Due Wednesday, February 16  

Astronomy C3602: Problem Set 2

1. One idea once used to explain the Hubble relation is the “tired light” hypothesis. This posits that the universe is not expanding, but that photons simply lose energy as they move through space (by some unexplained means), with the energy loss per unit distance being given by the law

\[
\frac{dE}{dr} = -KE
\]

where \( K \) is a constant. Show that this hypothesis gives a distance-redshift relation that is linear with \( z \) in the limit \( z \ll 1 \). What must the value of \( K \) be in order to yield a Hubble constant of \( H_0 = 70 \text{ km/s/Mpc} \)?

2. In a positively curved universe containing only matter (\( \Omega_0 > 1, \kappa = +1 \)), show that the present age of the universe is given by

\[
H_0 t_0 = \frac{\Omega_0}{2(\Omega_0 - 1)^{3/2}} \cos^{-1}\left(\frac{2 - \Omega_0}{\Omega_0}\right) - \frac{1}{\Omega_0 - 1}.
\]

3. One of the more recent speculations in cosmology is that the universe may contain a quantum field, called “quintessence,” which has a positive energy density and a negative value of the equation-of-state parameter \( w \). Assume, for the purposes of this problem, that the universe is spatially flat, and contains nothing but matter (with \( w = 0 \)), and quintessence with \( w = -1/2 \). The current density parameter of matter is \( \Omega_{m,0} \leq 1 \), and the current density parameter of quintessence is \( \Omega_{Q,0} = 1 - \Omega_{m,0} \). At what scale factor \( a_{mQ} \) will the energy density of quintessence and matter be equal? Solve the Friedmann equation to find \( a(t) \) for this universe. What is \( a(t) \) in the limit \( a \ll a_{mQ} \)? What is \( a(t) \) in the limit \( a \gg a_{mQ} \)? What is the current age of this universe, expressed in terms of \( H_0 \) and \( \Omega_{m,0} \)?

4. Suppose you wanted to create a static universe (\( \dot{a} = \ddot{a} = 0 \)) in which the gravitational attraction of matter is exactly balanced by the gravitational repulsion of quintessence (see previous problem) with equation-of-state parameter \( w_Q \). Within what range must \( w_Q \) fall for the effects of quintessence to be repulsive? For repulsive quintessence with energy density \( \epsilon_Q \), what is the necessary matter density \( \epsilon_m \) to produce a static universe. Will the curvature of the static universe be negative or positive? What will be the radius of curvature, expressed in terms of \( \epsilon_Q \) and \( w_Q \)?

NOTE: Solutions should be written clearly and completely with a concise description of the calculation and an explanation of the steps taken to get the result (this makes it easier to award partial credit). If you use textbooks, reference books or websites, please cite them where appropriate. Discussing the problems in groups is fine, but the final solution should be yours and yours alone.