1. Alternative Origin of Helium and the CMB

We showed in class that the “hot big bang” model explains the origin and mass fraction of cosmic helium, as well as the presence of the CMB. Suppose, however, that BBN did not occur (e.g. we were in a Steady State universe) and the universe instead consisted originally of hydrogen alone. In this case, we might argue that all of the observed helium (with a mass fraction of \( Y_{\text{He}} = 24\% \)) was created by fusion inside stars.

(a) How much energy was released per unit volume in the universe as a result of this helium fusion? You can take the baryon density as \( \Omega_{b,0} = 0.04 \), with \( H_0 = 70 \text{ km/s/Mpc} \). (Note: fusion of four H into one \(^4\text{He}\) releases 28.3 MeV.)

(b) Suppose that all of this energy is now in the form of radiation. Compare the resulting energy density to the present-day energy density of the CMB. Could the helium fusion account for the present-day CMB based on the energy density alone? Can you think of additional problems for this being the origin of the CMB?

2. Modified BBN

(a) If the neutron-to-proton fraction is given by \( f \) at the time of nucleosynthesis, show that the maximum helium mass fraction \( Y_{\text{He, max}} \) is given by \( 2f / (1 + f) \).

(b) Suppose the neutron decay time were \( \tau_n = 89 \text{ s} \) instead of \( \tau_n = 890 \text{ s} \), with all other physical parameters unchanged. Estimate \( Y_{\text{He, max}} \), the maximum possible mass fraction of \(^4\text{He}\), assuming that all available neutrons are incorporated into \(^4\text{He}\) nuclei.

(c) Suppose the difference in rest energy of the neutron and proton where \( Q_n = (m_n - m_p)c^2 = 0.129 \text{ MeV} \) instead of 1.29 MeV, with all other physical parameters unchanged. Estimate \( Y_{\text{He, max}} \).