Astronomy 3602: Homework #1

Due in class on Wednesday, September 29

Problem 1: Star Counts, Dust, and the Size of the Universe (30 points).

Suppose the Universe consisted of a sphere of radius $R = 100$ kpc, filled uniformly with stars identical to the Sun, and a space density of $n = 1\ \mathrm{pc}^{-3}$. The Sun's luminosity is $L_\odot = 3.8 \times 10^{26}$ W, and 1 pc $= 3.1 \times 10^{16}$ m.

(a) What is the total number of stars? Derive the distribution $N(> F)$ of the number of stars whose flux as seen on Earth is brighter than F. Use W m^{-2} as the unit for F, and make a sketch of $\log N$ vs $\log F$. [**10 points**]

(b) Suppose that the universe is filled with dust that absorbs the light of stars. The effect of the dust is to dim the flux of stars. More distant stars are more strongly dimmed, since they are seen through a thicker slab of intervening dust. We will mimic this dimming as follows. The flux $F_0 = L/4\pi r^2$ that a star at a distance r would have in the absence of dust is (i) unaltered for stars closer than $r = r_0$, but (ii) reduced by a factor of $(r_0/r)^2$ to the observed value of $F = (r_0/r)^2 \times F_0$ for stars that are farther than $r = r_0$. Here $r_0 = 1$ kpc is a constant. What is the distribution $N(> F)$ in the presence of dust? Make a sketch of $\log N$ vs $\log F$ as above. [**10 points**]

(c) The measured star counts $N(> F)$ can be used for a rough estimate of the size of the universe. One can associate the flux F_e at which $N(> F)$ deviates from a pure power–law with the distance $r_e = (L_\odot/4\pi F_e)^{1/2}$ beyond which the space density of stars "thins out". Suppose someone who does not know dust exists (such as William Herschel in 1785) tries to use this method. Would (s)he obtain an overestimate or an underestimate for the size? By what factor, in the example (b) vs. (a) above? [**10 points**]

Problem 2: Olbers' Paradox (20 points).

Suppose the Universe consisted of a random, statistically uniform distribution of stars in space identical to the Sun, with a radius of $R = R_\odot = 7 \times 10^{10}$ cm, and a space density of $n = 1\ \mathrm{pc}^{-3}$. As shown in lecture, if such a Universe were infinitely old, the surface brightness of the sky as seen by an observer on Earth would be infinitely large.

(a) How far, on average, would you have to look in such a Universe until your line of sight struck the surface of a star? Express your answer in parsecs. [**10 points**]

(b) Show that if the Universe has a finite age, the surface brightness on Earth would be finite. What is this surface brightness, if the Universe is $t_0 = 15$ Gyr old? Express your answer in the units $\mathrm{Wm}^{-2}\ \mathrm{sr}^{-1}$. The Sun's luminosity is $L_\odot = 3.8 \times 10^{26}$ W. [**10 points**]

Problem 3: Hubble Expansion (30 points).

Hubble found that the universe is "expanding linearly", i.e. the recession velocity v of a galaxy is related to its distance from us as $v = H_0 r$. Here H_0 is called the Hubble constant. Convince yourself that such a linear expansion has the following properties:

(a) When the positions and velocities are considered relative to any other galaxy, the expansion still has the same linear form. [**7 points**]

(b) If the galaxies do not change their velocities, then the linear expansion law is preserved. [**7 points**]

(c) The linear velocity pattern could also be explained by a "explosion" that imparts different velocities to individual galaxies at some definite moment in the past (while such a "velocity sorting" could explain the present-day linear expansion, it would violate the cosmological principle). [**6 points**]

(d) Show that the cosmological principle (i.e. homogeneity and isotropy) *require* that local motions obey Hubble's linear law. Hint: consider observers in two arbitrarily chosen galaxies, and then require that they infer the same expansion law for any arbitrary third galaxy. [**10 points**]

Problem 4: Tired Light Hypothesis (15 points).

[This is Problem 2.4 in Ryden.] A hypothesis once used to explain the Hubble relation is the "tired light hypothesis". This hypothesis postulates that the universe is not expanding, but that photons simply loose energy as they travel through space (by some unexplaid means), with the energy loss per unit distance being given by the law

$$\frac{dE}{dr} = -KE, \tag{1}$$

where K is a constant. Show that this hypothesis gives a distance-redshift relation that is linear in the limit of $z \ll 1$. What must the value of K be in order to yield a Hubble constant of $H_0 = 70$ km s Mpc^{-1} ? [Use the Hubble law $v = H_0 r$ with the non-relativistic relation for the Doppler redshift, $z \equiv (\lambda_{\rm obs} - \lambda_{\rm em})/\lambda_{\rm em} = v/c$. Here $\lambda_{\rm em}$ and $\lambda_{\rm obs}$ are the emitted and observed wavelengths, and c is the speed of light. You will also need the relation between a photon's energy and wavelength.]

Problem 5: Feedback (5 points).

Please rank the previous three problems overall on a scale of 1 (lowest) to 5 (highest) for (a) difficulty, (b) length, and (c) level of math involved. You will receive five points just for answering!