Problem 1

As a first step, let’s compute a measure of the fit for a given set of parameters. Before this data, many cosmologists thought that there was no cosmological constant (i.e. that $\Omega_\Lambda = 0$), and a typical set of parameters was $\Omega_\Lambda = 0$, $\Omega_m = 1$, $h = 0.5$, sometimes called the Standard Cold Dark Matter (SCDM) model. Compute $\chi^2$ for this choice of parameters:

$$
\chi^2 = \sum_{i} \left( \frac{\mu_i - \mu(z; \Omega_\Lambda, \Omega_m, h)}{\sigma_\mu} \right)^2,
$$

where the sum is over the $N$ SNIa.

Problem 2

Is this a good fit or not? To answer this question, first plot the data $\mu$ against $z$ (with error bars if you can) and the fitted relation $\mu(z; \Omega_\Lambda, \Omega_m, h)$. Next, compute how likely this value of $\chi^2$ is assuming that the errors are Gaussian distributed. You will need to know the number of degrees of freedom $\nu = N - M$, where $N$ is the number of data points (181) and $M$ is the number of fitted parameters (in this case, we have not fitted any parameters, so you may take $M = 0$).
Problem 3

For the next step, we will relax the no-cosmological constant assumption, and allow $\Omega_\Lambda$ to vary. For simplicity (and also because CMB data indicate this) we will stick with a flat universe (i.e. $\Omega_k = 0$), so that $\Omega_\Lambda + \Omega_m = 1$. We will also assume that other data let us fix the Hubble constant: $h = 0.71$ (in fact, the Cepheid data from the Hubble Key Project do a pretty good job of this). Now, compute and plot $\chi^2$ as a function of $\Omega_\Lambda$. Find the best fit value of $\Omega_\Lambda$, as well as the one-sigma uncertainty in this parameter.

Problem 4

Finally, we will allow all parameters except the Hubble constant (set $h = 0.71$). Compute $\chi^2$ for a grid of values of $\Omega_\Lambda$ and $\Omega_m$ ranging from 0 to 1 in both parameters. Assuming that you have matplotlib (or similar software) working, use this to create a contour plot of the $\chi^2$ values. Find the best fit value allowing both parameters to vary. Is this a good fit? Be quantitative in in your answer (and note that we have fitted two values, so $M = 2$).