

Astronomy GR6001: Problem Set #6

Due ONLINE on Monday, December 13, 2021

Problem 1 (20 points):

The broad emission lines of quasars are thought to arise from photo-ionized clouds near the quasar black hole, with very high density ($n_H \approx 10^{10} \text{ cm}^{-3}$) and column density ($N_H \approx 10^{22} \text{ cm}^{-2}$) and temperature $T \approx 1.5 \times 10^4 \text{ K}$. What is the frequency at which the cloud is optically thick to free-free absorption?

Problem 2 (40 points):

The Orion Nebula can be roughly described as a photoionized (by massive young stars) cloud of hydrogen, with a density of $n_e = 2 \times 10^3 \text{ cm}^{-3}$ and a temperature of $T = 8000 \text{ K}$. The approximate size of the nebula is $R = 1 \text{ pc}$, and its distance from us is $d = 0.5 \text{ kpc}$.

Compute, approximately, the flux density F_ν of the Nebula due to its free-free emission. Make a log-log plot of F_ν , in units of Jansky, against wavelength, from $100 \mu\text{m}$ to 100 cm . Hint: you have to include free-free self-absorption, and you have to use the radiative transfer equation.

Problem 3 (40 points):

The observed spectrum of the Crab Nebula between 10^8 and 10^{22} Hz may be approximated as two power-laws with spectral indices of 0.26 and 1.2, respectively, intersecting at 10^{15} Hz . At this intersect frequency, the flux density is $F_\nu = 4 \times 10^{-22} \text{ erg s}^{-1} \text{ cm}^{-2} \text{ Hz}^{-1}$. Assume that the Crab Nebula consists of uniform gas at a distance $d = 2 \text{ kpc}$ from us, and has a total volume of $V = 3 \times 10^{56} \text{ cm}^3$.

(a) Estimate the magnetic field strength and the total energy in relativistic particles in the Nebula, using an equipartition argument (i.e. assume that the total energy in the radiating particles and in the magnetic field are roughly equal).

(b) Estimate the synchrotron lifetime of electrons radiating at 10^8 , 10^{15} , and 10^{22} Hz , and compare this lifetime to the age of the Crab Nebula, 948 years. What can you conclude from this comparison?