Astronomy C2002: Problem Set 2

1. Calculate how far you could see through the Earth’s atmosphere if it had the opacity of the solar photosphere. Use $\kappa = 0.03 \text{ m}^2 \text{ kg}^{-1}$ for the sun’s opacity. [Problem 9.7 of Carroll & Ostlie (2007)]

2. A simple model for the late stages of a supernovae shock is a thin spherical shell, centered on the site of the explosion. Assume that the shell has expanded to a radius $R$ from the center, and has a width $\Delta R \ll R$. Assume further that the material filling this shell has a constant density $\rho$ and constant emission coefficient $j_\lambda$ and is optically thin (neglect absorption). Show that the observed specific intensity of the shell along a ray passing a distance $p$ from the center is approximately

$$I_\lambda = \frac{2j_\lambda \rho R \Delta R}{\sqrt{R^2 - p^2}}$$

for $p < R$ (and $I_\lambda = 0$ otherwise). Make a plot of $I_\lambda$ vs $p$ to demonstrate that the shell will appear brightest near $p \sim R$, thus showing that the supernovae shell will look like a “ring” on the sky.

3. In this problem, we will derive the “curve of growth” scalings between the equivalent width $W$ and the column density of absorbers $N$ that were claimed in class. When the profile of the line is dominated by Doppler broadening, we can write it as,

$$\tau(\lambda) = \tau_0 e^{-\alpha(\lambda-\lambda_0)^2}$$

where $\tau_0$ is the opacity at the center of the line (when $\lambda = \lambda_0$), and $\alpha = mc^2/2k_b T\lambda_0^2$ is a constant for a given line.

When the profile of the line is dominated by the “natural” broadening, the optical depth is a Lorentzian profile:

$$\tau(\lambda) = \tau_0 \left( \frac{Q^2}{(\lambda - \lambda_0)^2 + Q^2} \right)$$

where $Q$ is a parameter that we can take as a constant.

(a) First, plot both $\tau(\lambda)$ profiles, remarking on which profile has the larger “wings” (i.e. which has larger value of $\tau$ for large values of $\lambda - \lambda_0$).

(b) The equivalent width is an observed measure of the strength of the line, and can be written as

$$W = \int (1 - e^{-\tau(\lambda)})d\lambda$$
and has the same units as wavelength. Show that in the optically thin limit (when Doppler broadening dominates and $\tau \ll 1$), the equivalent width $W$ is directly proportional to the column density of absorbers $N$ (recall that $\tau_0 = N \sigma$, where $\sigma$ is the cross-section for each absorbing atom or molecule). *(hint: expand the exponential as a Taylor series.)*

(c) Show that in the intermediate regime, when $\tau_0$ is very large but natural broadening is still not important (so that ignore the Lorentzian), $W \propto (\ln N)^{1/2}$. *(hint: If you plot $1 - e^{-\tau}$ in this regime you will find that it looks like a square wave function – that is, it is approximately equal to 1 when $(\lambda - \lambda_0)^2$ is less than a certain value and zero everywhere else. If you assume it is exactly a square wave and substitute that into the integral for $W$, you can evaluate the integral for $W$ as a simple function of $\lambda - \lambda_0$.)

(d) Finally, in the highly optically thick regime, when natural broadening dominates, show that $W \propto N^{1/2}$. If you use the same trick as in the previous problem (but now using the Lorentzian instead of the Gaussian profile), this is easy.