Astronomy 3602: Homework #5

Due Wednesday, December 1

Problem 1: Hydrogen and Helium Recombination (40 points).

As discussed in the book, hydrogen recombined at a redshift of approximately $z = 1200$, with recombination starting already at $z \sim 1600$ (see Figure 8.5 in Ryden).

(a) What was the ratio of the number density of hydrogen–ionizing photons (i.e. photons with energies $E \geq$13.6eV) to the number density of hydrogen nuclei at $z = 1600$? Explain why this ratio does not have to equal unity at recombination.

(b) What was the ratio of the number density of helium–ionizing photons (i.e. photons with energies $E \geq$24.6eV) to the number density of helium nuclei at $z = 1600$? Does your answer imply that helium recombines before or after hydrogen?

For this problem, assume the following: at the present day, baryons contribute a fraction $\Omega_b = 0.04$ of the critical density, of which $Y_{\rm H} = 76\%$ by mass is hydrogen, and $Y_{\rm He} = 24\%$ is helium. Assume a Hubble constant of H_0 =70 km/s/Mpc. The temperature of the cosmic microwave background (CMB) today is $T_0 = 2.725$K. (Also a hint: the result of Exercise 2.5 in Ryden is useful for this problem.)

Problem 2: Reionization (30 points).

Recent observations by the *Planck* satellite have shown that $\approx 5\%$ of the CMB photons suffered a scattering with an electron on their way from redshift $z = 1100$ to Earth. The simplest explanation of this result is that the universe is kept fully ionized by starlight at all redshifts below $z_{\rm r}$. What is the value of $z_{\rm r}$ required to explain the electron scattering probability measured by *Planck*? This redshift corresponds to the epoch when the first stars formed. For simplicity, assume a flat, $\Omega_m = 1$ universe with $\Omega_b = 0.04$ and H_0 =70 km/s/Mpc. Compute $z_{\rm r}$ by assuming that all of hydrogen is ionized, but all helium is neutral, throughout the entire interval $0 < z < z_{\rm r}$. How does your answer change if you assume instead that all helium is in doubly ionized (He^{++}) form? (Hint: you will need Eq. 8.45 from Ryden; but please explain why and how this equation is relevant to this problem.)

Problem 3: Modified Nucleosynthesis (30 points).

Ryden estimates the maximum possible mass fraction of helium, $Y_{\rm He,max}$ in the universe, by assuming that at the time of nucleosynthesis, all available neutrons were converted into ^{4}He nuclei. Suppose that the neutron decay time, $\tau_n = 890$s, was ten times shorter, $\tau_n = 89$s. What would be the value of $Y_{\rm He,max}$? (Hint: we showed that nucleosynthesis occurred approximately 200 seconds after neutron–proton freeze–out).