require a medium to travel in. Lord Kelvin said, "it is as real as the air we breathe".

Michelson devised an experiment which raced light beams in two directions, travelling the same distance, and then recombined them. Because light is a wave, you can use the pattern of constructive and destructive interference to measure the distance travelled, and the speed. He did the experiment by racing the light beams in the direction of the Earth's orbital motion, in the opposite direction, and perpendicular. He expected to find values of 300,030 km/s in January, and 299,970 km/s in July – the detection of an "ether wind" created by the Earth's motion through the ether.

But when he measured it, he found 300,000 km/s, no matter what – in all directions, and no matter what time of year. All attempts, by Michelson, his colleague Edward Morley, and everyone else, gave the same result. As if the Earth were actually not moving at all!

Had this experiment been done in 1687, not 1887, the notion of an immobile Earth might have been considered. But the proofs of a moving Earth had accumulated far beyond the arguments of Galileo, Kepler, and Newton (phases of Venus, laws of planetary motion, laws of dynamics and gravitation). There was stellar aberration, stellar parallax, the Doppler effect, the shape of the Earth – and, for those unmoved by highbrow arguments, the Foucault pendulum, displayed in hundreds of science museums around the world.

So the null result of the Michelson-Morley experiment was truly a major crisis.

A few years into the new century, it called for new ideas on the nature of space and time. Very few scientists, or people for that matter, had written about space and time. What was there to say? Everyone knows what they are! Space is that giant container in which matter moves around. The container is really huge, but has no actual properties – beyond the obvious one of having three dimensions. Time is just a relentless tick-tick-tick process which plays out "in the background" of everything that exists in the world. It can't be slowed, or sped up, or changed in any way. End of story.

That was the framework adopted by the great Newton, and all physicists adopted it, probably without a moment's thought. But Newton himself was quite concerned about such things. His laws of dynamics required a precise definition of position, velocity, and acceleration… and therefore explicit assumptions about the meaning of space and time. The first edition of the *Principia Mathematica*, in 1687, did not address such matters. But it must have bothered him; after all, the full title of his work was *Mathematical Principles of Natural Philosophy* … so it was very desirable to define his terms, as he had carefully done with force, mass, and gravity. In later editions of the *Principia*, he added a "General Scholium" to address these matters. He wrote:

> "Absolute, true, and mathematical time, of itself and from its own nature, flows equably without regard to anything external, and by another name is called duration…. Absolute space, of its own nature, without regard to anything external, remains always similar and immoveable."

That proved to be good enough for ordinary life. It's good enough for a baseball outfielder, who knows that he can only throw with a certain speed… but by catching a fly ball on the run, advancing toward home plate, he will add his running speed to the throw, in hopes of nailing the runner at the plate. And it should have been good enough for the Michelson-Morley experiment to reveal the ether wind – the speed of the Earth's motion through the ether.

Around the year 1900, the null result of the Michelson-Morley experiment was well-known and mystifying. There were some wacky explanations floating around. As early as 1892, George Fitzgerald and Hendrik Lorentz suggested that the lengths of the Michelson apparatus were shrinking in the direction of the Earth's motion. Pretty desperate! They even worked out a formula for how much the lengths must shrink, in order to obtain a null result:

$$l = l_0 \sqrt{1-\left(\frac{v}{c}\right)^2} \qquad \text{(Equation 8.1)}$$

where l is the measured length, l_0 is the "true" length (measured if there were no motion), v is the speed (30 km/s) and *c* is the speed of light. This is the famous Lorentz-Fitzgerald contraction. But it attracted little attention prior to Einstein, because no one could understand the reason for the shrinkage. It was just a mostly-unexplained correction factor.*

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*But it has inspired a nice limerick:

There once was a fencer named Fisk,
Whose speed was exceedingly brisk.
So fast was his action,
The Fitzgerald contraction
Reduced his rapier to a disk.

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Enter Albert Einstein, a 26-year-old patent clerk – freshly equipped with a PhD in physics, but having not had much luck with his job search. Einstein recognized, as did Lorentz, that every measurement of motion involves space and time. In his opinion, Newton's notion of space and time might not be altogether useful in predicting the outcome of experiments, because it specifically referred to an "absolute" framework, independent of "sensible objects". That might, or might not, be good theology... but it could be considered suspect in the world of actual experiments and sensible objects.

In 1905 he published a remarkable short paper. It carried an obscure title ("On the Electrodynamics of Moving Bodies") and somewhat violated scientific norms by containing no citations. It didn't *prove* anything, but showed that some very profound conclusions follow if you make two postulates:

1. The speed of light is always the same, regardless of the motion of the observer or the source.
2. The laws of physics are the same in all inertial reference frames.

Let's unpack these seemingly innocuous postulates.

The first can be viewed as an empirical postulate dredged up in order to tolerate the null result of the Michelson-Morley experiment. And the second is practically the same as the law of inertia clearly expressed by Galileo: namely, that all the laws of mechanics are exactly the same for any two observers in a state of constant, unaccelerated motion. For example, imagine you're riding in an airplane at 1000 km/hour, and the ride is super-smooth. The flight attendant comes down the aisle and attempts to pour coffee into your cup. The plane is moving at 300 meters per second, so wouldn't it require a feat of great skill to actually deposit the coffee in your cup, rather than all over your lap? Yet these flight attendants get it right every time – not because they're well trained (although they are), but because of Galileo's law of inertia. They, you, the plane, the coffee pot, and your cup are all moving with the same speed, and nobody's accelerating. You might as well be standing still.

Let's change the example. You're a big shot with an executive jet, all to yourself, and you happen to be carrying a fully equipped physics lab. The windows are blacked out, and the ride is again super-smooth. Now you start doing physics experiments, measure the results, and compare them with the results you got back home in your physics lab. They're exactly the same. As long as your reference frame (the plane) is unaccelerated relative to your terrestrial lab, no mechanical experiment will tell you whether you are at rest or moving. The laws of mechanics are absolute, independent of the experimenter's motion. Or, to put it in a negative way, you can't tell your state of motion by any mechanical experiment you might dream up. You'll run the 100-yard dash in the same time (your executive jet is a really big plane), high-jump the same height, etc. Just as if you were still on the tarmac.

That's Galileo's* principle of relativity. But why is it called *relativity*? It asserts that the laws of mechanics are absolute, so “relativity” seems like a strange name. But here's the key: it asserts that *motion* is relative; the laws of physics and the outcome of experiments don't depend on your motion, but only on your acceleration. In physics we say that unaccelerated reference frames are inertial frames, and all inertial frames are equivalent (and indistinguishable).

Now you might think this is silly. Where are these inertial reference frames? Doesn't everything accelerate somewhat? The Earth rotates on its axis and orbits the Sun. The Sun rotates on its axis and orbits the center of the Milky Way. Where is even one inertial reference frame?

Probably nowhere. These experiments we discuss – with pouring coffee and hundred-yard dashes inside airplanes – are thought experiments (*gedankenexperiments*, in Einstein's phrase; German physicists seem to like long names). In learning physics, you definitely have to do real experiments; but to express the basic underlying principles, it frequently helps to envision how things should go in an idealized environment. This is because we believe (oh, do we ever!) that a really deep understanding of the physical world should be extremely simple. Even though our actual path to that understanding will be, if

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*Sometimes credited to Newton, since he formally and mathematically presented it, and explicitly connected it to forces.. But all the essentials are in Galileo's *Dialogue Concerning Two New Sciences,* so he usually gets the credit.

past experience is any guide, immensely tortuous.

Back to the history. Galilean relativity says that the laws of mechanics are the same in all inertial frames. But everyone thought that wouldn't apply to the laws of electromagnetism. Why? Because James Clark Maxwell showed that electromagnetic disturbances travel with a fixed speed… and that speed was the speed of light (*c*, the famous number), since light is an electromagnetic wave. So if you're moving towards the light source,, you'll measure the light to be moving at a slightly faster speed.

But Michelson, Morley, and many others did the experiment and found no variation in speed as the Earth wheeled around the Sun, or rotated on its axis. The measured speed was always c = 300,000 km/second.

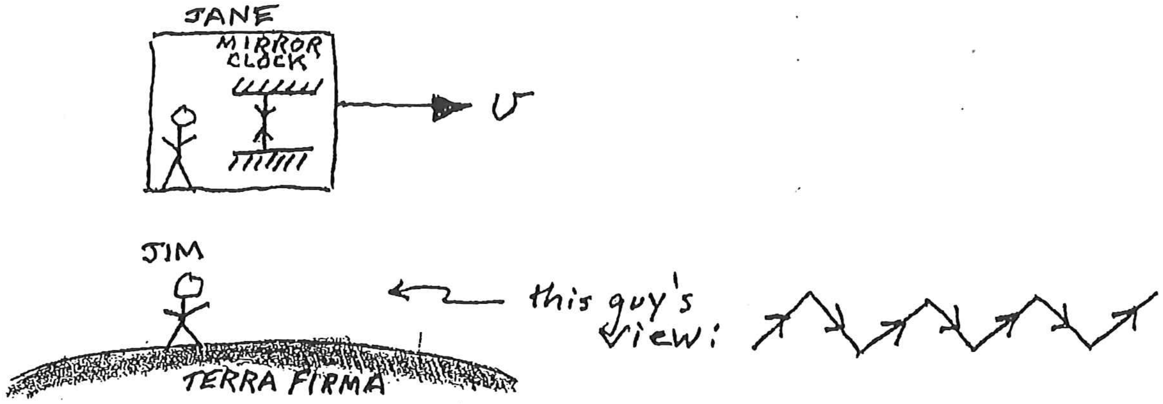
Einstein "explains" this by assertion with his first postulate. And it's automatically compatible with the second postulate, because it means that no physics experiment of any type – mechanical, electromagnetic, or anything else – can reveal your state of motion. There is no absolute motion. You can only talk about the motion you measure, and of course that means "relative to you".

Still, none of the foregoing even begins to explain or illustrate the strange, strange world that these postulates force upon us. Here's a strange one, which Einstein once said was the puzzle which started him thinking about these matters. Imagine you're looking at your bathroom mirror in the morning. There you are, perfectly framed right in the middle. Now the whole room, including you and the mirror, moves to the right with 90% of the speed of light. Would you still see yourself in the mirror? Maybe not, because by the time that light from your face reached the mirror, the mirror has moved out of the way. Only the light emitted far to the right could reach the mirror… then it reflects, and you'd see your image far to the left. Maybe you wouldn't see it at all, or it would be just hanging in space, detached from the mirror. Maybe you would just see the *side* of your face….

Pretty weird. An ordinary physicist might say, "well, that just proves that we're not moving very fast through space." But Einstein says "No, that violates the principle of relativity, the second postulate. Because you could then determine your absolute state of motion by measuring how far your image wanders from the mirror. That's not allowed. The image must stay in the center." To make sense of this, Einstein decided to explore a hypothesis crazier than anything previously mentioned; he thought there was something wrong with our concepts of either space, time, or simultaneity.

As it turns out, all three.

One more experiment. Jim and Jane possess identical clocks which measure time by bouncing a light beam back and forth between mirrors, as shown below. Each round trip of the light beam takes one second. A perfectly reasonable clock. Jim stays Earthbound, and Jane goes for a ride in space. She zooms past the stationary Jim with a speed of 0.9 c. As she flies past, Jim looks at Jane's clock and sees that it is running slow, since the light in Jane's clock has to travel a greater distance (and the speed of light is constant). He informs her of this when they meet again.



But Jane doesn't see anything amiss with her clock. During her ride, she checked the light-reflection clock with all other on-board clocks (pendulum, spring, atomic, even her pulse). Everything normal. But she saw Jim zooming by to the *left* at 0.9 c, and clearly saw that the ticks of his clock were running slow. Who's right, Jim or Jane? They can't both be right, can they?

This is an argument about "who is really moving?".Einstein said that since the motion is purely relative, there can be no physical effects distinguishing them. Otherwise, you could tell who is really moving – which violates the second postulate. Each sees the other's clock as running slow, and each is correct. Time is really proceeding at a different rate for Jim and Jane, relative to each other. The paradox arises because of Einstein's insistence that these concepts of space and time must be based on ***measurements***, and there is no cosmic referee to decide these disputes.

Einstein once put it this way: "No one has ever measured a time except at a place, nor measured a place except at a time." Thus are space and time intimately connected. There is no absolute space and no absolute time. There is such a thing as "proper" time and "proper" length – the time measured in your own reference frame, and the length of a meter stick you're holding in your hand.

The geometry in Figure 8.1 leads to

$$\Delta t = \frac{\Delta t_0}{\sqrt{1-(v/c)^2}} \qquad \text{(Equation 8.2)}$$

where Δt_0 is the proper time (measured in the rest frame, i.e. 1 second in our example), and Δt is the time measured by the moving observer (i.e. Jim's measurement of Jane's clock, and vice versa). Note that as v approaches c, Δt gets very long Later studies suggested that all clocks, not just the simple one considered here, obey the same formula. An hour glass, a pendulum, a vibrating atom, your pulse, your rate of hair growth, the lifetime of a radioactive nucleus… everything. It's known as relativistic time dilation, or simply ***time dilation***.

Although I've used very similar language, it's misleading to say "moving clocks slow down". Not according to the moving observer! (If they did, that would enable you to learn your state of absolute motion.) It's more accurate to say that different observers disagree about measurements of time… and since there is no time without a measurement, they disagree about time. Time is relative – not "flowing equably without relation to anything external", as Newton had said. For convenience, since we're accustomed to very poky speeds, we designate a measurement of time for $v = 0$ (often called "the lab frame") as a proper time. Proper time is good for nearly everything on Earth; but in the micro-world, and the extreme macro-world (distant galaxies, the vicinity of black holes, etc.), speeds close to *c* are encountered – in which case Eqations 8.1 and 8.2 are the law of the land.

A specialized but cute application of these ideas is to light itself. Shoot a light beam off into space. From our viewpoint, the light beam has a long journey ahead of it. But the light beam strongly disagrees. At v = c, it sees all lengths in direction of travel – ahead and behind it – as zero. So it sees the whole Universe as omnipresent, front and back. It gets everywhere instantly.

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Now all this talk of quarrels among twins, pouring coffee in airplanes, etc. is pretty interesting… BUT relativity theory would have remained mostly a philosophical curiosity if there were not actual and dramatic consequences for real physics experiments. Here are two real doozies..

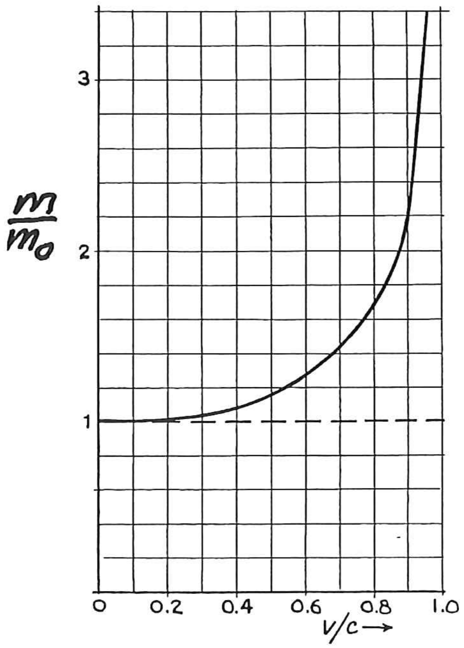
1. **Particle accelerators**. Physicists nowadays probe the structure of nuclei by firing high-speed particles (protons or electrons) at a target. The higher the speed, the better – because more energy is available to create new particles in the debris of the collisions. Many of these debris particles are just other protons – yawn – but every so often, they include one or two particles with a combination of charge and mass never previously seen. Hallelujah! They're not *stable* particles; otherwise the Universe would probably have created them long ago, and they'd still be found naturally. They quickly decay into other debris. But during their lifetime of a tiny fraction of a picosecond (plenty of time in the particle-physics world), their properties can be measured. The particle-physics zoo has a new member, and the experimenters pop the cork on the champagne.

Fascinating stuff. But here's the significance for our subject. We need to accelerate the original everyday protons up to very high energies, and we do this by successively applying an electric field to them. Around a billion applications of the electric field might do the trick. But as the proton goes faster and faster, we find empirically that it's harder and harder to accelerate them. The particle accelerators have to be really large – many miles long or across. The most famous accelerator in today's world is the Large Hadron Coillider, and it's so big that it actually spans two countries (France and Switzerland)!

The reason is that the mass of the proton grows enormously as the speed gets close to the speed of light. It takes a lot of power to keep accelerating these behemoths! The mass exactly tracks Einstein's equation:

$$m = \frac{m_o}{\sqrt{1-(v/c)^2}} \qquad \text{(Equation 8.3)}$$

where m_o is rhe rest mass. With this equation, look what happens to the mass as v gets close to c. The mass approaches a vertical line, becoming infinite when v = c (which is impossible, unless m_0 is zero, as with light itself)



So that's a real-world illustration of *relativistic mass*. Now fior length and time.

2. **Cosmic rays**. Geiger counters were invented by Hans Geiger, Rutherford's student, to detect elementary particles (mostly alpha-rays, beta-rays, gamma-rays). These rays mostly came from radioactive specimens which were being studied. But experimenters also found that even if you shield the Geiger counter from all known radiation sources, the clicks continued. There was an unexplained background of radiation. What was it, and where did it come from?

Some people thought it might come from space. In the 1930s, some physicists floated their detectors to great heights (~30 km) in the atmosphere, using very large ballons. just to see what it was like up there. They found that plenty of these particles – mostly protons and electrons – actually come from outer space. They became known as ***cosmic rays***.

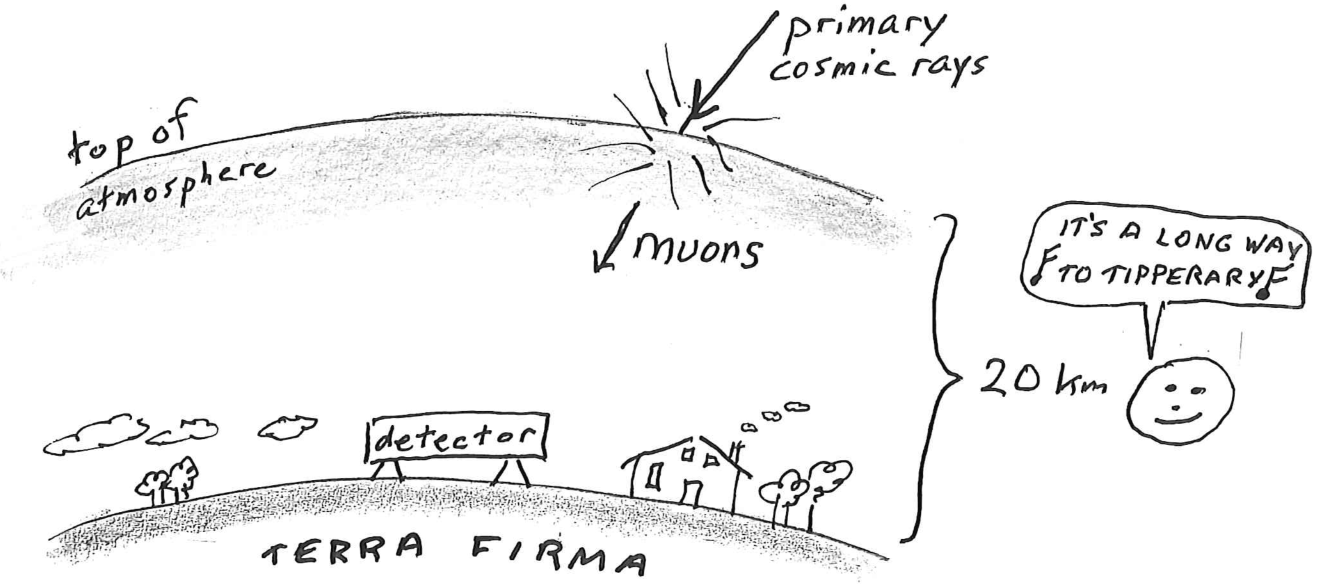
Protons and electrons are (more or less) eternal, so whenever they're shot off into space from anywhere in the Galaxy, some fraction of them will eventually make their way to Earth. But the cosmic-ray studies also revealed a particle similar to the electron but with a mass 200 times greater. As I.I. Rabi famously asked, “who ordered *that*?”

This new elementary particle was called the **muon** (Greek letter μ). One of its puzzling properties was its lifetime – just 2 microseconds. Now the speed limit in the Universe is the speed of light (300,000 km/sec). So if you just multiply speed x time, you find fthat muons can only travel 600 meters before they “die” (decay into something smaller, like electrons). So what are they doing up there?

Physicists quickly realized that muons can't truly come from outer space; they must be created in the upper atmosphere, as debris from the impact of cosmic-ray protons and electrons. So they should decay within about 600 m from where they formed, right? There shouldn't be any at sea level (unless you're within 600 m of a particle-physics laboratory). But cosmic-ray measurements from the ground show that muons are very abundant at the Earth's surface. Somehow they manage to travel all the way to the ground: about 20 km, although by all rights they should decay within the first 600 meters.

This is relativity, folks. The muon decays in 2 microseconds ***in its own rest frame***. But according to us, the muons are travelling very fast, around 0.99999 c. Therefore their clocks, as measured by us, are “slowed” by that “gamma”' factor, which is about 200. Their decay times are a clock – sort of like the threescore-and-ten that the Bible allots to humans. Thus we measure their decay times to be not 2 microseconds, but 400 microseconds… implying that they travel 200 times farther: not 600 m but 600x200 = 120,000 m. That's long enough to reach the ground.

But wait a minute. The muon may be small and transient, but that can also be said of humans. So let's consider its viewpoint. In its own rest frame, the muon is standing still, and the Earth is rushing towards it at 0.99999c. So it will measure the height of terrestrial objects to be shorter by that factor of 200. Therefore our 20 km atmosphere will seem to be only 100 m high. It's not an illusion. It ***is*** only 100 m high… and if the muons happen to be carrying a fully equipped physics lab, they can verify that their reference frame, as well as that of the Earth, is inertial. (Remember: the *laws* are identical; the measurements are not.) The muons can make it easily through so thin an atmosphere.



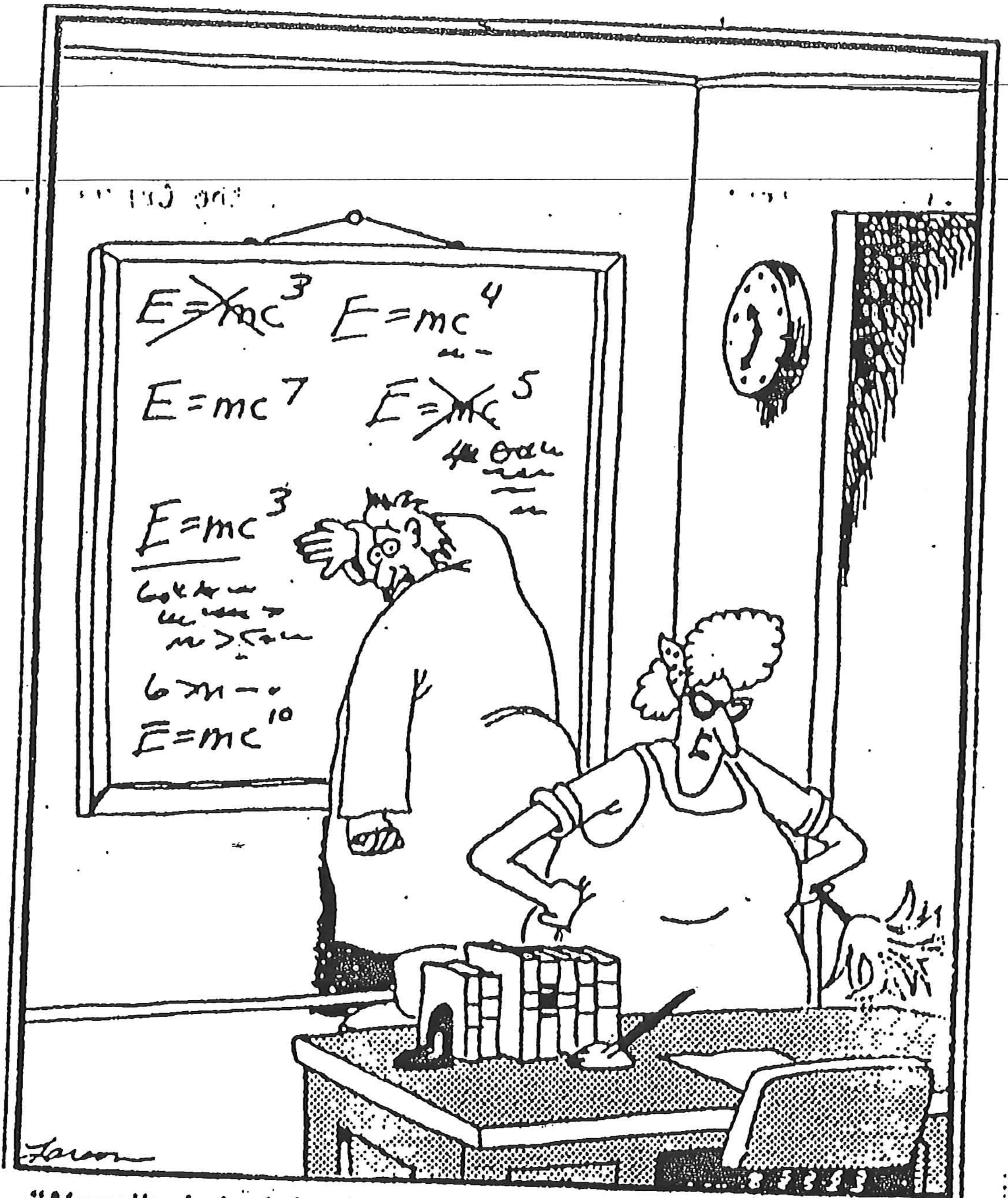
The muons and the humans describe their encounter in very different terms, but the result is the same (the muons easily reach the ground). The laws of physics are the same in all inertial reference frames: the speeding airplane, the speeding muon, the speeding Earth. So experiments carried out by observers in all these frames will have the same results, even though the observers will disagree sharply about measures of space and time (and mass).

That brings us to the last of the relativity equations, the most famous of them all:

$$E = mc^2 \qquad \text{(Equation 8.4)}$$

This represents the equivalence of mass and energy, as we discussed extensively in the previous chapter. It's strictly correct, although we are so used to thinking of mass and energy as different things that we often distinguish between them (e.g. a rock is mass, and light, or heat, is energy). Perhaps its greatest application in astronomy is in the liberation of energy in the manufacture of heavy elements, especially in the nuclear “burning” of hydrogen to form helium. The helium nucleus is 0.7% less massive than the four protons which are burned to produce it, so we say that we say that the burning is 0.7% efficient. Doesn't sound like much, but that simple 0.7% – multiplied by the speed of light, squared – is about 5 million times more powerful than chemical burning (e.g. TNT).

Derivation of $E = mc^2$ is tricky. But a possible solution is shown on the next page.



"Now that desk looks better. Everything's squared away, yessir, squaaaaaaared away."