Astronomy C2002: Problem Set 9

1. Assume that the current values for the cosmological parameters are: $\Omega_m,0 = 0.25$, $\Omega_{\Lambda,0} = 0.75$, the CMB temperature is $T = 2.728$ K and the current value of the Hubble constant $H_0 = 70$ km/s/Mpc.

(a) Find the equivalent mass density of radiation $\rho_{\text{rad}}$ (neglect any contribution due to neutrinos) and write it in terms of the critical density (i.e. find $\Omega_{\text{rad}}$).

(b) Evaluate $\Omega_m$, $\Omega_{\Lambda}$ and $\Omega_{\text{rad}}$ at the time of decoupling between photons and matter ($z = 1089$).

2. The proper distance to the particle horizon (the farthest observable point for an observer at time $t$) is given by:

$$d_h(t) = a(t) \int_0^t \frac{cdt'}{a(t')}.$$

(a) Calculate the horizon distance during the matter-dominated regime for a flat universe when $a(t) = Ct^{2/3}$, where $C$ is a constant. Evaluate $d_h$ assuming the universe is 13.7 Gyr old.

(b) Express and plot the above as a function of $z$.

(c) Repeat parts (a) and (b) for a flat universe which is dominated by a cosmological constant.

BONUS QUESTION

3 (a) Solve the Friedmann equation and find $a(t)$ for a spatially flat universe with $\Omega_{m,0} = 0.25$, $\Omega_{\Lambda,0} = 0.75$ (i.e. these are the values at the present day). Please do not neglect radiation. You will need to integrate this numerically. Please attach a printout of the computer code, or list the program commands (e.g. if you use a graphing calculator or a program such as Maple or Mathematica).

(b) Compare it to the case with $\Omega_{m,0} = 1$ and $\Omega_{\Lambda,0} = 0$. 
