Astronomy GR6001: Problem Set #1

Due in class on Monday, September 27, 2021

Problem 1 (15 points):

Astronomers often use a logarithmic frequency scale in their plots, since they are often interested in a range of frequencies extending over many orders of magnitude. In this case, for monochromatic quantities, such as F_ν, it is usually most convenient to plot the quantity νF_ν, rather than F_ν.

(a) Show that the units of νF_ν are the same as that of the total flux F.

(b) Show that if νF_ν is plotted against $\log \nu$, then equal areas under the plotted curve contribute equally to the total flux F.

(c) The quantity F_λ, denoting the flux per unit wavelength range, is often used as an alternative to F_ν. Show that $\nu F_\nu = \lambda F_\lambda$.

Problem 2 (25 points):

Show that the mean intensity, $J(r)$ at an arbitrary distance r away from a sphere of uniform surface brightness $I_\nu = B = \textit{constant}$ is given by

$$J(r) = \frac{B}{2}\left[1 - \sqrt{1 - \left(\frac{R}{r}\right)^2}\right] \tag{1}$$

Problem 3 (25 points):

Photons are produced in a uniform cloud of radius R at the rate Γ (photons per unit volume per unit time). Assume that the cloud is optically thin, i.e., neglect any absorption within the cloud (justified for hard enough X-ray photons in most real clouds).

(a) Find the specific number intensity I (photons cm^{-2} s^{-1} sr^{-1}) at a distance r from the cloud, as a function of the impact parameter b (measured from the center of the cloud). Note that I is defined based on the number of photons (rather than energy, as in the case of the specific intensity).

(b) Find the number flux F (photons cm^{-2} s^{-1}) at a distance r from the cloud in two different ways. First, use a simple conservation law. Second, explicitly integrate the specific number intensity. Verify that you get the same answer either way.

Problem 4 (30 points):

A simple model for the late stages of a supernova shock is a thin spherical shell, centered on the site of the explosion. Assume that the shell has expanded to a radius R from the center, and has a width $\Delta R \ll R$. Assume further that the material filling this shell has a constant emission coefficent j_ν and is optically thin (no absorption). Show that the observed surface brightness of the shell along a ray passing a distance p from the center is approximately

$$I_\nu = \frac{2 j_\nu R \Delta R}{\sqrt{R^2 - p^2}} \tag{2}$$

for $p < R$ (and $I_\nu = 0$ otherwise). Make a plot of I_ν vs p to demonstrate that the shell will appear brightest near $p \sim R$. Thus argue that the supernova shell will look like a "ring" on the sky. (Note: the above expression is inaccurate near $p \to R$ where it becomes singular. You need not compute corrections to the expression at $p \approx R$.)

Problem 5 (5 points):

Please rank the previous four problems overall on a scale of 1-5 for (a) difficulty, (b) length, and (c) level of math involved (or else feel free to provide feedback in some other format). You will receive five points just for answering!