Phenomenological Photofluiddynamics

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1 What It's About

In "The Brightest Stars" (1980) de Jager wrote

> Why is it that there are no stars brighter than a few million times the Sun, hotter than approximately a hundred-thousand degrees, ... What processes occur in stars ... that are presumably at the fringes of their stability?

This seems to be a surprising question to have posed in 1980 since there were already reasons to suspect that the supposed upper limit on stellar masses had been explained. In 1966, Ledoux had reviewed the position and noted that

> ... for given chemical composition and nuclear reaction, there is a critical mass above which the star becomes violently, vibrationally unstable under the influence of the nuclear reaction which overwhelms all the dissipative factors and leads to rapidly growing oscillations resulting in a an ejection of mass tending to bring the value of the mass back to the neighborhood of the critical mass.

Ledoux cited his estimate of the critical mass (1941) and the later one of Härm and Schwarzschild (1959) in the 50-60 $M_\odot$ range, with the possibility of perhaps 80 $M_\odot$ for metal poor stars. Of course, one may argue that the vibrations may not destroy the star, so de Jager's question had a point. On the other hand there was also Eddington's earlier work (1921) in which he derived the stellar luminosity above which the outward force of emerging radiation on the outer layers exceeds the surface gravity. We shall formulate this Eddington limit below. Together with the mass-luminosity relation, the Eddington limit had for years been considered to impose an upper bound on possible stellar masses. However, there are observations indicating that stars do exist above this limit and so it is worth examining the fluid dynamical issues that arise in the envelopes of massive stars. Perhaps that is what de Jager had in mind in raising this issue. The fluid dynamical processes that I shall discuss qualitatively here — photon bubbles and vortices — are what may allow a star to exist above the Eddington limit but, even with these, a very luminous star probably cannot keep all its mass indefinitely. So the question has to be rephrased as one about lifetimes of self-destruction and, as far as I am aware, *that* question has not been answered as yet. Whatever the ultimate resolution of this issue, there is plenty going on in the atmospheres of very luminous stars to intrigue those who are interested in fluid dynamics and the analogous case of disks is equally fascinating.

In the twenties, Struve and Elvey concluded from the profiles of spectral lines that there were large scale supersonic motions in the atmospheres of hot stars. There was no general agreement about the causes of these motions that Struve called macroturbulence and it was not even agreed that the term turbulence was appropriate. The source of these motions was attributed variously to pulsation, rotation and, as Huang and Struve (1960) later put it,

> turbulent motions are favored in regions of low density in hot stars and ... radiation pressure is probably important in creating turbulence.

Moreover, as we are dealing with ionized media, magnetic fields will also play a significant role. In the case of massive stars, with their convective cores, we may expect important dynamo action in the interiors giving rise to the protrusion of prevalent magnetic fields. Moreover, the observed fluid motions will probably produce tangled magnetic fields that are especially hard to deal with. Here, I conform to the preference of Jean-Paul Zahn, our dedicatee (with Silvano Bonazzola) and aim treat the purely fluid dynamical issues, reserving the inclusion of magnetic fields for a later discussion.

Even in the purely fluid dynamical case, the equations are complicated in this subject, and I shall not go much into such details either. Even less shall I treat solution methods, which are chiefly numerical for the complicated processes I shall discuss. Rather, my aim is to give some idea of what kind of dynamical activity we may expect in hot stars and disks and how they may rationalize some of the observed phenomena. For guidance, I shall refer to laboratory experiments that are qualitative analogues for the processes caused by the passage of radiation through matter, just as Bénard's early studies of laboratory convection served as guides to stellar convection a century ago. I believe too that the theoretical methods being used to solve some of the astrophysical ought to be tested against these laboratory experiments.

2 Photon Bubbles

About fifty years ago, I was a student in a graduate class on stellar atmospheres. We had come to the part on the computation of model atmospheres, a procedure that we were to carry out on mechanical calculators. Our particular assignment was to calculate the atmospheres of hot helium stars and each pair of students was given a different set of atmospheric parameters. The first students to finish the task found that their models blew up. As I am computationally impaired, I sought a way of avoiding the calculation and did so by convincing the teacher that the radiative fluxes in the models (as they were to be calculated by us) were too great for the given surface gravities. Thus I managed to avoid doing any numerical calculations (a practice that I have since kept up). None of us (including the professor, I suspect) knew of the Eddington limit (see below) at that time so we learned the hard way of this result and I was set on a course of research that occupied me for several decades. The whole exercise left me unhappy, especially on being told that the way to treat a convection zone in a star was to assume that it is marginally stable to convection. So, when I report below that something like this rule may play a role in stars beyond the Eddington limit, I shall be somewhat embarrassed. However, in this case, the rule of thumb has an empirical origin and it may be a useful first approximation in helping to describe states in excess of the Eddington limit. I came on the source of this notion in 1967 when, in a geophysical meeting in Newcastle, I learned of a subject that I feel suggests a way to imagine what is happening in hot stellar atmospheres. I shall describe that next.

2.1 Fluidized Beds

Figure 1: A bubbling bed.

When a fluid is forced to flow upward through a layer of particles, the drag on each particle tends to impel it upward. At a certain critical flow speed, the particles are levitated. A kind of phase change occurs in which the layer expands and the particles become free to move about as in a gas. Since the drag per particle decreases with a lowering of the number of particles per unit volume, the particles are not blown away but enter into a fluidized state. In the simplest fluidized state, the beds are homogenous are they are useful for many industrial purposes in which it is desired to establish intimate contact between chemicals that may be in the fluid and/or in the particles [62].

When the mass density of typical particle in the bed greatly exceeds the density of the driving fluid, the outflow does not remain smooth. Bubbles, or particle voids, form and interact strongly so that the bed resembles a boiling liquid. As we see in Figure 1, the homogeneity of the bed is destroyed. The picture shows a tube about eight cm across containing glass beads of about 0.1 mm diameter. Air is being forced up from below. If we fluidize such a layer using water, no bubbles form. On the other hand, if we fluidize a layer of lead pellets with water, bubbles do form.

To make this statement a bit quantitative, let ρ_f be the density of the fluid levitating a bed of particles and ρ_p be the density of an individual particle. (We shall not consider the more complicated case where there is a mixture of particle sizes and densities.) An important parameter of fluidized beds is the parameter $\xi = \rho_p/\rho_f$. Bubbles form when ξ is appreciably in excess of unity.

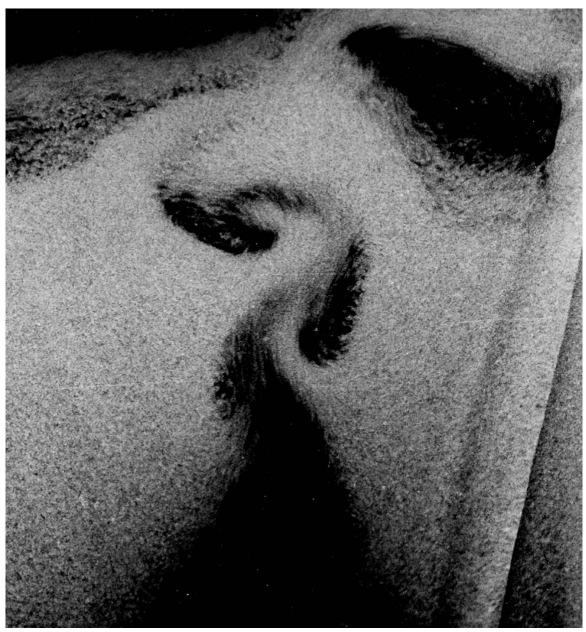

Figure 2: Sand fluidized by water in a wedge [65].

We should also take note of the possibility of modifying this last statement by introducing exceptional conditions. For example by introducing sloping side walls in a fluidized bed we can change the rules of void production. In Figure 2 we see a side view of a bed of sand fluidized by water flowing upward through a wedge-shaped container. In this case, even with a ξ of order unity localized voids form although they are not like the usual fluidization bubbles. Here the basic flow is related to what in fluid dynamics is called Jeffery-Hamel flow. Neither the basic state nor the primary instabilities of this flow have been analyzed for the fluidized case as far as I know.

2.2 The Eddington Limit

In the bubbling fluidized beds, we have a relatively light fluid moving upward through a second fluid with relatively dense particles and the drag on the particles is enough to levitate them. The photon gas coming from within a star (or disk) likewise exerts an outward force on the atmospheric particles and, when this radiative force just balances the weight of the particles, we are at the Eddington limit (1921).

Let $\boldsymbol{\mathcal{F}}$ be the frequency-integrated radiative flux, σ the Thompson cross-section per unit mass and c be the speed of light. The quantity $\sigma\boldsymbol{\mathcal{F}}/c$ has the dimensions of acceleration; it is the force per unit mass exerted on the material by the radiation. When the component of $\sigma\boldsymbol{\mathcal{F}}/c$ along the local direction of gravity equals the local acceleration of gravity, we are at the Eddington limit. For a spherically symmetric star with vertical component of the radiative flux, $\mathcal{F}$, the luminosity is $L = 4\pi R^2\mathcal{F}$ where R is the star's radius. When L is equal to the Eddington luminosity, $L_E = 4\pi R^2\sigma\mathcal{F}/c$, we are at the Eddington limit. In the spirit of fluid dynamics, we may define the *Eddington number*, $\varepsilon = L/L_E$.

That there are stars such as η Carinae with ε well above unity at least some of the time seems quite likely. Why do such stars exist? Or rather, we should ask, how long could they exist in the apparently superEddington state? This of course depends on the fluid dynamical activity in these objects. The analogy of the situation in very luminous stars to that of fluidized beds makes the suggestion that photon bubbles form in hot stellar atmospheres seem natural (Prendergast and Spiegel, 1973). Bubbling could allow a massive star to exist above the Eddington limit with much of the radiation escaping in the bubbles. Of course, there would be some mass loss, as we shall suggest below, and we need to learn the rate of that process. Bubbling in hot stars (and disks) could also provide a rationalization of the observed macroturbulence in these objects. It is therefore worth asking whether the formation of photon bubbles is likely and, at the moment, the most used approach to this question is by numerical simulations, which people have begun to carry out.

2.3 Beyond the Eddington Limit

When the fluidized state is first achieved, the bed is homogeneous and it generally remains so for small ξ. This is reasonable since the reduced gravity is zero in the fluidized state. For high ξ however, bubbling makes the bed inhomogeneous. There is an empirical rule of thumb in fluidization that says that all the flux of levitating fluid in excess of that needed to maintain fluidization escapes in the bubbles. If indeed photon bubbles occur in hot stars, may we expect a similar rule that allows the star to exceed the Eddington limit? If so, what is the formulation of the astrophysical rule?

Perhaps the rule as stated in the fluidized case is in reality an analogue of the zeroth order rule of stellar convection, namely that the instability serves to keep the system in a state of marginal stability. If that is true, the rule ought to be that the fluidized bed remains in the marginal state for bubble formation. When Prendergast and I first were thinking about photon bubbles, there did not appear to be a clear vision of how fluidization bubbles form. It was generally believed that instability caused the bubbles but the problem was that, though all fluidized beds beds were generally unstable, only beds with high ξ produced bubbles. High ξ did mean high growth rate, and that was thought to be the cause of the difference. However, it appears that the bubbles are not caused by the primary instabilities of the beds, which are likely to be drift instabilities. Rather, it the basic instabilities (mainly) set up vertically traveling plane waves that become unstable to secondary instabilities and form horizontal inhomogeneities from which bubles originate. Since it has been written that the literature of fluidization is larger even than that of Shakespearean scholarship, it may be best if I simply give a reference to a recent review (Guazzelli).

From this complex situation, we would like to extract a criterion for the onset of bubbling, presumably in the form a critical flow rate of levitating fluid. Any flux in excess of the critical rate, according to the above-mentioned empirical rule, would escape in bubbles. However, even now that the origin of bubbling in fluidized beds is better understood, there is, as far as I can make out, no clear cut criterion for the onset of bubbling. To get one, we may proceed as follows.

First, we would need to find the nonlinear oscillatory state produced by the primary instabilities of beds (or stars). Then we would need to study the stability of such nonlinear oscillatory states. Instabilities arising because of temporal variations in the control parameters are called parametric instabilities. To get an idea of how such a thing would go, consider a model of a spherical gas mass oscillating in an arbitrarily chosen, nonlinear, periodic and homologous manner. In studying this situation Poyet and I did not include radiative forces and we chose a model whose static state was convectively stable. When the oscillations were of sufficiently large amplitude, instability set in as is typical with Hill equations of the kind we found with our model. In effect, we found that oscillating stars with their varying effective gravity provokes (what we may call) *parametric convection*. Like ordinary convection, parametric convection will produce thermals, low density entities analogous to bubbles. The route to be taken is then clear both for the fluidization case, much of which has already been done there, though not in this way, as for the astrophysical case.

2.4 Photostatic Aspects

When a bed of particles becomes fluidized, it passes from a close-packed state to a homogenous two-fluid state. Here is a basic difference from the astrophysical state which is typically always fluid and it is worth looking into the static state of a radiating atmosphere below the Eddington limit to see what the situation is like as we let ε approach the critical limit of unity.

Consider a plane-parallel, non-nonrotating layer with vertical coordinate z assuming that the medium is fully ionized and neglecting absorption and Compton effect. The hydrostatic condition is

$$\frac{dp}{dz} = -g_* \rho$$

and the conditions of radiative equilibrium are (when the Eddington approximation is made in the matter frame)

$$\frac{d\mathcal{F}}{dz} = 0 \qquad\qquad \nabla E = -3\frac{\rho\sigma}{c}\mathcal{F}$$

where $\mathcal{F}$ is the vertical component of the radiative flux, E is the radiative energy density, σ is the Thompson cross-section per unit mass, c is the speed of light and the reduced gravity is

$$g_* = g - \frac{\rho\sigma}{c}\mathcal{F} \ .$$

We consider that the material medium is a perfect gas with

$$p = \mathcal{R}\rho T$$

and require that the gas pressure goes to zero as $z \to \infty$. At every point, we assume local equilibrium so that

$$E = aT^4$$

Then these equations are readily solved (Spiegel, ¿) and we find the temperature distribution

$$-\frac{g_\star z}{4\mathcal{R}\hat{T}} = \Theta - \frac{1}{2}\tan^{-1}\Theta - \frac{1}{2}\coth^{-1}\Theta$$

where

$$\Theta = \frac{T}{\hat{T}}$$

and the surface temperature $\hat{T}$ is basically an integration constant. In its upper regions, this solution is isothermal rather like the early version of what Teisserenc de Bort called the "isothermal region" of the earth's atmosphere in 1899. (It is now called the stratosphere.) Below, the model becomes polytropic.

When $g_* = 0$ we have the analogue of the fluidized state and this is the origin of the homogeneity of fluidized beds. Though there are issues of boundary conditions that we are glossing over here let us consider this uniform state and ask (as one does in fluidization theory) what the photostatic solutions looks like when a spherical hole is made in an otherwise uniform medium. The photostatic equations in this case are

$$\boldsymbol{\mathcal{F}} = -\frac{c}{3\rho\sigma}\nabla E \qquad\qquad \nabla\cdot\boldsymbol{\mathcal{F}} = 0 \ .$$

Let us use spherical coordinates centered on the hole and assume that, far from the hole, the radiative flow is constant. That is, as $r \to \infty$, $\boldsymbol{\mathcal{F}} \to \mathcal{F}_0 \hat{\mathbf{z}}$ where $\mathcal{F}_0$ is a constant and $\hat{\mathbf{z}}$ is a vertical unit vector. Also we suppose that E and $\hat{\mathbf{n}} \cdot \boldsymbol{\mathcal{F}}$ are continuous across the bubble surface at $r = r_0$ where $\hat{\mathbf{n}}$ is the unit normal on that surface. This problem is formally equivalent to an electrostatic problem and its solution (Spiegel) is like a dipole, namely

$$E = E_0 - \frac{3\rho\sigma}{c}\mathcal{F}_0 r_0 \left[\frac{r}{r_0} - \left(\frac{r_0}{r}\right)^2\right] \cos\theta$$

$$\boldsymbol{\mathcal{F}} = \nabla \left[\frac{cE}{3\rho\sigma}\right]$$

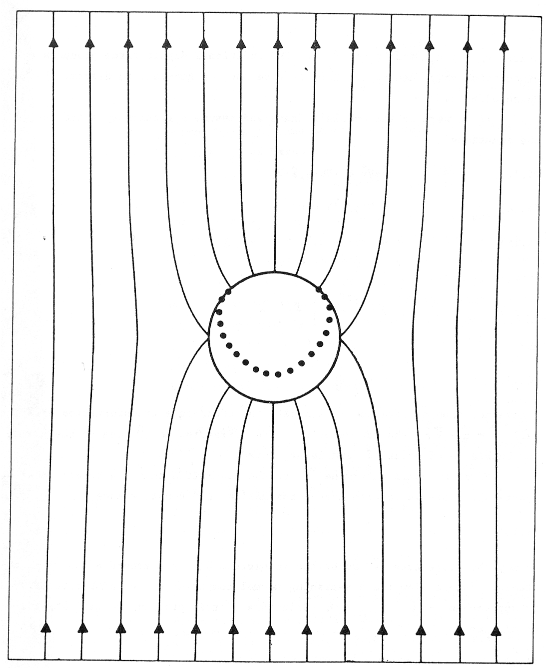

Figure 3: Radiative Flow in and Around a Spherical Hole.

Figure 3 shows the streamlines of the radiative flux caused by the spherical hole of the example. The flow from below tends to close the hole at the bottom as suggested by the dots. The flow out of the hole would open up the top, but to compute that we need to know to know the radiation field inside the hole. (I have asumed constant E in there.) From these rough considerations, we may see the hole as a propagating structure rather than a buoyant object. The secondary instability might change this view and bring it closer in character to the bubbles arising in convection in hot media. Convective photon bubbles were treated with mixing length by F.D. Kahn and his student V.S. Thorne in her Manchester thesis. (I thank Wal Sargent for telling me of this work.)

2.5 Instabilities

Bubbling in a fluidized bed is an outgrowth of a sequence of instabilities. Although instabilities occur for all values of ξ, the bubbling occurs only for ξ large enough to induce secondary instabilities. In astrophysical case, there are likewise instabilities for over nearly the full range of ε. Does this make for a similar dichotomy?

The instabilities that were found theoretically early in the fluidized studies are drift instabilities in the fluidized case (as Prendergast and I reported). It is not so clear that these can provoke the kind of oscillations that become unstable to bubble formation. Similarly, when the instabilities of stellar atmospheres with strong radiative forces were being discussed in the seventies, it also seemed that drift instability was the main event as was clearly brought out by Castor (see the papers in the Nice meeting). In his thesis at Columbia, Marzec chose boundary conditions that basically allowed only absolute instability and found none.

But there are also instabilities that arise when the radiative forces are omitted. Lou (...) has found instabilities in stellar model atmospheres confirming earlier reports of such by Graf (¿). In Lou's models, which are convectively stable, the heat transfer is by radiative condiction with constant conductivity, so there is apparently no κ-mechanism operating. Lou's instabilities appear with and without prevalent magnetic fields and appear to be absolute. Umurhan has verified the occurrence of these instabilities in the nonmagnetic case both numerically and by asymptotic methods. He has in particular considered the case where the heat flux is fixed on the upper and lower boundaries, so that the energy source of the instability cannot be the radiative flow. There is also related work by Asplund on cool supergiants.

Since the midseventies, there has been careful attention paid to the derivation of suitable equations for these problems in which the standard fluid equations are coupled to the lower moments of the transfer equations with the introduction of closure approximations at the level of the Eddington approximation (see for instance Simon, ... Hsieh and s. , Mihalas and M¿). Instability is found with equations at this level approximation (Shviv and Tao and S). Though disagreement has been reported in the results (Shaviv), it seems safe to assume that there are instabilities over the full range of values of ϵ, in analogy to the fluidization case. The recent trend in this has been to include magnetic fields (see Turner for literature) and this brings in further modes that are unstable and simulations in that case have produced photon bubbles.

It appears then that, just as instabilities appear for all ξ in the fluidized case, they appear for all ε in the stellar case. The analogy to fluidization seems to be holding up and, if there are indeed photon bubbles, we may call on the resulting vigorous motions to provide a natural explanation of the spectroscopic evidence for supersonic motions in the envelopes of hot stars and disks. The presumed bubbling could also provide an escape valve for the radiative flux in excess of the Eddington limit. This would follow if we may accept the empirical rule of fluidization for the transport by bubbles mentioned earlier. It would be useful to get a theoretical handle on this rule in both the laboratory and the stellar contexts. However, there is an aspect of the problem that may be more significant.

3 Photovortices

3.1 The Effect of Rotation

An article dedicated to Jean-Paul Zahn, as this is, would be incomplete without a discussion of the effects of rotation, though I shall not consider the large scale flows of his expertise. Rather, I want to build on another aspect of this subject, namely that rotating, turbulent cosmic fluids form coherent vortices as in the Great Red Spot of Jupiter and in dust devils [58] and tornados. The meaning of the term coherent here is that the vortices live longer than might have been anticipated on the basis of simple dimensional arguments [71]. A true understanding of the formation process of these objects by turbulence is lacking despite many laboratory and numerical studies of this process. One reason for this difficulty is that it is hard to observe the vortex formation process directly. On the other hand, it might be simpler to think about this issue if we could replace the turbulence by bubbling and so observe the vortex formation process more readily in the merging of bubbles. Since vigorously bubbling fluids have something in common with turbulent fluids, this may not be an unreasonable thing to expect.

In experiments in rotating tanks of turbulent water performed in Grenoble, vortices were observed by introducing small gas bubbles into the flow [67]. The bubbles were attracted into the vortices and so rendered them plainly visible. In the Grenoble experiments, the bubbles were so small that they did not feed back on the underlying flow. However, when the bubbles are large and can modify the flow, they would still be attracted to vortices, while contributing to their development. This process has been seen experimentally. A simple way to preform such experiments is to drop Alka Seltzer tablets into a rotating tank of water as was done by Turner and by Gough and Lynden-Bell. The bubbles iproduced in this way typically gather to form strong vortices in the center of the tank and these are maintained till the bubbling stops. This promises to be a fertile field of study for, when vortices are formed by bubbling, the process can be seen more directly than in a turbulent fluid and this advantage should lead to a greater understanding.

In Woods Hole, Karl Helfrich, Jack Whitehead and I have begun to experiment with vortex formation by bubbling in a rotating tank. Getting bubbles to merge into a single vortex is feasible but we have not yet succeeded in generating a field of vortices. The latter possibility is significant for many issues beyond our immediate concern here. Bubbles have some dynamics in common with the rising convective thermals generated in the earth's atmosphere above warm water or land. The latent heat of water vapor that energizes the convection also appears through ionization in the stellar case. Thus, in studying vortex formation in bubbling fluids, we may also hope, with Turner, to learn something about how tropical hurricanes form.

In another context, it has been found that by rotating incinerators one can raise the rate at which air flows through the systems and so accelerate the burning process [82]. We may ask whether the known rapid rotation of hot stars will similarly increase the escape rate of their radiation and so allow the Eddington limit to be exceeded. In both cases the enhanced outflows would result from the formation of vortices just as in the experiments on bubbling fluids. The difference is that here the bubbles are not introduced by external means but are integral to the processes we are studying. Once the vortices are formed, they can serve as conduits through which the lighter flluid (air or radiation) can escape. This would offer an explanation of some observations that point to the existence of spots on hot stars [60] and quasar disks [**?**]abram) since the escaping radiation would be beamed by vortices [64]. The main task would then be to see whether indeed the vortex formation takes place and to try to understand the mechanism. This can best be done in the case of accretion disks.

3.2 Vortices in Disks

Though I have dealt mainly with the stellar problems so far, it is at least as important to understand the influence of rotation on the internal dynamics of accretion disks. Accretion disks arise in the formation of planetary systems and in evolving binary stars. They are the sources of the prodigious outpouring of radiation from quasars located in cores of galaxies where matter is being captured by massive black holes. In many instances, especially the last, they are frequently quite hot and radiative forces on the matter play a role in their fluid dynamics.

Hot disks are generally considered to be turbulent because of a linear instability called the magnetorotational instability. In the present discussion we call on photon bubbling to provide disorder. In the simplest case of disks with no radiative forces and no magnetic fields, it is a matter of current debate whether turbulence arises. Direct simulation of Keplerian flows where the 'turbulence' is introduced as an *imposed* random velocity field) does produce vortices as in the example shown in Figure 6. These are anticyclonic vortices, that is, they rotate in the opposite direction to the main flow. Cyclonic vortices, on the other hand, are quickly sheared out in the Keplerian flow, which is linearly stable.

If we prefer not to introduce the turbulence by hand, but the medium is a good electrical conductor, we may introduce a magnetic field so that instability occurs. Then current vortices form as seen in Figure 5, but I have said I shall not discuss magnetic issues much here, so let us turn to the next subtopic.

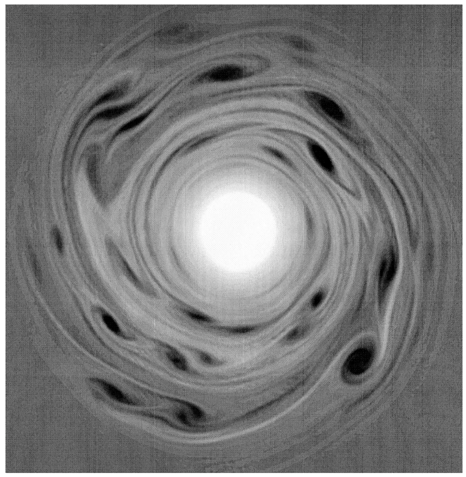

Figure 4: Vortices on an Accretion Disk [59]

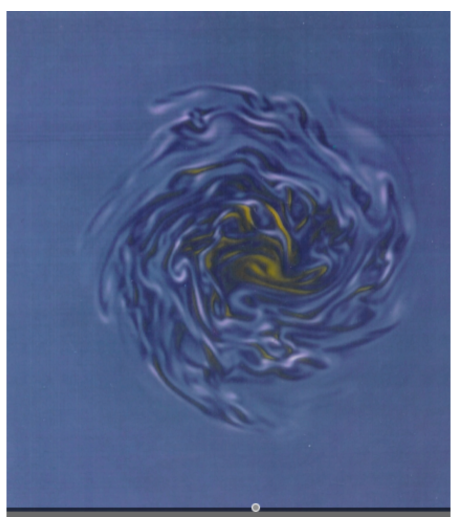

Figure 5: Magnetic Structures on an Accretion Disk [59]

3.3 Agglomeration by Vortices

Even if we do not have a clear source of turbulence in a disk, we may be safe in assuming that disks form in an initially turbulent state. Simulations of Keplerian flows with a superposed disordered perturbation on its initial vorticity field also give rise to the formation of anticyclonic vortices. Not only do vortices form quickly in such simulations, but particles into the flow are drawn into the vortices in a short time. To understand this process we may look at the equation of motion of a dust particle in the swirling flow.

Suppose we use a coordinate system with constant angular velocity $\mathbf{\Omega}$. The motion of a particle at position $\mathbf{x}$ is described by

$$\frac{d^2\mathbf{x}}{dt^2} = -2\mathbf{\Omega} \wedge \frac{d\mathbf{x}}{dt} - \mu\left(\frac{d\mathbf{x}}{dt} - \mathbf{u}(\mathbf{x}, t)\right) + \left(\Omega^2 - \frac{GM}{|\mathbf{x}|^3}\right)\mathbf{x}\ .$$

Here, the first term on the right is the Coriolis force felt by the particle, μ is the friction coefficient that produces the drag of the fluid velocity $\mathbf{u}$, and the last term is the difference between the centrifugal and graviational forces on the particle.

Suppose there is vortex and we transform to a polar coordinate system, (r, θ) with origin at the vortex center. The radial component of the new equation is then

$$\frac{d^2r}{dt^2} = r\dot{\theta}^2 + 2\Omega r\dot{\theta} - \mu(\dot{r} - u_r) + 3\Omega^2 r \sin\theta$$

where we have linearized the the last term for simplicity. As a particle circles around a local vortex, this last term will cancel out on average and the drag term will become small as the particle adjusts to the local fluid velocity. (These purely qualitative statements are made to give an idea of what happens; in real life, we solve the full equation of motion numerically.) So we are left staring at the first two terms on the right. We may simplify these by assuming that the particle's angular velocity around the vortex is approximately the local vorticity ω. So, with $\omega \approx \dot{\theta}$, those two terms boil down to $r\omega(\omega + 2\Omega)$ where. By definition, Ω is positive. So if ω is negative and of sufficient magnitude, the particle will be drawn toward what is an anticyclonic vortex. It will probably not get all the way in since the force vanishes as r gets small and that is why the bubbles marking the vortices in the Grenoble experiment seem to form rings around the vortices. And this brings us to the point of interest here — bubbles are attracted to vortices of suitable sign just as material particles are. This is because bubbles have added mass and do behave somewhat like particles. However, very small gas bubbles in liquids do tend to rise straight up, as Dom Perignon reported in the time of Newton.

3.4 Light Beams

If vortices form, they will produce inhomogeneities that allow radiation to escape through regions of low density. Let us look at an idealized example in a polytopic nonrotating layer

(Dowling and Spiegel). We consider a vertical tube of vorticity of circular cross-section. The mean properties of our simple vortex, in cylindrical coordinates (r, θ, z), are governed by these equations:

$$\frac{dp}{dz} = -g\rho$$
$$\frac{dp}{dr} = \rho\frac{v^2}{r}.$$

For a polytropic medium, $dh = dp/\rho$, where h is the specific enthalpy. From from the first of these equations we learn that

$$h(r, z) = -gz + f(r),$$

where f is an arbitrary function of r. According to the second equation, f satisfies

$$f' = \frac{v^2}{r}.$$

The latter result implies that v is independent of z.

We adopt the standard potential vortex, with $v = v_0 r/r_0$ for $r/r_0 < 1$ and $v = v_0 r_0/r$ for $r/r_0 > 1$, where r_0 and v_0 are constants. For $z = 0$, we require that the surface becomes flat as $r \to \infty$. Then we get

$$f = v_0^2 \begin{cases} \frac{r^2}{2r_0^2} - 1, & \text{if } r < r_0; \\ -\frac{r_0^2}{2r^2}, & \text{if } r > r_0. \end{cases}$$

The vortex deforms the atmosphere's surface from the plane $z = 0$ into

$$\frac{gz}{v_0^2} = \begin{cases} 1 - \frac{r^2}{2r_0^2}, & \text{if } r < r_0; \\ \frac{r_0^2}{2r^2}, & \text{if } r > r_0. \end{cases}$$

So the vortex represents a pit in the stellar atmosphere with a depth $z_0 = z(r = 0) = v_0^2/g$. The contours of the thermodynamic quantities follow that of the surface, but are vertically displaced.

For a polytrope with $p = K\rho^\Gamma$, we have $h = [\Gamma K/(\Gamma - 1)]\rho^{\Gamma-1}$. If the medium is also a perfect gas with $p = \Re\rho T$, we have $h = c_p T$. Now the radiation pressure is $p_{\rm rad} = \frac{1}{3}aT^4$, so the total pressure is $P = p + p_{\rm rad} = p(1 + \beta)$, where β is a constant times $\rho^{3\Gamma-4}$. For the plausible condition (for hot stars), $\Gamma \approx 4/3$, the dependence of pressure on temperature is virtually unchanged and the vortex structure survives as described even when the effects of radiation pressure are admitted.

As the medium is polytropic, the contours of ρ and of T are coincident; for $\Gamma = 3/4$, T^3/ρ is a constant. In that case, we have the simple approximation that the radiation flows down thw temperature gradient. Since $h = c_p T$, we have, for $r < r_0$, the isotherms

$$T = \frac{g}{c_p}z + \frac{n_0^2}{c_p}\left(\frac{r^2}{2r_0^2} - 1\right).$$

The main observational consequence of having a strong, deep vortex in the atmosphere is the attendant modification of the radiation field. Like the hydrogen bubbles of the aforementioned Grenoble experiments, photons are light particles and they too are attracted to the vortex. But here, the description of the process is different than the one we gave in the previous section. As we did earlier in this discussion, we turn to the photostatic equations with $E = aT^4$. Then we find

$$\boldsymbol{\mathcal{F}} = -\frac{56\sigma_\star T^3}{3\sigma\rho}\nabla T$$

where $\sigma_\star$ is the Stefan-Boltzmann constant.

Since T is approximately the potential for $\boldsymbol{\mathcal{F}}$, we see from the formula just given for the isotherms, that radiation flows toward the vortex core and continues on into the surface depression made by the vortex, as illustrated in Figure 3. As described in (Dowling and S.), a strong vortex will produce a deep thermal hole, which will serve as a sort of light well. Photons gushing from it will sweep material from the vortex core along with it. This motion will seed a stellar wind without any need for resonance lines. This may related to the issues of the evolution of metal-poor stars mentioned by Maeder in the Bonazzola-Zahn-fete.

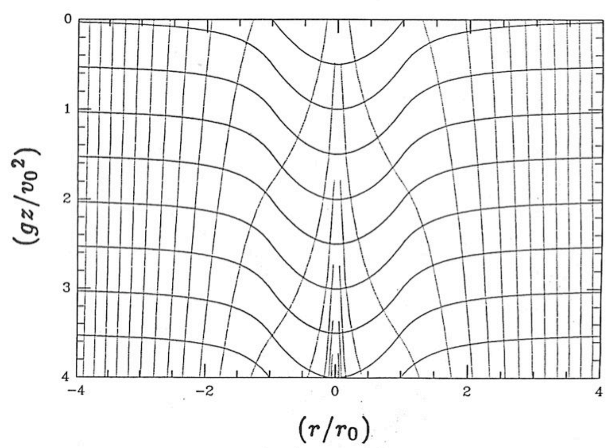

Figure 6: Beaming Through a Photovortex [59]

We have not resolved the quantitative problems involved in this process. The local emerging radiative flux is enhanced over its undisturbed value by a factor of about $(z_0/r_0)^2$, that is about $[v_0^2/(gr_0)]^8$. This is the quantity that the theory has to produce, but it is not a quantity that can be reliably estimated until we know whether we are dealing with a single giant vortex or many small ones. We have illustrated the focussing effect on outgoing radiation due to the low-temperature cores associated with a small, intense vortex. This produces a spot on the stellar surface.

4 Some Expectations

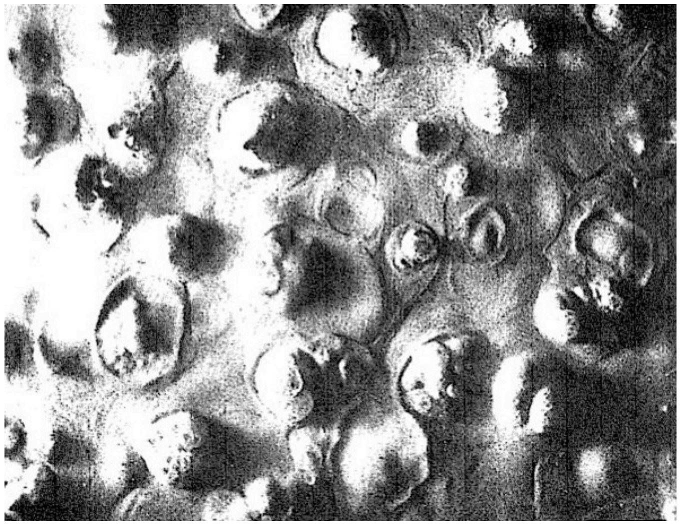

Figure 7: A bubbling bed seen from above.

velocities, genertion of coronas and the startup of winds. It is therefore worthwhile to begin thinking seriously about rotatino radiative dynamics. This problem, central to astrophysics, has been remarkably neglected so far. And so has its analogue in the collapse of the surpernova core. As the core spins up in the collapse, it too should be prone to vortex formation and the neutrinos will beam out. In all these questions, both the jovian and the solar observations will guide our thinking. The effects of rapid rotation are evident in the former and the role of magnetic fields are crucial in the latter.

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